STATISTICAL ANALYSIS OF AUSTRALIAN TEMPERATURE DATA

By S. C. Das*

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Summary

The mean summer maximum temperatures of five Australian inland localities for a period of 65 years from 1891 to 1955 are analysed and it is shown that the temperatures have an overall parabolic trend.

I. Introduction

In a previous paper (Das 1956) the statistical analysis of Australian pressure data was considered in order to test a hypothesis, regarding a shift in the high pressure belt, which arose out of Deacon's (1953) work. It was shown there that the belt is slowly moving southward. In this paper we attempt to make a statistical analysis of the mean summer maximum temperatures of five Australian inland localities to verify how far the temperature data conform to the above hypothesis. The five localities chosen for the purpose were Bourke, Hay, Broken Hill, Narrabri, and Walgett. Broken Hill has data dating back to 1891, while the other localities have data for a few more years. The mean summer maximum temperatures as given in the data are the averages of the mean monthly maxima for the months of December, January, and February. For example, the figure given for 1891 is the average of the mean monthly maxima for December 1890, January and February 1891.

II. Analysis of the Data

In order to examine whether the data follow a trend, a polynomial regression line of the form $y=A'+B'\xi_1'+C'\xi_2'$, or of the form $y=A'+B'\xi_1'+C'\xi_2'+D'\xi_3'$ depending on the tests of goodness of fit of temperature on time, has been fitted to the data for individual localities. Following the accepted notation used in Fisher and Yates's tables, ξ_1', ξ_2', and ξ_3' stand for orthogonal polynomials of degree one, two, and three respectively in t, where t is the number of years measured from the centre point of the series. The results are shown in Table 1.

The second, third, and fourth columns give the three regression coefficients and the next three columns give the corresponding t values for testing the significance of these regression coefficients. The first regression coefficient is significant for each locality and the second is significant only in the case of Hay. The last column gives the corresponding regression equations. It is evident from the regression equations that in all of the stations the mean summer

* Australian National University, Canberra ; present address : Ravenshaw College, Cuttack, Orissa, India.

maximum temperatures exhibit slow linear decreases except for Hay where the trend is parabolic.

In testing $t_{C'}$ for Hay for significance we must, of course, regard it as having been chosen as the largest of the five statistics. Nevertheless it is clearly still significant at the 5 per cent. point even after allowing for this. With the exception of Bourke all the values of $t_{C'}$ are positive, moreover, which suggests that a stronger conclusion regarding the effect of the variable ξ_2' on the temperatures may be got by a joint test of significance of the five $t_{C'}$ values.

TABLE 1

Locality	B'	C'	D'	$t_{B'}$	$t_{C'}$	$t_{D'}$	Regression Equations (°F)
Bourke	—0·06983	—0·00003		3·58	0·09		$y=97\cdot4-0\cdot06983t$
Hay	—0·02353	0·00402	0·00003	3·60	2·94	1·28	$y=88\cdot5-0\cdot02353t+0\cdot00402t^2$
Broken Hill ..	—0·06979	0·00180		4·40	1·90		$y=90\cdot4-0\cdot06979t$
Narrabri ..	—0·03227	0·00033		3·72	1·06		$y=93\cdot7-0\cdot03227t$
Walgett ..	—0·02081	0·00151		2·38	0·78		$y=94\cdot7-0\cdot02081t$

We may consider the model

$$\begin{pmatrix} \varkappa_{1t} \\ \varkappa_{2t} \\ \vdots \\ \varkappa_{5t} \end{pmatrix} = \mathbf{B} \begin{pmatrix} \xi_{0t}' \\ \xi_{1t}' \\ \xi_{2t}' \end{pmatrix} + \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \vdots \\ \varepsilon_{5t} \end{pmatrix}$$

or

$$\boldsymbol{\varkappa}_t = \mathbf{B}\boldsymbol{\xi}_t + \boldsymbol{\epsilon}_t,$$

where $\varkappa_{jt}$ is the temperature at the jth station at time t and $\mathbf{B}$ is a (5×3) matrix of regression coefficients. The ε_t's are presumed to be serially independent and to have a joint normal distribution with an arbitrary (non-singular) correlation matrix. We then wish to test the hypothesis that the last column of $\mathbf{B}$ is composed entirely of zeros. The likelihood ratio test of this hypothesis is the partial canonical correlation of the vector $\boldsymbol{\varkappa}_t$ with ξ_{2t}' when the effects of the other elements in $\boldsymbol{\xi}_t$ have been removed (Bartlett 1947). In this simple case the partial canonical correlation reduces to the partial multiple correlation of ξ_{2t}' with $\varkappa_{1t}, \varkappa_{2t}, \ldots, \varkappa_{5t}$ when the effects of means and ξ_{1t}' have been removed. The test statistic may be eomputed as

$$F = \frac{1-L}{L} \cdot \frac{n-7}{5},$$

where

$$L = \frac{|\,\mathbf{C}_{\varkappa\varkappa\cdot1} - [\mathbf{C}_{\varkappa2\cdot1}\mathbf{C}_{\varkappa2\cdot1}']C_{22\cdot1}^{-1}\,|}{|\,\mathbf{C}_{\varkappa\varkappa\cdot1}\,|},$$

and

$$\mathbf{C}_{\varkappa\varkappa\cdot 1}=\mathbf{C}_{\varkappa\varkappa}-[\mathbf{C}_{\varkappa 1}\,.\,\mathbf{C}_{1\varkappa}]C_{11}^{-1},$$

$$\mathbf{C}_{\varkappa 2}=\left[\sum_{t=1}^{65}\varkappa_{jt}\xi_{2t}'\right],\quad \text{(a matrix of 5 rows)}$$

$$\mathbf{C}_{\varkappa\varkappa}=\left[\sum_{t=1}^{65}(\varkappa_{it}-\bar{\varkappa}_i)(\varkappa_{jt}-\bar{\varkappa}_j)\right],\quad \text{(a } 5\times 5 \text{ matrix)}$$

$$\mathbf{C}_{1\varkappa}=\left[\sum_{t=1}^{65}\xi_{1t}'\varkappa_{jt}\right],\quad \text{(a matrix of 5 columns)}$$

$$C_{11}=\sum_{1}^{65}\xi_{1t}'^{2},\quad C_{22\cdot 1}=C_{22}=\sum_{1}^{65}\xi_{2t}'^{2}.$$

In the present case

$$F=\frac{58}{5}\cdot\frac{1-L}{L}=3\cdot 70.$$

This is distributed in the F distribution with 5 and 58 degrees of freedom and is therefore highly significant. There is no doubt therefore that there is some overall second degree effect whose influence appears to be to slow down and reverse the effect of the decline in maximum summer temperatures.

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IV. References

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