SOME OBSERVATIONS OF WIND VELOCITY AUTOCORRELATIONS IN THE LOWEST LAYERS OF THE ATMOSPHERE

By R. J. Taylor*

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Summary

From wind velocity observations in the lowest layers of the atmosphere, mean square velocity differences over a time interval at a point fixed in space are derived and their variation with the time interval is considered. The magnitude of these mean square differences is related to the rate of viscous dissipation of energy and to the shearing stress. On the average, fair agreement with predictions from the dimensional arguments of the theory of local isotropy is shown even though the results pertain to eddy sizes outside the inertial subrange as usually defined.

I. Introduction

The theory of local isotropy attributed to Kolmogoroff and first expounded in English by Batchelor (1947) is generally taken as applying only to the " viscous " and " inertial " subranges of eddy sizes and special interest attaches to the second of these, that is, to those eddies which are small enough to be statistically decoupled from the larger, directionally biassed eddies and yet not so small that their motion is appreciably affected by viscous forces. The theory is not intended to apply to eddies larger than those in this subrange. Nevertheless autocorrelations with regularities of the type predicted by this theory have been observed in the atmosphere (e.g. Taylor 1952*a* ; MacCready 1953) up to eddy sizes several times the height of observation where one would expect the influence of the underlying surface in imposing a directional bias to outweigh any inherent tendency of the motion towards isotropy.

It must be realized that the theory of local isotropy in its usual form is divisible into two parts : first, the two separate hypotheses of statistical decoupling and local isotropy proper and, secondly, the dimensional arguments based on them. It would appear that, while these hypotheses are a logical basis for the dimensional arguments, they may not be the only possible ones and that these arguments may be valid for other reasons even outside the domain where a Kolmogoroff " inertial subrange " can be expected to exist. The purpose of this paper is to present a series of results which, though showing wide scatter, in the mean exhibit this type of unexpected agreement with the dimensional arguments of the theory of local isotropy.

II. Results

As part of the programme of field work of the C.S.I.R.O. Division of Meteorological Physics, a large number of photographically recorded galvanometer traces of the speed, azimuth, and inclination to the horizon of the wind has

* Division of Meteorological Physics, C.S.I.R.O., Melbourne.

been made. The techniques involved and other analyses of these records are described elsewhere. By means of a process of mechanical computation, it is possible to rewrite this information in the form of continuous traces of u, v, and w; respectively the down-wind, cross-wind, and vertical components of velocity. Each of the records used here represents 5 min of recording time.

Autocorrelation coefficients for a number of different time lags have been calculated from the traces of u, v, and w for a total of nine 5-min runs and are exhibited in Table 1 together with relevant meteorological data. The time obtained by dividing the height of observation by the mean wind speed has also been tabulated as an indication of the time of passage of an eddy of down-wind dimension equal to the height of observation.

Since the original records were made with critically damped galvanometers having natural periods of approximately 2 sec, it might be suspected that false autocorrelations are thereby introduced into the record. This was investigated by considering the equation describing the response, $\theta(t)$, of a critically damped galvanometer of natural period $2\pi/\omega$ to a fluctuating input $u'(t)$, where θ and u' are to be considered as excursions from mean values. This equation is

$$\ddot{\theta}+2\omega\dot{\theta}+\omega^2\theta=\omega^2 u'(t).$$

We define functions

$$R_u(\sigma)=\overline{u'(t)u'(t+\sigma)},$$

similar functions involving v and w,
and

$$\rho(\sigma)=\overline{\theta(t)\theta(t+\sigma)},$$

where a bar denotes a mean taken over the whole record. These represent the time lag covariances existing in the original quantity and in the galvanometer trace respectively. It follows then that

$$\begin{aligned}R=\omega^{-4}[\overline{\ddot{\theta}(t)\ddot{\theta}(t+\sigma)}&+2\omega\{\overline{\ddot{\theta}(t)\dot{\theta}(t+\sigma)}+\overline{\dot{\theta}(t)\ddot{\theta}(t+\sigma)}\}\\&+\omega^2\{\overline{\ddot{\theta}(t)\theta(t+\sigma)}+4\overline{\dot{\theta}(t)\dot{\theta}(t+\sigma)}+\overline{\theta(t)\ddot{\theta}(t+\sigma)}\}\\&+2\omega^3\{\overline{\dot{\theta}(t)\theta(t+\sigma)}+\overline{\theta(t)\dot{\theta}(t+\sigma)}\}+\omega^4\overline{\theta(t)\theta(t+\sigma)}].\end{aligned}$$

We can now make use of a type of argument introduced by G. I. Taylor (1922) to investigate certain properties of a fluctuating quantity. Following him, we suppose that a value of ρ is calculated by reading off values of θ from the record at a large but finite number of points and evaluating the form for ρ given above. Then we can expect the value of ρ to remain unaltered if a different set of points, displaced in time by a constant small amount δt from the first set, is used instead, that is,

$$\begin{aligned}\overline{\theta(t)\theta(t+\sigma)}&=\overline{\theta(t+\delta t)\theta(t+\delta t+\sigma)}\\&=\overline{\{\theta(t)+\delta t\dot{\theta}(t)\}\{\theta(t+\sigma)+\delta t\dot{\theta}(t+\sigma)\}}\\&=\overline{\theta(t)\theta(t+\sigma)}+\delta t\{\overline{\dot{\theta}(t)\theta(t+\sigma)}+\overline{\theta(t)\dot{\theta}(t+\sigma)}\}.\end{aligned}$$

TABLE 1
AUTOCORRELATION COEFFICIENTS

Date	Run No.	Time (E.S.T.)	Height z (m)	$\bar{u}$* (m sec^{-1})	$z/\bar{u}$ (sec)	Temp. Diff.† 2–32 m (°F)		Time Lag σ (sec) 1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3	4	5	6	8	10	15	20	30	40
8.xi.51	1	1241	1·5	2·18	0·69	−0·4	u	0·92	0·81	0·74		0·62	0·52		0·34	0·24	0·18	0·05	−0·10	−0·17	
							v	0·83	0·75	0·67		0·56	0·46		0·38	0·39	0·35				
							w	0·40	0·27	0·17		0·06	0·02		−0·01	0·00					
8.xi.51	2	1251	29	7·85	3·7	−0·9	u	0·94	0·93	0·88		0·87	0·84		0·76	0·68	0·64	0·58	0·57	0·57	
							v	0·93	0·89	0·84		0·77	0·71		0·63	0·54	0·53	0·46			
							w	0·74	0·62	0·52		0·35	0·27		0·09	0·02	0·01				
8.xi.51	3	1300	1·5	4·47	0·34	−0·5	u	0·73	0·63	0·59		0·50	0·41		0·28	0·29	0·20	0·01	0·03	0·06	
							v	0·67	0·34	0·42		0·31	0·26		0·19	0·16	0·17				
							w	0·30	0·17	0·15		0·04	0·01		−0·01	−0·07	0·03				
8.xi.51	4	1308	29	7·70	3·8	−1·0	u	0·92	0·89	0·87		0·85	0·82		0·79	0·75	0·72	0·66	0·60	0·55	0·45
							v	0·87	0·81	0·76		0·73	0·68		0·60	0·57	0·54	0·47	0·39	0·31	
							w	0·82	0·61	0·49		0·25	0·07		0·01	0·07					
9.i.52	1	1422	29	4·25	6·8	−1·3	u	0·93	0·86	0·80		0·73	0·62		0·49	0·41	0·32	0·26	0·05	0·05	
							v	0·83	0·72	0·64		0·48	0·44		0·35	0·17	0·02	−0·11	−0·08	0·14	
							w	0·95	0·87	0·79		0·73	0·68		0·55	0·37	0·29	0·21	0·16	−0·02	
9.i.52	2	1438	1·5	3·91	0·38	−0·5	u	0·75	0·61	0·51		0·40	0·28		0·11	0·07	0·06	−0·04	−0·03		
							v	0·83	0·75	0·63		0·55	0·46		0·41	0·33	0·27	0·11	0·09	0·03	
							w	0·29	0·15	0·13		0·13	0·14		0·01	0·15	0·14				
9.i.52	3	1549	29	4·60	6·3	−1·0	u	0·81	0·71	0·60		0·47	0·38		0·33	0·28	0·17	0·13	0·23		
							v	0·76	0·70	0·63		0·51	0·42		0·31	0·31	0·39				
							w	0·96	0·93	0·82		0·82	0·80		0·64	0·62	0·48	0·29			
9.i.52	4	1557	1·5	3·82	0·39	−1·6	u	0·81	0·72	0·62		0·51	0·45		0·28	0·12	0·02	−0·04	0·04		
							v	0·81	0·72	0·60		0·46	0·28		0·14	0·09	−0·02	−0·08	0·03		
							w	0·53	0·21	0·18		0·09	0·08		−0·11	0·01					
9.x.51	2	1624	7	4·18	1·7	Not re-corded	u	0·81	0·65	0·60	0·57	0·49	0·41	0·37							
							v	0·88	0·83	0·80	0·77	0·77	0·71	0·70							
							w	0·51	0·38	0·19	0·12	0·06	0·07	0·08							

* At height z.
† Negative sign indicates lapse.

Therefore

$$\overline{\dot{\theta}(t)\theta(t+\sigma)}+\overline{\theta(t)\dot{\theta}(t+\sigma)}=0.$$

By further applications of this principle and also by relating the superior dot (which, of course, denotes $\mathrm{d}/\mathrm{d}t$) to the operator $\mathrm{d}/\mathrm{d}\sigma$ we simplify the expression for $R(\sigma)$ and obtain

$$R=\rho-\frac{2}{\omega^2}\frac{\mathrm{d}^2\rho}{\mathrm{d}\sigma^2}+\frac{1}{\omega^4}\frac{\mathrm{d}^4\rho}{\mathrm{d}\sigma^4},$$

so that R can be calculated numerically from a graph showing ρ as a function of σ.

In practice, however, it was found that the terms in $\mathrm{d}^2\rho/\mathrm{d}\sigma^2$ and $\mathrm{d}^4\rho/\mathrm{d}\sigma^4$ were small except near $\sigma=0$ where the derivatives of the ρ curve are hard to assess. For $\sigma\geqslant1$ sec, it has therefore been taken that $R(\sigma)=\rho(\sigma)$, but there remains the possibility that a significant error may exist in the mean square of u', that is, in $R(0)$.

From the autocorrelation coefficients and mean square velocity fluctuations were calculated mean square velocity differences

$$D_u(\sigma)=\overline{\{u(t)-u(t+\sigma)\}^2}=2\overline{u'^2}\left\{1-\frac{R}{\overline{u'^2}}\right\},$$

and similar functions $D_v(\sigma)$ and $D_w(\sigma)$. The logarithms of D were plotted against those of σ and a typical example of such a graph is shown in Figure 1.

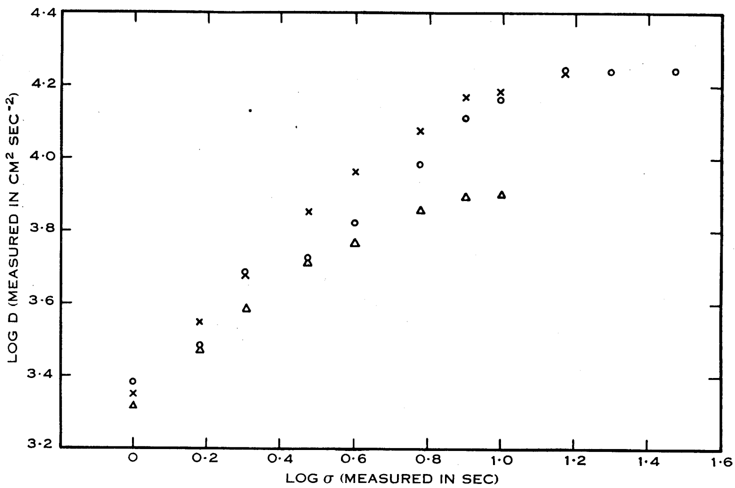

Fig. 1.—Variation of mean square velocity difference with time lag. ○, $\overline{(u-u_\sigma)^2}$; ×, $\overline{(v-v_\sigma)^2}$; △, $\overline{(w-w_\sigma)^2}$.

When $R/\overline{u'^2}$, the correlation coefficient between $u(t)$ and $u(t+\sigma)$, is large, a comparatively small error in $\overline{u'^2}$ will have a disproportionately large effect on the value of D. In some of the graphs of log D against log σ, the values of D for $\sigma=1$ sec and $\sigma=1\cdot5$ sec appear to be rather too small to be consistent with the trend of D for somewhat larger σ. This effect has been attributed to underestimation of $\overline{u'^2}$ and the points concerned have been ignored in considering the variation of D with σ.

With the reservation expressed in the previous paragraph, every graph showed a fairly straight section rising at slope p, representing the index in the proportionality

$$D \propto \sigma^p,$$

followed by a section of continually decreasing slope and occasionally one of negative slope. In Table 2 are presented the values of p for the straight sections of the curves and the approximate limiting values of σ beyond which the slopes begin to decrease. Values of $z/\bar{u}$ from Table 1 are repeated here so that they may be compared with these limiting σ.

TABLE 2

VARIATION OF MEAN SQUARE VELOCITY DIFFERENCE WITH TIME LAG

Date		8.xi.51			8.xi.51			8.xi.51	
Run No.		1			2			3	
Mean sq. differences in:	u	v	w	u	v	w	u	v	w
p	1·05	0·87	0·51	0·78	1·03	0·83	0·54	0·72	0·19
Limiting σ (sec)	4	4	2	10	4	3	6	3	4
$z/\bar{u}$ (sec)		0·69			3·7			0·34	
Date		8.xi.51			9.i.52			9.i.52	
Run No.		4			1			2	
Mean sq. differences in:	u	v	w	u	v	w	u	v	w
p	0·51	0·52	0·90	0·77	0·67	0·72	0·58	0·83	*
Limiting σ (sec)	>40	6	4	10	10	10	6	4	<1½
$z/\bar{u}$ (sec)		3·8			6·8			0·38	
Date		9.i.52			9.i.52			9.x.51	
Run No.		3			4			2	
Mean sq. differences in:	u	v	w	u	v	w	u	v	w
p	0·84	0·64	†	0·51	0·97	0·25	0·61	0·77	0·68
Limiting σ (sec)	3	4	†	10	4	6	4	3	2½
$z/\bar{u}$ (sec)		6·3			0·39			1·7	

* Indeterminate: not enough points on straight portion of graph.
† Indeterminate: points too badly scattered for reasonable estimate.

It is apparent from Table 2 that the quoted values of p apply up to limiting values of σ which are, in almost every case, several times as large as $z/\bar{u}$ and which occasionally rise to 10 or 20 times as large. This is especially so in the case of u, though unexpectedly large limiting σ for v and w are by no means absent.

In Table 3, means and standard errors of the means of p are given for u, v, and w separately and for all of them taken together. There is some basis for suggesting that the observed difference between the mean p for v and that for w may be significant. If this is so, it is probably to be attributed to the restricting action of the underlying surface on the vertical component and to the relative increase in constraint with increasing scale of motion.

III. Comparison with Theory

In considering the dependence of D on parameters of the flow, the theory applicable to the inertial subrange immediately suggests either ε and σ or ε, $\bar{u}$, and σ (where ε is the rate of dissipation of kinetic energy per unit mass) as possible groups of defining parameters. In laboratory studies of turbulent

Table 3

Values of p in $D \propto \sigma^p$

Component	u	v	w	All
Mean p	0·69	0·78	0·58	0·69
Standard error of mean p	0·06	0·05	0·10	0·04

structure, correlations of this type are invariably taken as equivalent to spatial correlations over a distance $\bar{u}\sigma$ and MacCready (1953) and other authors have extended this assumption of equivalence to the atmosphere. If this is assumed, dimensional analysis indicates that

$$D \propto \varepsilon^{2/3} \bar{u}^{2/3} \sigma^{2/3}.$$

The present writer (1952*a*) previously took the view that D should be considered as a function of ε and σ alone. Robinson (1953) took up an analogous position in considering energy spectra as a function of a time variable (frequency) rather than a space variable (wave number) and he adduced evidence in support of this attitude. If D depends only on ε and σ, then dimensional analysis leads us to

$$D \propto \varepsilon\sigma.$$

However, the observed mean p for all components offers strong support for believing that D is proportional to $\sigma^{2/3}$. The variation of D with ε, $\bar{u}$, and σ (if only these three are admitted as significant variables) now becomes determinate on dimensional grounds, D being proportional to the two-thirds powers of ε and $\bar{u}$, as well as of σ.

A further comparison of these observations with the theory can be made by taking advantage of an expression given by Obukhov and Yaglom (1951)

$$6\nu \frac{\partial}{\partial r}\{\overline{(u-u_r)^2}\} + |\, S\,| \{\overline{(u-u_r)^2}\}^{3/2} = 4\varepsilon r/5,$$

where ν is the kinematic viscosity, u and u_r are the velocity components, directed along r, at two points distant r from one another, and S is the skewness in the distribution of $(u-u_r)$. This equation was given in another form by Batchelor (1947) but Obukhov and Yaglom point out that, in the terms of the theory, S must be an absolute constant within the limits of eddy size to which the theory applies. To make use of this expression we must assume that the mean square velocity differences here considered are equivalent to $\overline{(u-u_r)^2}$ with r given by $\bar{u}\sigma$. Only $D_u(\sigma)$ is of the necessary "longitudinal" form; D_v and D_w are concerned with velocity components at right angles to the distance $\bar{u}\sigma$.

Following Obukhov and Yaglom, we accept a value of $-0 \cdot 4$ for S and neglect the term containing ν in the inertial subrange. We have then

$$0 \cdot 4 D^{3/2} = 4 \varepsilon \bar{u} \sigma / 5,$$

and from the constant-p portion of the graphs referred to above it is possible to make an estimate of ε for each occasion.

The present writer (Taylor 1952*b*) has brought forward reasons why ε can be taken as equal to $(\tau/\rho)(\partial \bar{u}/\partial z)$ (where τ is the shearing stress and ρ the density) in steady conditions and on occasions when the heat flux is not too large. From the values of ε obtained as described above, values of τ were thus computed, making use of a wind gradient calculated by logarithmic interpolation between mean wind speeds measured simultaneously at two heights. These were then compared with values of τ determined at the same time from the covariance between vertical and horizontal components. The results of this comparison are displayed in Table 4.

TABLE 4

COMPARISON OF SHEARING STRESSES

Date	Run No.	Shearing Stress (dyn cm^{-2}) From Autocorrelations	Shearing Stress (dyn cm^{-2}) From cov(u,w)
8.xi.51	1	0·19	0·92
	2	3·50	1·41
	3	0·49	2·26
	4	*	(−0·86)
9.i.52	1	*	(−0·57)
	2	1·36	1·06
	3	4·31	2·05
	4	0·99	1·81
9.x.51	2	0·24	0·15
Mean..	.. ..	1·58	1·38

* The existence, on these occasions, of an upward flux of momentum indicates conditions under which the approximation $\varepsilon = (\tau/\rho)(\partial \bar{u}/\partial z)$ cannot be expected to apply. These occasions are therefore omitted from the comparison.

It is worthy of note that there is relatively good agreement, on the average, between the shearing stresses calculated by the two different methods and this agreement offers support for the suggestion that the observed proportionality between D and $(\bar{u}\sigma)^{2/3}$ is, indeed, a continuation of a similar proportionality at smaller eddy sizes where Obukhov and Yaglom's expression can be expected, on solid theoretical grounds, to apply. For the individual runs, it will be noticed that the two shearing stresses often differ quite widely, although the difference is never as great as an order of magnitude.

From the two types of comparison made above, it would appear that properties which might be predicted for the inertial subrange can also apply, on the average, to eddies of down-wind dimension many times the height of observation.

It is, perhaps, not obvious that these eddies are in fact outside the range of local isotropy. Robinson (loc. cit.) claims to have shown equipartition of kinetic energy between vertical and horizontal components in the range 3–8 c/s at $1 \cdot 5$ m above the ground but no evidence of equipartition at lower frequencies appears to emerge from his observations. Webb (1955) has carried out some spectrum analyses, and from his results it is possible, on two occasions, to demonstrate the existence of a band of relatively low frequency having approximate equipartition between two components. In neither case, however, does this band appear to be an extension of a region of more exact equipartition at higher frequencies.

Eddies within the range of local isotropy must, moreover, show not only equipartition of kinetic energy but also zero correlation between two velocity components, so that they cannot contribute to the turbulent flux of momentum. The contribution of various eddy sizes to this flux has been studied by Deacon (1955) who concludes, from an analysis of six measurements of momentum flux over 5-min periods, that the greatest contribution to the flux at a height of 28 m is from eddies ranging in period from 5 sec to 3 min. At the average wind speed existing for these measurements, this indicates that the smallest eddy contributing to the momentum flux at this height has a down-wind dimension of at most 40 m. This figure is supported by a result of Panofsky (1953) who gives a stress spectrum analysis for a height of 23 m. At the existing wind speed, the highest frequencies contributing to the shearing stress represent eddies of dimension about 40 m.

Again, a flux measurement at 7 m which Deacon has analysed shows maximum contribution to the flux by eddies of period 16 sec and a small contribution from those of period as short as 1 sec, with a mean wind speed of $3 \cdot 27$ m sec^{-1}. This indicates that eddies of down-wind dimension about 3 m make some contribution to the momentum flux at this height.

In order to obtain lengths from the present results to compare with those quoted, the limiting σ (for autocorrelations in u) as shown in Table 2 were multiplied by the appropriate mean wind speeds. At heights of $1 \cdot 5$ and 29 m, where there are several observations, there is considerable variability in $\bar{u}\sigma_{\text{lim}}$ but the average values are 24 and 110 m respectively. For the one result at 7 m, $\bar{u}\sigma_{\text{lim}}$ is 17 m. Further work on the spectrum of the momentum flux is in progress but it already seems clear that considerations both of this spectrum and of the partition of kinetic energy among components indicate that the observations here dealt with pertain to eddies outside the locally isotropic range.

The continuity relationship between autocorrelation coefficients derived by de Karman and Howarth (1938) can be used as a test of isotropy. The analogue of this relationship for local isotropy has been stated by Batchelor (1947) and is

$$D_n = D_l + \frac{r}{2}\frac{\partial}{\partial r} D_l,$$

where D_l is a "longitudinal" mean square velocity difference and D_n a "transverse" one between two points separated by r. Here, D_l corresponds to D_u, D_n to D_v or D_w, and $r=\bar{u}\sigma$. By putting $D_l=D_u$, values of D_n were calculated in the form given above and from them were derived values of D_v/D_n and D_w/D_n (which will be equal to unity in the locally isotropic range). There was a considerable amount of scatter but mean values of these ratios at 1·5 m and 29 m are given in Table 5. A mean was not calculated unless there were at least three results to contribute to it.

TABLE 5

TEST OF LOCAL ISOTROPY

σ (sec)	1	$1\frac{1}{2}$	2	3	4	6	8	10	15
D_v/D_n at 29 m	0·88	0·90	0·89	1·02	0·98	0·92	0·93	0·87	0·98
D_w/D_n at 29 m	0·47	0·67	0·85	0·88	0·84	0·94	0·98	1·28	
D_v/D_n at 1·5 m	0·41	0·42	0·43	0·43	0·47	0·46	0·45	0·42	
D_w/D_n at 1·5 m	0·42	0·35	0·30	0·26	0·24	0·22	0·19		

Only in the case of D_v/D_n at 29 m is the theoretical value at all nearly approached with any consistency though it might be considered (neglecting the somewhat suspect values of D for $\sigma=1$ and $1\frac{1}{2}$ sec) that D_w/D_n at this height is not significantly different from unity. However, it is more likely that the steady growth of D_w/D_n at 29 m from 0·47 to 1·28 is a reflection of the difference between the mean observed p for u and w at this height (0·72 and 0·82 respectively). Evidence in Table 5 for the existence of local isotropy is very doubtful at 29 m and quite absent at 1·5 m, and the previous conclusion, that the observations here dealt with represent eddies outside the locally isotropic range, is supported.

IV. Conclusions

It is generally accepted that, in the inertial subrange, the rate of viscous dissipation of energy is the only parameter influencing the structure of the flow, the statistical characteristics of which are uniquely determined by it. In the results here analysed, however, this dependence on ε alone does not appear to exist. On the average, predictions of the theory of local isotropy are fulfilled surprisingly well over a range of eddy sizes to which they were never intended to apply but there are considerable discrepancies in individual observations.

This suggests that ε, though it no longer possesses all the power it has in the inertial subrange, remains an important parameter in the range of eddy sizes considered, even though other variables (wind profiles, stability, and the like) are beginning to have an effect. The evidence shows that the rate of viscous dissipation of kinetic energy has a considerable amount of influence in defining the structure of eddies of a size important in micrometeorological investigations and deserves more attention in this type of work than it has hitherto received.

V. Acknowledgments

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