

FORMULAE FOR THE MEAN LOSSES OF ENERGY IN COLLISIONS OF SLOW ELECTRONS MOVING IN DIATOMIC GASES

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Summary

Formulae are derived for the mean losses of energy in collisions with molecules by electrons moving in a steady state of motion in a diatomic gas in an electric field. Although the theory is incomplete and crude, yet the formulae describe the experimental measurements closely in the smaller ranges of electronic energy.

Such formulae find application in the theory of radio wave interaction.

I. INTRODUCTION

Although the results of the measurements of the mean losses of energy experienced by slow electrons in a steady state of motion among the molecules of diatomic gases have been available for some time, there do not appear to exist any formulae, with a reasonable physical basis, that represent the experimental results over any significant range of the mean energies of the electrons. Recently, the problem has been treated by the methods of quantum mechanics by Gerjuoy and Stein (1954, 1955) and it has also been discussed by Massey and Burhop (1952).

Because such formulae would be of practical value, especially in situations inaccessible to direct laboratory measurement (as, for instance, when the mean energies of agitation of the electrons do not differ greatly from that of the gas molecules), the aim of the investigation that follows is to seek satisfactory formulae by means of crude theory, rather than to present a rigorous theory of the mechanism of the losses of energy.

Recent measurements of the diffusion of slow electrons in deuterium made in this department by Miss B. I. H. Hall (1955), are significant in this context. It was found that the collisional cross sections of the molecules in collisions with electrons moving with any given speed U were the same in deuterium and in hydrogen, as would be expected from the identity of the electronic shells of their molecules. However, the mean energy ΔQ_H lost by an electron with energy Q in collision with a molecule of hydrogen is twice the mean energy ΔQ_D lost by an electron with the same energy Q in collisions with molecules of deuterium, for all values of the energy Q . Since the ratio of the mean losses of energy $\Delta Q_H/\Delta Q_D$ is the same as the inverse ratio M_D/M_H of the masses of molecules, it is suggested that the only kinds of exchanges of energy in collisions concerned are those that are inversely proportional to the masses of the molecules. Two types of collision

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that exhibit this behaviour are perfectly elastic collisions between electrons and molecules and collisions that produce changes in the rotational states of the molecules.

II. PERFECTLY ELASTIC COLLISIONS

Were electrons with energy Q to collide with molecules with energy Q_0 as if each were a perfectly elastic sphere then the mean energy lost by an electron in a collision, when $Q > Q_0$ and ratio of their masses $m/M \ll 1$, is

$$\Delta Q_e = (2m/M)(Q - Q_0), \quad \dots\dots\dots (1)$$

and the proportion of its energy lost in a collision is, on the average,

$$\eta = \Delta Q_e / Q = 2m/M(1 - 1/k_T), \quad \dots\dots\dots (2)$$

where $k_T = Q/Q_0$ (Townsend's energy factor).

Also

$$\eta k_T = Q_e/Q_0 = 2m/M(k_T - 1). \quad \dots\dots\dots (3)$$

If M is the mass of a molecule of hydrogen then $2m/M = 5.44 \times 10^{-4}$.

III. ROTATIONAL STATES IN HYDROGEN

The energies ϵ_J associated with the rotational states of a diatomic molecule are

$$\epsilon_J = (h^2/8\pi^2 I)J(J+1), \quad \dots\dots\dots (4)$$

where h is Planck's constant, I the moment of inertia of the molecule, and J an integer.

The rotational angular momentum of the molecule in state J is

$$P_J = \{J(J+1)\}^{1/2} h/2\pi. \quad \dots\dots\dots (5)$$

The relative abundances p_J of molecules of hydrogen in the various states J are :

parahydrogen ($J=0, 2, 4, \dots$)

$$p_J = (2J+1) \exp(-\epsilon_J/kT),$$

orthohydrogen ($J=1, 3, 5, \dots$)

$$p_J = 3(2J+1) \exp(-\epsilon_J/kT),$$

where k is Boltzmann's constant.

TABLE 1
RELATIVE POPULATIONS OF STATES IN HYDROGEN

Molecule	Parahydrogen			Orthohydrogen		
	0	2	4	1	3	5
J	0	2	4	1	3	5
Relative population ..	1	0.825	2.4×10^{-2}	4.95	0.57	5.4×10^{-2}

When $T=288^\circ\text{K}$; $h^2/(8\pi^2 I) \simeq Q_0/5$; $Q_0=3/2$. $kT=5.96 \times 10^{-14}$ ergs.

The relative populations of the states with J small at $T=288^\circ\text{K}$ are therefore the values given in Table 1.

Energy steps

$$\Delta \varepsilon_{J'J} = \varepsilon_{J'} - \varepsilon_J = (J' - J)(J' + J + 1) \times 0.198 Q_0.$$

Whence

$$\varepsilon_{10} = 0.39 Q_0; \quad \varepsilon_{21} = 0.79 Q_0; \quad \varepsilon_{32} = 1.19 Q_0; \quad \varepsilon_{43} = 1.58 Q_0.$$

Steps of angular momentum

$$\Delta P_{J'J} = P_{J'} - P_J = [\{J'(J' + 1)\}^{\frac{1}{2}} - \{J(J + 1)\}^{\frac{1}{2}}] h / 2\pi$$

$$\begin{aligned} P_{10} &= 1.49 \times 10^{-27}; & P_{21} &= 1.08 \times 10^{-27}; & P_{32} &= 1.06 \times 10^{-27}; & P_{43} &= 1.06 \times 10^{-27} \\ P_{20} &= 2.56 \times 10^{-27}; & P_{31} &= 2.15 \times 10^{-27}; & P_{42} &= 2.12 \times 10^{-27}; & P_{53} &= 2.11 \times 10^{-27} \end{aligned}$$

erg sec.

IV. NECESSARY CONDITIONS FOR PRODUCTION OF A TRANSITION

$$\Delta P_{J'J} \quad (J' > J)$$

These are :

$$(a) \quad Q \geq \Delta \varepsilon_{J'J};$$

(b) the angular momentum of the colliding electron about the centre of the molecule when approaching it must equal or exceed $\Delta P_{J'J}$.

In practice, if condition (b) is fulfilled then so also is condition (a).

Suppose that when the impact parameter r_i of the electron's orbit about the molecular centre exceeds a critical distance r_c the mutual interaction of the molecule and the electron has vanished so that the change $\Delta P_{J'J}$ cannot take place. It is therefore necessary that the speed U of an electron should exceed U_c if the transference of momentum $\Delta P_{J'J}$ is to be possible, where $U_c = \Delta P_{J'J} / m r_c$.

If the speed U of the electron exceeds U_c then the increment $\Delta P_{J'J}$ is possible provided $r \leq r_i \leq r_c$ where $r = \Delta P_{J'J} / m U$. Let the probability that an electron of speed U should produce the increment $\Delta P_{J'J}$ in the angular momentum when these conditions are fulfilled be $p(U)$.

V. THE MEAN ENERGY LOST BY AN ELECTRON IN A COLLISION

Let the speeds U of the electrons in collisions with molecules be distributed according to some law that the proportion of speeds that exceed U but do not exceed $(U + dU)$ is $f(U) dU$. The total number of collisions per second per electron in which the conditions for the possibility of a transference of angular momentum $\Delta P_{J'J}$ are satisfied is

$$\begin{aligned} N &= n \pi r_c^2 \int_{U_c}^{\infty} (1 - r^2/r_c^2) U f(U) dU \quad \dots \dots \dots (6) \\ &= n \pi r_c^2 I, \end{aligned}$$

where n is the number of molecules in unit volume, $U_c = \Delta P_{J'J} / m r_c$, and $r = \Delta P_{J'J} / m U$.

Also $r/r_c = U_c/U$ and I is the integral.

The total loss of energy in these collisions is

$$n \pi r_c^2 \Delta \varepsilon_{J'J} \int_{U_c}^{\infty} p(U) (1 - r^2/r_c^2) U f(U) dU = n \bar{p} A_c \Delta \varepsilon_{J'J} I,$$

where $\bar{p}$ is the mean value of the probability $p(U)$ averaged over all speeds $U > U_c$; and $A_c = \pi r_c^2$.

This total loss of energy averaged over all collisions of this type contributes to the mean energy lost by an electron in any collision the amount

$$\Delta Q_{J'J} = (\bar{p}A_c/A)(\Delta\varepsilon_{J'J}/\bar{U})I, \quad \dots\dots\dots (7)$$

where A is the mean collisional cross section of the molecules when the mean speed of the electrons is $\bar{U} = \int_0^\infty Uf(U)dU$.

The total mean energy lost by an electron in a collision taking into account elastic and rotational losses is

$$\Delta Q = \Delta Q_e + \Sigma_{J'J} \Delta Q_{J'J}. \quad \dots\dots\dots (8)$$

VI. SPECIAL DISTRIBUTIONS $f(U)$

It is convenient to obtain the formula for $\Delta Q_{J'J}$ in the special cases in which $f(U)$ corresponds respectively to laws of distribution of Maxwell and Druyvesteyn.

(a) *Maxwell's Distribution*

$$f(U) = (4/\alpha^3\sqrt{\pi})U^2 \exp(-U^2/\alpha^2), \quad \bar{U} = 2\alpha/\sqrt{\pi},$$

where α is the most probable speed.

The integral in equation (6) becomes

$$I = I_1 - I_2,$$

where

$$I_1 = 2\alpha/\sqrt{\pi} \int_{(U_c/\alpha)^2}^\infty e^{-y} y dy = (2\alpha/\sqrt{\pi})(1 + U_c^2/\alpha^2) \exp(-U_c^2/\alpha^2),$$

$$I_2 = \frac{2U_c^2}{\alpha\sqrt{\pi}} \int_{(U_c/\alpha)^2}^\infty e^{-y} dy = (2U_c^2/\alpha\sqrt{\pi}) \exp(-U_c^2/\alpha^2).$$

Whence

$$I = I_1 - I_2 = (2\alpha/\sqrt{\pi}) \exp(-U_c^2/\alpha^2) = \bar{U} \exp(-U_c^2/\alpha^2).$$

It follows from equation (7) that at $T = 288^\circ\text{K}$,

$$\begin{aligned} \Delta Q_{J'J} &= (\bar{p}A_c/A)\Delta\varepsilon_{J'J} \exp(-U_c^2/\alpha^2) \\ &= (\bar{p}A_c/A)(J' - J)(J' + J + 1)(0.198Q_0) \exp(-U_c^2/\alpha^2). \quad \dots\dots (9) \end{aligned}$$

The contribution of this source of loss to the total mean proportional loss of energy in a collision is

$$\begin{aligned} \eta_{J'J} &= \Delta Q_{J'J}/Q = \Delta Q_{J'J}/(Q_0 k_T) \\ &= (a/k_T) \exp(-U_c^2/\alpha^2), \end{aligned} \quad \left. \begin{aligned} & \\ & \end{aligned} \right\} \dots\dots (10)$$

$$a = 0.198\bar{p}A_c(J' - J)(J' + J + 1)/A.$$

where

Since $U_c = \Delta P_{J'J}/mr_c = Ch/2\pi mr_c = 1.05 \times 10^{-27} C/mr_c$ and $Q = Q_0 k_T = 3m\alpha^2/4$, where $C = \{J'(J'+1)\}^{\frac{1}{2}} - \{J(J+1)\}^{\frac{1}{2}}$ and $m = 0.912 \times 10^{-27}$ g, it follows that, when the temperature $T = 288^\circ \text{K}$,

$$\left. \begin{aligned} U_c^2/\alpha^2 &= \frac{1.50 \times 10^{-14} C^2}{r_c^2 k_T} = b/k_T, \\ b &= 1.50 \times 10^{-14} C^2/r_c^2. \end{aligned} \right\} \dots\dots\dots (11)$$

where

Consequently, equation (10) may be written

$$\eta_{J'J} = (a/k_T) \exp(-b/k_T). \dots\dots\dots (12)$$

Also

$$\eta_{J'J} k_T = \Delta Q_{J'J}/Q_0 = a \exp(-b/k_T). \dots\dots\dots (13)$$

The measured value of $\eta_{J'J} k_T$ is not represented by this formula for there exist inverse transitions in which electrons receive energy from the molecules. These are such that in thermal equilibrium with $k_T = 1$, the effective loss of energy in collisions is, on the average, zero. The effective values of $\eta_{J'J} k_T$ are therefore assumed to be

$$\left. \begin{aligned} \eta_{J'J} k_T &= a[\exp(-b/k_T) - \exp(-b)], \\ \eta_{JJ'} &= (a/k_T)[\exp(-b/k_T) - \exp(-b)]. \end{aligned} \right\} \dots\dots (14)$$

These are then summed over all transitions $J \rightarrow J'$ to give the total effective losses $\eta_r k_T$ in collisions that arise from rotational transitions, thus,

$$\eta_r k_T = k_T \sum \eta_{J'J} = \sum a_{J'J} \exp[(-b_{J'J}/k_T) - \exp(-b_{J'J})]. \dots\dots (15)$$

(b) *Druyvesteyn's Distribution*

$$f(U) = \frac{4U^2}{\alpha^3 \Gamma(3/4)} \exp(-U^4/\alpha^4).$$

Then

$$\bar{U} = \alpha/\Gamma(3/4) = 0.816\alpha, \quad \bar{U}^2 = \alpha^2 \Gamma(5/4)/\Gamma(3/4) = 0.75\alpha^2,$$

whence $\alpha^2 = 2.7Q/m$. The most probable speed is $\alpha/\sqrt{2}$. It follows that

$$U_c^2/\alpha^2 = 75 \times 10^{-16} (C^2/r_c^2 k_T), \dots\dots\dots (16)$$

where C is defined as in equation (11).

It may be shown that the formulae that here correspond to equation (14) are

$$\left. \begin{aligned} \eta_{J'J} k_T &= c[F(d/k_T) - F(d)], \\ F(d/k_T) &= 2/\sqrt{\pi} \exp(-d^2/k_T^2) - 2(d/k_T) \operatorname{erfc}(d^2/k_T^2), \end{aligned} \right\} \dots\dots (17)$$

where

$$d = 75 \times 10^{-16} (C/r_c)^2, \quad c = 0.198 \sqrt{(\frac{1}{2}\pi)} \bar{p} A_c (J' - J)(J' + J + 1)/A.$$

The coefficients a and c that appear in formulae (14) and (17) are, in general, not constants, but are functions of k_T through the term $\bar{p}/A$ that appears in

equation (7) so that a and c would be more correctly given in the form, $\text{constant} \times \varphi(k_T)/\psi(k_T)$. Although the experimental dependence of A upon k_T is known for some gases there is no information about $\bar{p}(k_T)$. In what follows it is supposed that $\bar{p}(k_T)$ is in effect a constant.

VII. ENERGY LOSSES OF ELECTRONS IN HYDROGEN WHEN $k_T < 5$

According to Section II, the sum of the relative populations of the states $J=1, 2$, and 3 is 6.35 , whereas that of the state $J=0$ is unity. Moreover, the increments of angular momentum ΔP_{12} , ΔP_{23} , and ΔP_{34} are about 1.07×10^{-27} erg sec, whereas ΔP_{01} is appreciably greater with a value of 1.49×10^{-27} erg sec.

It is to be expected therefore that, as k_T is increased from $k_T=1$ (thermal equilibrium) to, say, $k_T=5$, the electrons lose energy chiefly in exciting the transitions $P_1 \rightarrow P_2$, $P_2 \rightarrow P_3$, $P_3 \rightarrow P_4$. It is reasonable to group these transitions and to assign to them a common value of $r_c U_c = 1.07 \times 10^{-27} m$ erg sec.

In hydrogen the collisional cross section A is closely represented by $A = 0.816 (10.7 + k_T) \times 10^{-16} \text{ cm}^2$, when $1.73 < k_T < 5.1$. Consequently the following formula is suggested for η_r :

$$\eta_r = \frac{\alpha}{k_T(10.7 + k_T)} [\exp(-b/k_T) - \exp(-b)]. \quad (18)$$

η_r is a maximum where

$$b/k_T = [1 + k_T(10.7 + k_T)][1 - \exp\{-b(1 - 1/k_T)\}]. \quad (19)$$

A first approximation to b is $[1 + k_T/(10.7 + k_T)]k_T$. A second and adequate approximation is

$$b = k_T[1 + k_T/(10.7 + k_T)][1 - \exp\{-(1 + k_T/(10.7 + k_T))(k_T - 1)\}]. \quad (20)$$

When b is found from equation (20), α may be found from the experimental value of η_{max} . In hydrogen when $1 < k_T < 5$ it was found that

TABLE 2
COMPARISON OF EXPERIMENTAL AND CALCULATED VALUES OF η_r

k_T	..	..	1.34	1.73	2.5	3.4	4.2	5.1	5.9	6.8
$\eta k_T \times 10^4$	..	..	7.16	18.3	41.2	62.2	78.0	95.3	114	129
$(\eta k_T)_e \times 10^4$	..	..	1.85	3.96	8.16	13.1	17.4	22.3	26.8	35.8
$(\eta k_T)_r \times 10^4$	..	..	5.31	14.4	33.0	49.1	60.6	73.0	84.2	84.2
$\eta_r \times 10^4$	..	..	3.98	8.25	13.2	14.4	14.4	14.3	14.3	14.3
$\eta_r \times 10^4$ (calc.)	..	..	4.05	8.30	12.7	14.4	14.4	14.3	12.7	11.5

$\alpha = 0.271$; $b = 4.49$. A comparison is made in Table 2 between η_r calculated from equation (18) with these values of α and b and the experimental results of Crompton and Sutton (1952). The figures in the second row are the measured values of ηk_T and the third row gives the corresponding values of $(\eta k_T)_e$, the contribution to ηk_T of the losses in elastic

collisions (Section II). The figures in the fourth row are the result of subtracting the third row from the second and represent $(\eta k_T)_r$, the contribution of the rotational losses. The fifth row is η_r . The last row contains the values of η_r calculated from equation (18). Agreement is good within the range $1 < k_T < 5$. When k_T exceeds 5 the formula fails because additional sources of loss of energy become important and because A increases more slowly with k_T than is suggested by the formula $A = 0.816 (10.7 + k_T) \times 10^{-16} \text{ cm}^3$. The appropriate adaptations of formulae (21) based on Druyvesteyn's distribution do not describe the observations as accurately as does formula (18). The critical radius r_c is, according to equation (11), given by $r_c = (1.52 \times 10^{-14}/b)^{1/2} C$. In transitions $J' \rightarrow J=1$; $J=1, 2, 3$, the value of C is about 1.02. With $b = 4.49$ the value of r_c is $5.6 \times 10^{-8} \text{ cm}$. If it is supposed that the appropriate transitions are $J \rightarrow J' = J+2$, then the value of r_c is approximately twice as great.

VIII. ENERGY LOSSES IN OXYGEN

Although the assumption that the distribution function $f(U)$ is that of Maxwell appears to be satisfactory for the motion of electrons in hydrogen when $k_T < 5$, yet it is to be anticipated that Druyvesteyn's distribution function is more nearly correct at larger values of k_T . This expectation is confirmed by an analysis of some unpublished measurements by R. W. Crompton and D. J. Sutton in this department of the motions of electrons in oxygen in which the variation of A with k_T is small. It is found that the dependence of η upon k_T is very closely described by formula (17) based on Druyvesteyn's function and not at all well by formula (14) based on Maxwell's, over the range of values of k_T from 8 to 30, the curve of $\eta v. k_T$ in oxygen differs markedly from those of hydrogen and nitrogen in that the first maximum is clearly marked and lies near $k_T = 14.4$ instead of $k_T = 4$. A consequence of this displacement of the maximum to a relatively large value of k_T is that the constant term $F(d)$ in equation (17) is negligibly small. Thus in oxygen

$$\left. \begin{array}{l} \text{where} \quad \eta_r = (c/k_T) F(d/k_T) = (c/d) x F(x), \\ \quad \quad \quad x = d/k_T, \\ \text{and} \quad \quad \quad F(x) = (2/\sqrt{\pi}) \exp(-x^2) - 2x \operatorname{erfc}(x^2). \end{array} \right\} \dots\dots\dots (21)$$

The function $x F(x)$ possesses a maximum value 2.03 at $x = 0.435$, consequently, if the experimental maximum is $(\eta_r)_{\max}$ at $(k_T)_{\max}$, then, $d = 0.435(k_T)_{\max}$ and $c/d = (\eta_r)_{\max}/2.03$.

The large value of $(k_T)_{\max}$ in oxygen is to be attributed to the fact that, because of the absence of nuclear spin, only the rotational states corresponding to $J=1, 3, 5, \dots$ exist, the most populous states at $T=288^\circ \text{K}$ being those for which J is of the order of 8. The smallest of the increments of angular momentum require transitions $J \rightarrow J+2$, and when J is as large as 8 all $\Delta P_{J, (J+2)} \rightarrow 2h/2\pi = 2.102 \times 10^{-27} \text{ erg sec}$. Thus, in formula (17), $C=2$ and

$$\begin{aligned} r_c^2 &= 75 \times 10^{-16} C^2/d = 3 \times 10^{-14}/0.435(k_T)_{\max} \simeq 4.8 \times 10^{-15}, \\ r_c &\simeq 7 \times 10^{-8} \text{ cm}. \end{aligned}$$

The particular form of equation (21) that describes the proportional energy loss in oxygen is :

$$\eta = 4.17 \times 10^{-2} (6.25/k_T) F(6.25/k_T). \dots\dots\dots (22)$$

Table 3 gives a comparison of the predictions of this formula and the measured values of η of Crompton and Sutton, on the assumption that the distribution function $f(U)$ is that of Druyvesteyn.

TABLE 3
COMPARISON OF PREDICTED AND MEASURED VALUES OF η

Small Values of k_T										
k_{TD}	..	..	..	4.1	5.3	6.2	7.2			
$10^3 \times \eta_D$	..	..	..	3.09	4.14	5.10	5.75			
$10^3 \times \eta$ (calc.)	..	..	..	1.04	2.75	4.17	5.29			

Large Values of k_T												
k_T	..	..	7.9	8.8	9.5	10.1	11.0	12.7	15.8	19.4	22.3	36.0
$10^3 \times \eta$	..	..	6.45	6.92	7.30	7.7	8.02	8.35	8.35	8.03	7.70	6.40
$10^3 \times \eta$ (calc.)	..	..	6.2	6.90	7.35	7.7	8.0	8.34	8.36	8.08	7.66	5.0

It can be seen that the agreement between the calculated and measured values is close when $8 < k_T < 30$ but that divergences occur outside these limits. At small values of k_T the distribution function $f(U)$ is presumably tending towards that of Maxwell so that formula (22) is not applicable. The experimental curve for η v. k_T exhibits a minimum at $k_T = 36$ with a progressive rise as k_T increases. There is therefore an additional process producing loss of energy whose effects are not taken account of by formula (22).

IX. ENERGY LOSSES IN NITROGEN

It was found (Crompton, Huxley, and Sutton 1953) that in nitrogen and air the collisional cross section A is proportional to the mean speed $\bar{U}$ of the electrons and therefore to k_T . In nitrogen, when $k_T < 9$, $A = 2.86 k_T^{\frac{1}{2}} \times 10^{-16}$. In this gas, therefore, it is to be expected that η would be represented by a formula of the form

$$\eta = \frac{\alpha}{k_T^{\frac{3}{2}}} [\exp(-b/k_T) - \exp(-b)]. \dots\dots\dots (23)$$

Because measurements of η are lacking for values of k_T less than that at which η attains its maximum value in the range of small values of k_T it is not possible to ascribe values to α and b with any confidence.

X. THE MEAN RATE R AT WHICH AN ELECTRON LOSES ENERGY

The mean energy effectively lost by an electron in unit time in producing rotational changes follows from equation (6) and the following equation.

Since in both the Maxwell and Druyvesteyn forms of the distribution function the mean speed $\bar{U}$ appears as a factor in the expression for the total energy lost per second, this rate of loss may be written in the form

$$\left. \begin{array}{l} R/n = \alpha Q^{\frac{1}{2}} [\exp(-\beta/Q) - \exp(-\beta/Q_0)], \text{ Maxwell's distribution} \\ \text{or} \\ R/n = \gamma Q^{\frac{1}{2}} [F(\delta/Q) - F(\delta/Q_0)], \text{ Druyvesteyn's distribution} \end{array} \right\} \quad (24)$$

where α , β , γ , and δ are constants independent of Q and Q_0 and the function F is defined in equation (17).

The forms of these expressions that are adapted to the laboratory measurements at $T=288^\circ\text{K}$ are

$$\left. \begin{array}{l} R/n = ak_T^{\frac{1}{2}} [\exp(-b/k_T) - \exp(-b)], \\ R/n = ck_T^{\frac{1}{2}} [F(d/k_T) - F(d)], \end{array} \right\} \quad \dots\dots\dots (25)$$

where a and c are constants, and $b = \beta/Q_{288}$, $d = \delta/Q_{288}$, and $\delta = 0.435\beta$. Q_{288} is Q_0 at $T=288$.

The quantity R/n is doubly convenient. First, it is independent of the mean collisional cross section A ; secondly, it is immediately derivable from the laboratory measurements of the drift speeds W of electrons in a steady state of motion through a gas in a steady electric field Z .

In terms of the drift speed W and electronic charge e , R is given by $R = ZeW$. Let p be the pressure of the gas in millimetres of mercury at $T=288^\circ\text{K}$, and let n be the number of molecules of the gas in a cubic centimetre.

Then $n = 3.35 \times 10^{16}p$ and it follows that

$$(3.35 \times 10^{16} \times 300/4.8 \times 10^{-10})R/n = 2.09 \times 10^{28}R/n = (Z/p)W, \quad \dots \quad (26)$$

where Z is expressed in volts per centimetre.

Since k_T is a function of Z/p it follows that both R/n and $(Z/p)W$ may be found as experimental functions of k_T and the result may be compared with the theoretical formulae (25). If equations (26) correspond to reality, then the experimental quantity $W(Z/p)k_T^{-\frac{1}{2}}$ should be related to k_T , when $T=288^\circ$ (the laboratory temperature) as follows:

$$\left. \begin{array}{l} W(Z/p)k_T^{-\frac{1}{2}} = \text{constant} [\exp(-b/k_T) - \exp(-b)], \text{ Maxwell's} \\ \hspace{15em} \text{distribution} \\ W(Z/p)k_T^{-\frac{1}{2}} = \text{constant} [F(d/k_T) - F(d)]. \hspace{2em} \text{Druyvesteyn's} \\ \hspace{15em} \text{distribution} \end{array} \right\} \dots \quad (27)$$

It is convenient to test these expressions against the measurements of electronic motions in oxygen because in this gas, not only are the terms $\exp(-b)$ and $F(d)$ negligible but a single type of rotational loss is present over a wide range of the parameter k_T (Section VIII).

Table 4 shows that good agreement is obtained with the first of formulae (27) when $k_T < 9$, and with the second when $10 < k_T < 30$.

It would appear therefore, that the transition from Maxwell's law of distribution to that of Druyvesteyn takes place in the range of values of k_T $8 < k_T < 10$.

TABLE 4
COMPARISON OF CALCULATED AND OBSERVED VALUES OF y

Smaller values of k_T ; $y = (Z/p) W k_T^{-\frac{1}{2}} = 1.55 \times 10^6 \exp(-14.4/k_T)$						
k_T (Maxwell)	4.7	6.0	7.1	8.2	9	10
$(y \text{ observed}) \times 10^{-4}$..	7.4	13.0	19.9	26.1	32.3	39.1
$(y \text{ calculated}) \times 10^{-4}$	7.1	14.0	20.3	26.6	31.2	36.6

Larger values of k_T ; $y = (Z/p) W k_T^{-\frac{1}{2}} = 1.94 \times 10^6 F(6.25/k_T)$									
k_T (Druyvesteyn) ..	8.8	9.5	10.1	11.0	12.7	15.8	19.4	22.3	36.0
$(y \text{ observed}) \times 10^{-5}$..	4.2	4.91	5.72	6.42	7.86	9.89	11.6	12.6	17.5
$(y \text{ calculated}) \times 10^{-5}$	4.57	5.20	5.80	6.48	7.86	9.89	11.6	12.4	15.9

The particular forms assumed by the expressions (24), as derived from the experimental measurements at $T=288$ are :

$$\left. \begin{aligned} R/n &= 3.04 \times 10^{-16} Q^{\frac{1}{2}} \exp(-8.58 \times 10^{-13}/Q), & k_T < 8, \\ R/n &= 3.80 \times 10^{-16} Q^{\frac{1}{2}} F(-3.72 \times 10^{-13}/Q), & 10 < k_T < 30. \end{aligned} \right\} \dots (28)$$

In equation (9) write $R = v \Delta Q_{J'J}$ and put $v = nA\bar{U}$, $\bar{U} = 4.33 \times 10^{13} Q^{\frac{1}{2}}$, $J' - J = 2$, $J = 8$, $(\Delta \epsilon_{J'J})_{\text{oxygen}} = 0.198 Q_0/16$. Comparison with equation (28) suggests that $\bar{p} = 3 \times 10^{-2}$.

The quantity R for air is of fundamental importance in the theory of radio wave interaction, but this aspect of the study is discussed in another paper.

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XII. REFERENCES

- CROMPTON, R. W., and SUTTON, D. J. (1952).—*Proc. Roy. Soc. A* **215**: 467.
 CROMPTON, R. W., HUXLEY, L. G. H., and SUTTON, D. J. (1953).—*Proc. Roy. Soc. A* **218**: 507.
 GERJUOY, E., and STEIN, S. (1954).—Westinghouse Research Laboratories Sci. Res. Pap. 60-94439-1-P1.
 GERJUOY, E., and STEIN, S. (1955).—Westinghouse Research Laboratories Sci. Res. Pap. 60-94439-1-P2.
 HALL, BARBARA I. H. (1955).—*Aust. J. Phys.* **8**: 468.
 MASSEY, H. S. W., and BURHOP, E. H. S. (1952).—"Electronic and Ionic Impact Phenomena." p. 279. (Clarendon Press: Oxford.)