COUNTS OF EXTRAGALACTIC RADIO SOURCES AND UNIFORM MODEL UNIVERSES

By G. C. McVittie*

[*Manuscript received April* 5, 1957]

Summary

The radio sources are assumed to be galaxies sharing in the red-shift phenomenon. The counts of such sources to successive limits of flux density are interpreted in terms of the model universes of general relativity. The flux density in each infinitesimal interval of frequency is assumed to be proportional to a power of the frequency and to a multiplicative parameter, both these quantities being regarded as functions of the red-shift, i.e. of the time of travel of the radiation. All sources at the same distance from the observer are assumed to be identical radiators; this permits the introduction of a standard comparison source. The data in the Sydney catalogue can then be interpreted in terms of radio sources of constant strength (i.e. which are independent of the red-shift) if they are similar to NGC 1275, taken as standard. If they are similar to Cygnus A, the data imply that the sources were radiating more strongly when their radiation left them than is Cygnus A now. This type of conclusion appears to be unavoidable for the data of the Cambridge catalogue. It is briefly explained how to modify the theory to take account of a variation with time of the space-density of sources and also of a mixture of comparison sources.

I. Introduction

It has been known for 20 years that counts of the numbers of galaxies to successive limits of apparent magnitude at optical wavelengths would lead to important conclusions about the nature of the astronomical universe. Observations of this kind can be interpreted in terms of the uniform model universes of general relativity and the theoretical method of attack is known (McVittie 1956). The optical data, however, suffer from the defect that the apparent magnitudes of faint galaxies are very difficult to determine, and progress in this direction is likely to be slow. In contrast, the analogue of the apparent magnitude of a radio source can be measured with, relatively speaking, very great accuracy and speed. In the catalogues of radio sources published by the Cambridge (Shakeshaft *et al.* 1955) and the Sydney (Mills and Slee 1957) observers respectively, there are numerous sources classified as extragalactic. It is true that, whereas the flux density (equivalent to the apparent magnitude of an optical source) of such a source can be measured satisfactorily, it is not so easy to distinguish real from spurious sources as it is with an optical telescope. This is, of course, because of the as yet imprecise directivity of the radio telescopes employed. Preliminary as the Cambridge and Sydney catalogues may be, it is still worth while to work out the theory of the distribution in space of extragalactic radio sources on the assumptions (*a*) that these sources are galaxies

* University of Illinois Observatory, Urbana, Ill., U.S.A.

of some kind and that the red-shift phenomenon applies to them even if they have not been identified with optical objects; (*b*) that the uniform model universes of general relativity provide the appropriate theory for the purpose; (*c*) that the dispersion in intrinsic flux density of the sources, at each instant of cosmic time t (see equation (**2.01**)), is negligible. Clearly, (*c*) is a bold assumption since, even amongst those sources that have been identified as colliding galaxies, the flux density of Cygnus A (IAU 19N4A) observed at the Earth, is 100 times or more that of NGC 1275 (IAU 03N4A). Yet, from their optical red-shifts, the distance of the former object is three times that of the latter. In Section V a suggestion for removing assumption (*c*) is made. The theory will be presented in terms of observable quantities, flux densities, red-shifts, numbers of radio sources, etc., rather than in terms of the derivative concept of distance. Though the procedure may seem roundabout, it avoids all the complexities inherent in the notion of the distance of an object whose red-shift may be large and which is located in a universe the nature of whose geometry is not known *a priori* (McVittie 1957).

The analysis of the spatial distribution of radio sources is less complicated than that for optical sources because the former have a relatively simple spectral energy distribution function. The flux density from a radio source is usually given in watts m^{-2} $(c/s)^{-1}$ and, if ν is the frequency, the amount of energy crossing a unit area in unit time at the point of observation, in the infinitesimal frequency interval $|d\nu|$ is proportional to $\nu^{x}|d\nu|$ where the spectral index x is a number lying between $-0\cdot6$ and $-1\cdot0$ (Ryle 1955). It is true that recent work by Adgie and Smith (1956) indicates that, for Cygnus A, x varies with the frequency between these limits. Nevertheless we shall assume that x is a constant. Converting to wavelength, this flux is therefore $\lambda^{-x-2}d\lambda$, which it is convenient to write as

$$\lambda^{p-1}d\lambda, \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (\mathbf{1.01})$$

so that p is a number lying in the range $0 \geqslant p \geqslant -0\cdot4$.

II. Uniform Model Universes

In this section we shall summarize various results on uniform model universes that will be needed. The proofs will be found in the relevant sections of "General Relativity and Cosmology" (McVittie 1956; hereinafter referred to as GRC). The metric is (GRC Section 8.2)

$$ds^2 = dt^2 - \frac{R^2(t)(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2)}{c^2(1+kr^2/4)^2}, \quad \ldots\ldots \quad (\mathbf{2.01})$$

where s, t have the physical dimensions of time, r, θ, φ are dimensionless spatial coordinates, R has the dimensions of length and is an undetermined function of t, and k, the space-curvature constant which determines the nature of the geometry, can be made to have the values $+1$, 0, or -1 by a suitable choice of the unit of r. It is convenient to locate the observer O at $r=0$ and to assume that all his observations are made at the same instant t_0. The gravitational effects of the distribution of galaxies are idealized by replacing the galaxies by a continuous perfect fluid of density ρ and pressure P where (GRC (**4.225**), (**8.210**), (**8.211**))

$$8\pi G\rho = 3(kc^2 + R'^2)/R^2 - \Lambda, \quad \ldots\ldots\ldots\ldots\ldots \quad (\mathbf{2.02})$$

$$8\pi GP/c^2 = -2R''/R - (R'/R)^2 - kc^2/R^2 + \Lambda, \quad \ldots\ldots \quad (\mathbf{2.03})$$

a prime denoting a derivative of R and Λ being the cosmical constant. Thus the distribution of perfect fluid is spatially uniform because ρ and P depend on the time alone, apart from constants, and it is in this sense that spatial uniformity is to be understood in a theory based on (**2.01**).

Sources of radiation are defined (GRC Section 8.4) to have world-lines along which $t=s$ and r, θ, φ are constant. A source P_i located at $r=r_i$, emitting radiation at time t_i which reaches O at time t_0, has r_i, t_i connected by (GRC (**8.409**))

$$c\int_{t_i}^{t_0} \frac{\mathrm{d}t}{R(t)} = -\int_{r_i}^{0} \frac{\mathrm{d}r}{1+kr^2/4}, \quad \ldots\ldots\ldots\ldots \quad (\mathbf{2.04})$$

an equation derived from the motion of the light ray. The red-shift in the spectrum of the radiation from P_i, as observed at O, is (GRC (**8.412**))

$$\delta = R_0/R_i - 1, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (\mathbf{2.05})$$

where $R(t_0)=R_0$, $R(t_i)=R_i$. Here $\delta=\mathrm{d}\lambda/\lambda$, where λ is the wavelength of the radiation received at O, the wavelength of emission at P_i being λ_i. Since the red-shift is common to all lines in the spectrum of P_i (Lilley and McClain 1956 ; Minkowski and Wilson 1956), we have also (GRC (**8.601**), (**8.602**))

$$\lambda = (1+\delta)\lambda_i, \qquad \mathrm{d}\lambda = (1+\delta)\mathrm{d}\lambda_i. \quad \ldots\ldots\ldots \quad (\mathbf{2.06})$$

The distance between O and P_i depends partly on the operational procedure that is used to measure it and is not an absolute quantity as in classical theory (GRC Section 8.5). In cosmology the type of distance normally employed is luminosity-distance D, which is such that the intensity of the radiation from a source falls off as the inverse square of D. It can be proved that for a source such as P_i (GRC (**8.517**))

$$D = \frac{R_0^2 r_i}{R_i(1+kr_i^2/4)}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (\mathbf{2.07})$$

The total number of sources P_i that have r-coordinates in the range $0 \leqslant r \leqslant r_i$ is (GRC (**8.702**))

$$N = \frac{4\pi\alpha}{Q}\int_0^{r_i} \frac{r^2\mathrm{d}r}{(1+kr^2/4)^3}, \quad \ldots\ldots\ldots\ldots \quad (\mathbf{2.08})$$

where α is an absolute constant and $1/Q$ is the fraction of the whole celestial sphere over which the observer O counts the sources. This result depends on the assumption that each source has fixed (r, θ, φ) coordinates but it does not imply that each source radiates in exactly the same way. Since each r in the integrand of (**2.08**) is connected with the corresponding time of emission by (**2.04**), it follows that the formula for N makes allowance for the different times of travel to O of the radiation from the sources in each successive shell centred at O. Thus, since the distribution of sources is changing its scale with time while remaining similar to itself, the number N will not correspond to that in an instantaneous picture of a " uniform " distribution of sources at rest in a Euclidean space, even if $k=0$, which is the condition for a space of this kind.

There are two ways of comparing the foregoing formulae with observation. The first is to pre-assign R as a function of t and also to pick particular values of k or of Λ, or of both these constants. This method usually employs the device of assuming a special form for P in (**2.03**) and determining R from the resulting differential equation. Examples will be found in the work of Hoyle and Sandage (1956) or of Shakeshaft (1954); the second author, apparently influenced by the creation of matter theory of Bondi and Gold (Bondi 1952), presupposes that R is an exponential function of t and that $k=0$. The second method is to use the observations themselves to determine, as far as possible, $R(t)$ and the constants k and Λ. It will be used here and it depends on the assumption that $R(t_i)$ can be expanded in a Taylor series in terms of the time of travel, t_0-t_i, of the radiation from the source to the observer (GRC Sections 9.1 and 9.2). Elimination of the time of travel between (**2.04**), (**2.05**), and (**2.07**) will then give D as a power series in δ (GRC (**9.210**)), namely,

$$D=\frac{c}{h_1}\delta\left(1+\frac{h}{2}\delta+. . .\right), \quad \text{. . . . .} \quad (\mathbf{2.09})$$

where

$$h_1=R'_0/R_0, \quad h=(h_1^2+h_2)/h_1^2, \quad h_2=R''_0/R_0, \quad \text{. . . .} \quad (\mathbf{2.10})$$

and a subscript denotes that the functions are evaluated at $t=t_0$. Inverting we have

$$\delta=\frac{h_1}{c}D\left(1-\frac{h}{2}(h_1D/c)+. . .\right). \quad \text{. . . . .} \quad (\mathbf{2.11})$$

In a similar way it follows that (GRC Section 9.3)

$$N=\frac{4\pi\alpha}{3QR_0^3}D^3\left\{1-3(h_1D/c)+3\left(2+\frac{h}{2}+\frac{2}{5}\,\frac{c^2}{a^2h_1^2}\right)(h_1D/c)^2+. . .\right\}, \quad \text{. . . . .} \quad (\mathbf{2.12})$$

where

$$a^2=4R_0^2/k. \quad \text{. . . . .} \quad (\mathbf{2.13})$$

It is to be noticed that the coefficient of each successive term in these expansions involves a higher derivative of R, evaluated at the instant t_0, than the preceding one. The formula (**2.12**) gives, of course, the number of sources whose luminosity-distances do not exceed D.

Observations of the red-shifts of galaxies versus their apparent magnitudes in the optical range yield numerical values of the Hubble parameter h_1 and of the acceleration parameter h_2, essentially through fitting the data to formula (**2.09**) (Humason, Mayall, and Sandage 1956; McVittie 1957). For our present purpose, the numerical value of h_1 is happily not required; it lies between $8{\cdot}75$ and $4{\cdot}64\times10^{-18}$ sec^{-1}, according to the method of interpretation of the data that is employed (McVittie 1957). The value of h_2 is still more uncertain; if we write $h_2=-q_0h_1^2$, the number q_0 is given as $2\cdot5\pm1$ by Hoyle and Sandage (1956), whilst McVittie (1957) concludes that $2\cdot7\leqslant q_0\leqslant5\cdot6$. Thus in (**2.10**) we write

$$h_2=-q_0h_1^2, \quad h=-(q_0-1), \qquad \text{(2.14)}$$

where q_0 is likely to be greater than $1\cdot5$ and less than $5\cdot5$. Incidentally, the Bondi and Gold creation of matter theory requires that $q_0=-1$ (Hoyle and Sandage 1956) and therefore this theory does not apparently agree with the presently available optical data on red-shifts. Again, if ρ_0, P_0 denote the values of the density and pressure at time t_0, it is usually accepted that P_0/c^2 is negligibly small compared with ρ_0. Thus by (**2.03**) and (**2.14**)

$$\Lambda=(-2q_0+1)h_1^2+c^2k/R_0^2, \qquad \text{(2.15)}$$

and then (**2.02**) becomes

$$4\pi G\rho_0=(q_0+1)h_1^2+c^2k/R_0^2. \qquad \text{(2.16)}$$

It therefore follows, since $\rho_0>0$, that

$$(q_0+1)h_1^2>-c^2k/R_0^2. \qquad \text{(2.17)}$$

It should be remarked that the equation

$$c^2k/R_0^2=(2q_0-1)h_1^2$$

found by Hoyle and Sandage (1956) is not so much a consequence of the observational data as a result of their *a priori* assumption that $\Lambda=0$.

III. The Number of Radio Sources

We shall convert the formula (**2.12**) into one involving the limiting flux density from a radio source corresponding to the limiting luminosity-distance D. The first step is to connect the flux density S_i of a radio source lying in the shell of radii $r_i+\mathrm{d}r$ and r_i with its luminosity-distance D_i. The assumption will be made that, for an observer at unit distance from the source and observing the radiation shortly after its emission, the flux density in the infinitesimal range $\mathrm{d}\lambda_i$ of wavelength is, by (**1.01**),

$$C(\delta_i)\lambda_i^{p(\delta_i)-1}\mathrm{d}\lambda_i, \qquad \text{(3.01)}$$

C and p being functions of δ_i which can be expressed as

$$C(\delta_i)=C_0\left(1+C_1\delta_i+\frac{C_2}{2}\delta_i^2+\ldots\right), \qquad \text{(3.02)}$$

$$p(\delta_i)=p_0\left(1+p_1\delta_i+\frac{p_2}{2}\delta_i^2+\ldots\right), \qquad \text{(3.03)}$$

where C_0, C_1, C_2, . . . and p_0, p_1, p_2, . . . are constants. Here δ_i is to be regarded as a replacement for the time t_i of emission of the radiation. Thus the flux density is the same for all sources with the same r_i but may differ for different r_i. In view of the remarks made in Section I, it will be assumed that

$$0\geqslant p_0\geqslant-0\cdot4. \qquad \text{(3.04)}$$

The flux density in watts m^{-2} measured at O in the range λ' to λ of *observed* wavelength is, by (**2.06**) and (**3.01**),

$$I_i = \frac{C(\delta_i)}{D_i^2(1+\delta_i)^{p(\delta_i)}} \int_\lambda^{\lambda'} \sigma(\lambda)\lambda^{p(\delta_i)-1} d\lambda,$$

where $\sigma(\lambda)$ is the " extinction factor " which allows for the imperfect recording by the apparatus of the incoming energy. Dropping the subscript i and assuming that $\sigma(\lambda)$ is approximately constant over the range of integration, we have

$$I = \frac{C(\delta)\sigma}{D^2(1+\delta)^{p(\delta)}p(\delta)}(\lambda'^{p(\delta)} - \lambda^{p(\delta)}).$$

In this formula there occur the constant C_0 of (**3.02**), which depends on the strength of the source as this would be measured at unit distance from it shortly after the time of emission of the radiation, and the luminosity-distance D of the source from the observer. Neither of these quantities is, in general, known for a radio source whose flux density is measured. For a given flux density the source may be weak and nearby or strong and remote. Following the practice of optical astronomy, where the flux density corresponds to apparent magnitude, we introduce the analogue of absolute magnitude. To this end it will be supposed that a typical, or standard, source has been found whose I, denoted by I_s, has been measured with the same apparatus and which also has a known red-shift, δ_s. This standard source is presumed to have the same constants, $C_0, C_1, C_2, \ldots$ and $p_0, p_1, p_2, \ldots$, as have the other sources that are being studied. Denoting quantities referring to the standard source by the subscript s, we have

$$\frac{I}{I_s} = \frac{C(\delta)}{C(\delta_s)} \frac{(1+\delta_s)^{p(\delta_s)}}{(1+\delta)^{p(\delta)}} \frac{p(\delta_s)}{p(\delta)} \frac{\lambda'^{p(\delta)} - \lambda^{p(\delta)}}{\lambda'^{p(\delta_s)} - \lambda^{p(\delta_s)}} \frac{D_s^2}{D^2}. \quad \ldots\ldots \quad (\mathbf{3.05})$$

If S, S_s are the flux densities in watts $\mathrm{m}^{-2}\,(\mathrm{c/s})^{-1}$ then

$$S = \frac{I}{\nu - \nu'} = \frac{I\lambda\lambda'}{c(\lambda' - \lambda)},$$

and so

$$S/S_s = I/I_s, \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (\mathbf{3.06})$$

for the same range of wavelength for the standard source as for the other.

The factors on the right-hand side of (**3.05**) will now be expanded in power series in δ and in δ_s, except for the factor involving the ratio of luminosity-distances. It is convenient to do this for $\ln(I/I_s)$ in the first instance, and we find, after some calculation,

$$\ln\frac{C(\delta)}{C(\delta_s)} = C_1(\delta - \delta_s) + (C_2 - C_1^2)(\delta^2 - \delta_s^2)/2 + \ldots,$$

$$\ln\frac{(1+\delta_s)^{p(\delta_s)}}{(1+\delta)^{p(\delta)}} = -p_0(\delta - \delta_s) - p_0(2p_1 - 1)(\delta^2 - \delta_s^2)/2 - \ldots,$$

$$\ln\frac{p(\delta_s)}{p(\delta)} = -p_1(\delta - \delta_s) - \frac{(p_2 - p_1^2)}{2}(\delta^2 - \delta_s^2) - \ldots,$$

$$\ln\frac{\lambda'^{p(\delta)} - \lambda^{p(\delta)}}{\lambda'^{p(\delta_s)} - \lambda^{p(\delta_s)}} = l_1(\delta - \delta_s) + (l_2 - l_1^2)(\delta^2 - \delta_s^2)/2 + \ldots,$$

where

$$\left.\begin{aligned} l_1 &= p_0 p_1 \frac{\lambda'^{p_0}(\ln \lambda') - \lambda^{p_0}(\ln \lambda)}{\lambda'^{p_0} - \lambda^{p_0}}, \\ l_2 &= \{p_0 p_2(\lambda'^{p_0}(\ln \lambda') - \lambda^{p_0}(\ln \lambda) + \\ &\qquad p_0^2 p_1^2(\lambda'^{p_0}(\ln \lambda')^2 - \lambda^{p_0}(\ln \lambda)^2)\}/(\lambda'^{p_0} - \lambda^{p_0}). \end{aligned}\right\} \quad .. \quad (\mathbf{3.07})$$

Thus (**3.05**) is

$$\ln (I/I_s) = -2 \ln (D/D_s) - 2b_1(\delta - \delta_s) + b_2(\delta^2 - \delta_s^2) + \ldots,$$

where

$$b_1 = (p_0 + p_1 - C_1 - l_1)/2, \quad \ldots\ldots \quad (\mathbf{3.08})$$

$$b_2 = (p_0 - 2p_0 p_1 + p_1^2 - p_2 - C_1^2 + C_2 - l_1^2 + l_2)/2. \quad \ldots\ldots \quad (\mathbf{3.09})$$

Hence it follows that

$$D \exp (b_1 \delta - b_2 \delta^2/2) = D_s \exp (b_1 \delta_s - b_2 \delta_s^2/2) \,.\, (I_s/I)^{\frac{1}{2}}, \quad .. \quad (\mathbf{3.10})$$

where the exponentials are to be expanded to the order δ^2 or δ_s^2 only. Define a variable y by

$$\begin{aligned} y &= (I_s/I)^{\frac{1}{2}} D_s \exp (b_1 \delta_s - b_2 \delta_s^2/2) \\ &= (I_s/I)^{\frac{1}{2}} D_s \{1 + b_1 \delta_s + (b_1^2 - b_2)\delta_s^2/2\}, \end{aligned} \quad \ldots\ldots \quad (\mathbf{3.11})$$

and then (**3.10**) is

$$y = D\{1 + b_1 \delta + (b_1^2 - b_2)\delta^2/2 + \ldots\}. \quad \ldots\ldots \quad (\mathbf{3.12})$$

But now by (**2.11**) we obtain

$$y = D\{1 + b_1(h_1 D/c) + (b_1^2 - b_2 - b_1 h)(h_1 D/c)^2/2 + \ldots\}, \quad .. \quad (\mathbf{3.13})$$

and so inverting this series by successive approximations

$$D = y\{1 - b_1(h_1 y/c) + (3b_1^2 + b_2 + b_1 h)(h_1 y/c)^2/2 + \ldots\}, \quad .. \quad (\mathbf{3.14})$$

which expresses D as a power series in the ratio of flux-densities parameter y. Introducing the constant A by

$$10^A = \frac{4\pi\alpha}{3QR_0^3} D_s^3 \{1 + b_1 \delta_s + (b_1^2 - b_2)\delta_s^2/2\}^3,$$

and taking the logarithmic forms of (**2.12**) and (**3.14**), the number of radio sources with flux density ratios up to y are given by

$$\ln N = \ln 10^A + 3 \ln (I_s/I)^{\frac{1}{2}} - 3(1 + b_1)(h_1 y/c) + 3b_3(h_1 y/c)^2 + \ldots,$$

or, if $E = \ln 10 = 2 \cdot 303$,

$$\log N = A + 3 \log \left(\frac{I_s}{I}\right)^{\frac{1}{2}} - \frac{3}{E}\{(1 + b_1)(h_1 y/c) - b_3(h_1 y/c)^2 + \ldots\}, \quad .. \quad (\mathbf{3.15})$$

where, using (**2.13**) also,

$$b_3=\tfrac{1}{2}+b_1^2+b_1+\tfrac{1}{2}b_2+\tfrac{1}{2}h(1+b_1)+\frac{1}{10}\,\frac{c^2k}{R_0^2h_1^2}. \qquad \textbf{(3.16)}$$

The non-logarithmic form of (**3.15**) is

$$N=10^A(I_s/I)^{3/2}\left[1-3(1+b_1)(h_1y/c)+3\left\{b_3+\frac{3}{2}(1+b_1)^2\right\}(h_1y/c)^2+\ldots\right], \qquad \textbf{(3.17)}$$

and (**3.15**) and (**3.17**) are the final forms of the formulae for log N and for N respectively.

In actual computations, the relation (**3.06**) is used to calculate (I_s/I) through the flux densities measured in watts m^{-2} $(\mathrm{c/s})^{-1}$. Terms in h_1y/c may be computed from (**3.11**), (**2.09**), (**2.14**), and (**3.06**) which give

$$h_1y/c=(S_s/S)^{\frac{1}{2}}\delta_s \quad \text{(first approximation)}, \qquad \textbf{(3.18)}$$

$$=(S_s/S)^{\frac{1}{2}}\delta_s\{1+(2b_1-q_0+1)\delta_s/2\} \quad \text{(second approximation)}. \qquad \textbf{(3.19)}$$

Again, if equation (**3.10**) is multiplied by h_1/c and (**2.09**), (**2.14**), and (**3.06**) are used, we obtain

$$\delta_s\{1+(2b_1-q_0+1)\delta_s/2\}(S_s/S)^{\frac{1}{2}}=\delta\{1+(2b_1-q_0+1)\delta/2+\ldots\}, \qquad \textbf{(3.20)}$$

an equation that can be used to calculate the red-shift δ corresponding to S. But this is possible only if the series that have been employed are rapidly converging and this need not be so if δ is large. In this connexion, it should be noted that in special and general relativity the relative velocity of source and observer tends to c as δ tends to infinity (Dingle 1950). This is in contrast to the classical case where a relative velocity c corresponds to $\delta=1$.

If all terms on the right-hand side of (**3.15**) after the second are omitted, and (**3.06**) is also used, we obtain

$$\log N=B-3/2\log S, \qquad \textbf{(3.21)}$$

where B involves S_s, D_s, and δ_s. This equation states the " $-3/2$ law " for the distribution of radio sources provided that it is assumed that the standard source is the same for all sources, i.e. that its flux density does not depend on S. The omission of the terms in (**3.15**) is justifiable in two quite different ways. Firstly, it may be supposed that h_1y/c, $h_2(y/c)^2$, etc. are negligibly small, which means that the red-shifts and the other parameters involving the motions of the sources are regarded as being small. In addition, as the last term of (**3.16**) demonstrates, powers of ky^2/R_0^2 are to be regarded as negligible. This is equivalent to asserting either that $k=0$ (Euclidean space) or, if k equals $+1$ or -1 (spherical or hyperbolic space), that the square and higher powers of the ratios of the luminosity-distances of the sources to R_0 are small. Thus this method of obtaining (**3.21**) from (**3.15**) implies that the sources are all, cosmically speaking, near to the observer and so moving slowly in a model universe (**2.01**) where R_0 is large.

But there is a second way in which the " $-3/2$ law" can be obtained. It has already been noticed that, in the infinite series such as (**3.12**) or (**3.15**), the coefficient of each successive term contains a higher derivative of R, calculated at the instant t_0, than the preceding one. Of these derivatives, the first and second are determinable from the presently available data on optical red-shifts and both these derivatives occur in the coefficients of the second-order terms of the series. Thus *one* of these series could be selected and the coefficients of the third and higher order terms could be equated to zero by a suitable choice of the third and higher order derivatives of R. In addition, in either (**3.15**) or (**3.17**), the coefficient of the second-order term could be made to vanish by a suitable choice of the constant $c^2k/(10R_0^2h_1^2)$, provided that the condition (**2.17**) were not thereby violated. Thus a " first-order log N model universe" will be defined as one in which, by a suitable choice of the derivatives of R combined with $b_3=0$, formula (**3.15**) becomes exactly

$$\log N = A + 3 \log (I_s/I)^{\frac{1}{2}} - \frac{3}{E}(1+b_1)(h_1 y/c)\,; \qquad \ldots\ldots\ldots\ldots \quad (\mathbf{3.22})$$

and a " second-order log N model universe" will be one in which the formula becomes exactly ($b_3 \neq 0$)

$$\log N = A + 3 \log (I_s/I)^{\frac{1}{2}} - \frac{3}{E}(1+b_1)(h_1 y/c) + \frac{3b_3}{E}(h_1 y/c)^2. \quad \ldots \quad (\mathbf{3.23})$$

Similarly a " first-order N model universe" would be one in which (**3.17**) would reduce exactly to

$$N = 10^A (I_s/I)^{3/2}\{1 - 3(1+b_1)(h_1 y/c)\}, \qquad \ldots\ldots\ldots\ldots \quad (\mathbf{3.24})$$

with a corresponding definition for a second-order N model. This reduction of one series to a simple form does not also reduce all the other series to a finite number of terms. For example, in a first-order log N model ($b_3=0$), though (**3.15**) has become (**3.22**), it does not follow that the coefficients of the terms of order greater than the first in (**3.17**) vanish also. Now suppose that, in a first-order log N model (**3.22**), it turns out that the constant b_1 is approximately equal to -1. Then this model will also reproduce the " $-3/2$ law" (**3.21**) even though the red-shifts of the sources may be large and the constant R_0 not necessarily large. These two ways of obtaining (**3.21**) indicate that the observational determination of a " $-3/2$ law" for a set of sources tells us by itself very little either about the nature of their motions relative to the observer or about the curvature of space.

In a first-order log N model, $b_3=0$, and so the formula for determining the sign of k is, by (**2.14**) and (**3.16**),

$$c^2k/R_0^2 = 5h_1^2\{(1+b_1)q_0 - 2 - 2b_1^2 - 3b_1 - b_2\}, \qquad \ldots\ldots \quad (\mathbf{3.25})$$

and then by (**2.15**) and (**2.16**) we also have

$$\Lambda=\{(3+5b_1)q_0-9-10b_1^2-15b_1-5b_2\}h_1^2, \quad .. \quad (\mathbf{3.26})$$

$$4\pi G\rho_0=\{(6+5b_1)q_0-9-10b_1^2-15b_1-5b_2\}h_1^2, \quad .. \quad (\mathbf{3.27})$$

from which the value of the cosmical constant and of the present value of the density could be calculated given h_1, b_1, b_2, and q_0. Since the density must be positive we must have

$$(6+5b_1)q_0-10b_1^2-15b_1-5b_2>9. \quad \ldots\ldots\ldots\ldots \quad (\mathbf{3.28})$$

By (**3.06**) and (**3.11**), log y^{-2} differs from log S only by a constant, and so from (**3.15**) we find that

$$\frac{\mathrm{d}\log N}{\mathrm{d}\log S}=\frac{\mathrm{d}\log N}{\mathrm{d}\log y^{-2}}=-\frac{y}{2}\frac{\mathrm{d}\ln N}{\mathrm{d}y}$$

$$=-\frac{3}{2}\{1-(1+b_1)(h_1y/c)+2b_3(h_1y/c)^2+.\ .\ .\}. \quad .. \quad (\mathbf{3.29})$$

Thus the terms in this power series are proportional to those of the series in (**3.15**). Hence, in a first-order log N model using (**3.06**) and (**3.18**) also,

$$\frac{\mathrm{d}\log N}{\mathrm{d}\log S}=-\frac{3}{2}\{1-(1+b_1)\delta_s(S_s/S)^{\frac{1}{2}}\}. \quad \ldots\ldots\ldots\ldots\ldots \quad (\mathbf{3.30})$$

IV. Comparison with Observation

At the present time the observational data that could be used in combination with the foregoing formulae are of a preliminary nature. Nevertheless it is useful for illustrative purposes to discuss them and we begin with those of Mills and Slee (1957) on counts of radio sources of Class II (extragalactic), which we reproduce in columns 1 and 2 of Table 1. As Mills and Slee point out, the

Table 1

Observational data, Class II sources (Mills and Slee)

Category	Range of S (W m^{-2} (c/s)$^{-1}$) (1)	No. of Sources in Range (2)	Limit of S (3)	N (4)
(*a*)	$<10\times10^{-26}$	42	7×10^{-26}	311
(*b*)	10– 19·9	177	10	269
(*c*)	20– 39·9	68	20	92
(*d*)	40– 79·9	19	40	24
(*e*)	80–159·9	5	80	5

weakest category (*a*) is very incomplete and the strongest category (*e*) is probably statistically deficient. In column 3 are given the lower limits of the ranges of S; in the Mills and Slee catalogue, the Class II sources are, unfortunately, not separately indicated but, taking all sources, galactic and extragalactic, it seems likely that 7×10^{-26} W m^{-2} (c/s)$^{-1}$ is the lower limit for category (*a*). In column 4 are the cumulative totals for the sources listed in column 2. Suppose that we wish to interpret the data in columns 3 and 4 by means of a first-order log N model. With the aid of (**3.06**) and (**3.18**) we can write (**3.15**) as

$$A-\frac{3}{E}\{\delta_s(S_s/S)^{\frac{1}{2}}\}b_1=\log N-3\log(S_s/S)^{\frac{1}{2}}+\frac{3}{E}\delta_s(S_s/S)^{\frac{1}{2}}. \quad \ldots\ldots \quad (\mathbf{4.01})$$

To make use of this formula, however, a standard source must be selected and this will be done in two different ways as follows.

(a) NGC 1275 (IAU 03N4A) as Standard (Mills and Slee)

Let it be assumed that all the Class II sources have, on the average, the same $C(\delta)$ and $p(\delta)$ as this pair of colliding galaxies. The optical red-shift is $\delta_s=0\cdot018$ (McVittie 1957) and the flux density measured with the Sydney

TABLE 2

NGC 1275 AS STANDARD (MILLS AND SLEE)

Category	S	N	log N	$(S_s/S)^{\frac{1}{2}}$	$3\log(S_s/S)^{\frac{1}{2}}$	$\frac{3}{E}\delta_s(S_s/S)^{\frac{1}{2}}$
(*a*)	7×10^{-26}	311	2·49	5·86	2·30	0·14
(*b*)	10	269	2·43	4·90	2·07	0·12
(*c*)	20	92	1·96	3·46	1·62	0·08
(*d*)	40	24	1·38	2·45	1·17	0·06
(*e*)	80	5	0·70	1·73	0·72	0·04

instrument is $S_s=240\times10^{-26}$ W m^{-2} (c/s)$^{-1}$ (Mills 1952). The relevant items in the computations of the terms involving S and N in (**4.01**) are shown in Table 2. The equations of condition for A and b_1 are shown in Table 3.

TABLE 3

EQUATIONS FOR A AND b_1

Category	Equation	A for $b_1=-0\cdot1$	A for $b_1=-1\cdot12$
(*a*)	$A-0\cdot1419b_1=0\cdot3321$	0·32	0·17
(*b*)	$A-0\cdot1187b_1=0\cdot4782$	0·47	0·35
(*c*)	$A-0\cdot0839b_1=0\cdot4289$	0·42	0·33
(*d*)	$A-0\cdot0593b_1=0\cdot2723$	0·27	0·21
(*e*)	$A-0\cdot0420b_1=0\cdot0252$	0·02	$-0\cdot02$

A least squares solution of equations (*a*) to (*d*) indicates that $A=0\cdot27$ and $b_1=-1\cdot12$ and therefore (**4.01**) becomes

$$\log N=0\cdot27-\frac{3}{2}\log(S/S_s)+0\cdot003(S_s/S)^{\frac{1}{2}}, \quad \ldots\ldots \quad (\mathbf{4.02})$$

which is shown in graphical form as curve I in Figure 1. The physical significance of this result will be discussed below. If we assume that $b_1=-0\cdot1$, and that

$\log N=1\cdot 0$ corresponds to $\log (S_s/S)=0\cdot 5$ (point X of Fig. 1) then, in place of (**4.02**), we find

$$\log N=0\cdot 29-\frac{3}{2}\log (S/S_s)-0\cdot 022(S_s/S)^{\frac{1}{2}}, \quad \ldots\ldots \quad (\mathbf{4.03})$$

which gives the curve II of Figure 1. In Figure 1 the points for categories (*a*) to (*e*) are shown for the values of S/S_s deduced from the fifth column of Table 2. As was to be expected, the category (*e*) does not fit either curve satisfactorily.

In view of the uncertainties in the data it would be difficult to assert that either of the curves I or II is to be preferred to the other. Since, for curve I, b_1 lies fairly close to -1, the formula (**4.02**) closely mimics the " $-3/2$ law " (**3.21**) and indeed this is the reason, on the present interpretation, why Mills

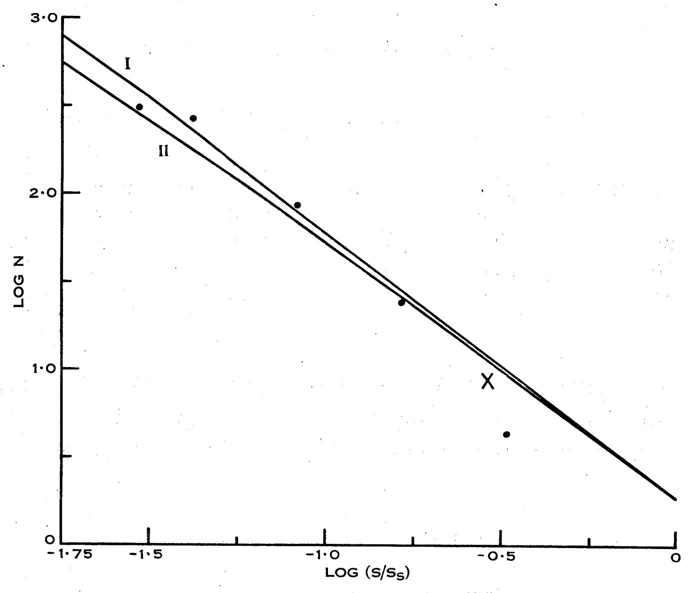

Fig. 1.—Plot of $\log N$ against $\log (S/S_s)$: Sydney catalogue with NGC 1275 as standard source.

and Slee regard their data as in accord with this law. But from the physical point of view the two curves have widely different implications. Considering curve II first, for which $b_1=-0\cdot 1$, it is to be noticed that, if there are no secular changes in the radiative properties of the sources, then in (**3.02**) and (**3.03**), C and p are simply the constants C_0 and p_0. Further, by (**3.07**), (**3.08**), and (**3.09**), it follows that

$$b_1=b_2=p_0/2, \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (\mathbf{4.04})$$

and therefore, by (**3.04**), $0\geqslant b_1\geqslant -0\cdot 2$. Thus curve II, or formula (**4.03**), represents the case of no secular changes in the radiative properties of the sources. The condition (**3.28**) for the positiveness of the density is verified for the range

$0 \geqslant b_1 \geqslant -0\cdot 2$ and $b_1 = b_2$, provided that $q_0 > 1\cdot 5$. The space-curvature constant k has an indeterminate value in this model, since we have from (**3.25**)

$$\begin{aligned} c^2k/R_0^2 &= 5h_1^2(0\cdot 8q_0 - 1\cdot 28), &&\text{if } b_1 = b_2 = -0\cdot 2, \\ &= 5h_1^2(q_0 - 2), &&\text{if } b_1 = b_2 = 0, \end{aligned}$$

and therefore k depends critically on the value of q_0 that is adopted, in the range $1\cdot 5 \leqslant q_0 \leqslant 5\cdot 5$. Assuming that δ can be calculated from the first approximation to (**3.20**), it follows that the red-shift for the weakest sources (a) of Table 2 is of the order of $0\cdot 1$, i.e. that their distances lie between that of the Corona Borealis ($\delta = 0\cdot 07$) and of the Boötes cluster ($\delta = 0\cdot 13$) of galaxies.

Turning next to curve I, or formula (**4.02**), it is no longer possible to avoid the hypothesis of secular changes, since (**4.04**) is inconsistent with (**3.04**) when $b_1 = -1\cdot 12$. The simplest interpretation is now to assume that p is independent of δ, and therefore equal to p_0, and that C is a linear function of δ. This means that, in (**3.02**) and (**3.03**)

$$\begin{aligned} C_1 \neq 0, \quad C_n &= 0 \quad (n = 2, 3, 4, \ldots), \\ p_n &= 0 \quad (n = 1, 2, 3, \ldots), \end{aligned}$$

and therefore, by (**3.07**), (**3.08**), and (**3.09**),

$$b_1 = (p_0 - C_1)/2, \quad b_2 = (p_0 - C_1^2)/2. \qquad \ldots\ldots\ldots (\mathbf{4.05})$$

For the sake of definiteness, let p_0 have the mean value for the range (**3.04**), namely $p_0 = -0\cdot 2$. Then $b_1 = -1\cdot 12$ and (**4.05**) give

$$C_1 = 2\cdot 04, \quad b_2 = -2\cdot 18. \qquad \ldots\ldots\ldots\ldots (\mathbf{4.06})$$

Thus

$$C(\delta) = C_0(1 + 2\cdot 04\delta),$$

and therefore, even for $\delta = 0\cdot 1$, C/C_0 is $1\cdot 2$ which would mean that even the strongest of the sources of Table 2 were, at the moment their radiation left them, emitting some 20 per cent. more powerfully than is NGC 1275 at present. Since the time of travel of the radiation from a source for which $\delta = 0\cdot 1$ is of the order of 400 million years, it would be necessary to suppose that colliding galaxies of the type of NGC 1275 were subject to a very rapid—cosmically speaking—attenuation in their radiative properties.

The condition (**3.28**) for $b_1 = -1\cdot 12$, $b_2 = -2\cdot 18$ is satisfied for any positive q_0; the space-curvature formula (**3.25**) gives

$$c^2k/R_0^2 = 5h_1^2(-0\cdot 12q_0 + 1\cdot 03),$$

a result which suggests, rather inconclusively, that k is positive for q_0 lying in the range $1\cdot 5 \leqslant q_0 \leqslant 5\cdot 5$, so that space is spherical.

The rapid rate of diminution in the strength of a source implied by curve I (formula (**4.02**)) seems rather implausible; until observation definitely disproves it, it would be better to accept curve II (formula (**4.03**)) in spite of the fact that it departs more from the " $-3/2$ law " than does the other curve. But secular changes in radiative properties are avoided and all sources are regarded as similar to NGC 1275. All the 311 sources are relatively close to our Galaxy, even those of category (*a*) being less remote than is the Boötes cluster of galaxies. If they are all indeed NGC 1275 type galaxies it is strange that so few of them have been identified with optical objects.

(*b*) *Cygnus A* (*IAU 19N4A*) *as Standard* (*Mills and Slee*)

Alternatively let it be assumed that the radio sources of Table 1 have, on the average, the same $C(\delta)$ and $p(\delta)$ as the pair of colliding galaxies called Cygnus A, for which (McVittie 1957 ; Pawsey, personal communication 1957)

$$S_s=19{,}000\times10^{-26}\ \mathrm{W\ m^{-2}\ (c/s)^{-1}},$$
$$\delta_s=0\cdot056,$$

and that we again use a first-order log N model. We adopt (**4.01**) again and this assumes that (**3.18**) is still a valid approximation for h_1y/c. Then Tables 2 and 3 are replaced by Tables 4 and 5 respectively.

TABLE 4

CYGNUS A AS STANDARD (MILLS AND SLEE)

Category	S	N	log N	$(S_s/S)^{\frac{1}{2}}$	3 log $(S_s/S)^{\frac{1}{2}}$	$\frac{3}{E}\delta_s(S_s/S)^{\frac{1}{2}}$
(*a*)	7×10^{-26}	311	2·49	52·10	5·15	3·80
(*b*)	10	269	2·43	43·59	4·92	3·18
(*c*)	20	92	1·96	30·82	4·47	2·25
(*d*)	40	24	1·38	21·79	4·01	1·59
(*e*)	80	5	0·70	15·41	3·56	1·12

TABLE 5

EQUATIONS FOR A AND b_1

Category	Equation	A for $b_1=-0\cdot1$	A for $b_1=-1\cdot07$
(*a*)	$A-3\cdot801b_1=+1\cdot143$	$+0\cdot76$	$-2\cdot92$
(*b*)	$A-3\cdot180b_1=+0\cdot692$	$+0\cdot37$	$-2\cdot71$
(*c*)	$A-2\cdot248b_1=-0\cdot255$	$-0\cdot48$	$-2\cdot66$
(*d*)	$A-1\cdot590b_1=-1\cdot045$	$-1\cdot20$	$-2\cdot75$
(*e*)	$A-1\cdot124b_1=-1\cdot740$	$-1\cdot85$	$-2\cdot94$

A least squares solution gives $A=-2\cdot8$, $b_1=-1\cdot07$ and therefore (**4.01**) becomes

$$\log N=-2\cdot8-\frac{3}{2}\log(S/S_s)+0\cdot005(S_s/S)^{\frac{1}{2}}. \quad \ldots. \quad (\mathbf{4.07})$$

If we choose $b_1=-0\cdot1$ and make log $N=0\cdot75$ at log $(S/S_s)=-2\cdot375$ (point X on Fig. 2), then (**4.01**) reads

$$\log N=-1\cdot75-\frac{3}{2}\log(S/S_s)-0\cdot066(S_s/S)^{\frac{1}{2}}. \quad \ldots. \quad (\mathbf{4.08})$$

It is clear from Figure 2 that curve I, which represents (**4.07**), fits the data, but that curve II (formula (**4.08**)) is quite unacceptable. Thus secular changes cannot now be avoided. Adopting the method that led up to (**4.06**) through (**4.05**), we now find that

$$C_1=1\cdot 94, \quad b_2=-1\cdot 98, \qquad \text{(4.09)}$$

from which it is easy to verify that (**3.28**) is satisfied for all positive q_0. From (**3.25**) we obtain

$$c^2k/R_0^2=-5h_1^2(0\cdot 07q_0+0\cdot 10),$$

a result which, again with very low weight, suggests that k is negative and that space is hyperbolic. The values of $(S_s/S)^{\frac{1}{2}}$ in Table 4 are now so large that the

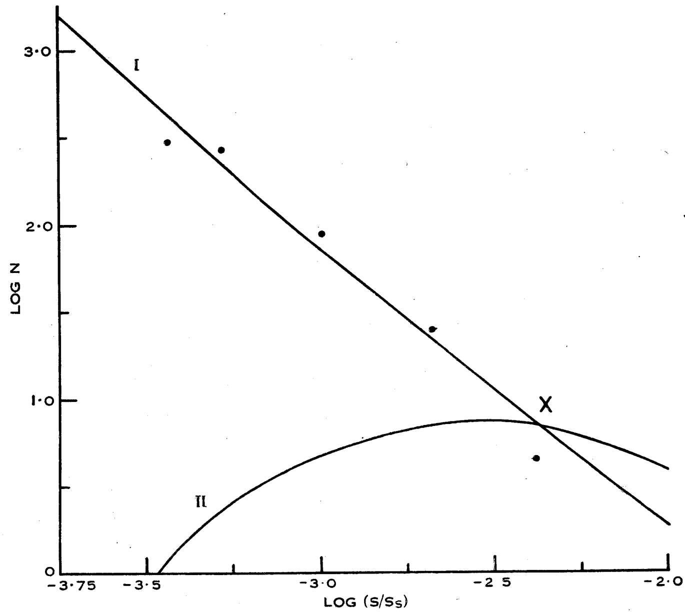

Fig. 2.—Plot of log N against log (S/S_s) : Sydney catalogue with Cygnus A as standard source.

red-shifts can no longer be calculated from (**3.20**) ; higher order terms would have to be taken into account. However, if in this formula we set $\delta_s=0\cdot 056$, $b_1=-1\cdot 07$, $q_0=2\cdot 3$, and $(S_s/S)^{\frac{1}{2}}=5$, we find $\delta=0\cdot 36$, which almost equals the greatest red-shift ($\delta=0\cdot 4$) detected by Baum (1957) by photoelectric means. But even for the strongest sources (e) of Table 4, $(S_s/S)^{\frac{1}{2}}$ is over three times as large as that needed to give $\delta=0\cdot 36$. Thus one may speculate that the red-shift for these sources would be of the order of unity. With this interpretation of the data it is not to be expected that optical identifications of the radio sources would have been made. Since, by (**4.09**),

$$C(\delta)=C_0(1+1\cdot 94\delta),$$

it follows that for $\delta=1\cdot 0$, C/C_0 is nearly equal to three. Thus it would seem that even the strongest sources of Table 4 must be assumed to have been radiating, at the time of emission of the radiation by which they are now observed, nearly three times as strongly as is Cygnus A. Hence we have in an acuter form the situation that confronted us when NGC 1275 was taken as standard source and Curve I of Figure 1 was taken as representing the data.

(c) *NGC 1275 (IAU 03N4A) as Standard (Shakeshaft* et al.)

In the Cambridge catalogue (Shakeshaft *et al.* 1955) the radio sources that are reckoned to be extragalactic are also not individually indicated. But it is possible to identify them by the prescription given, viz. they are the radio sources less than 20′ in angular diameter and lying outside the band of $\pm 12°$ in galactic latitude. The present writer's analysis of the catalogue leads to the results displayed in Table 6 for the indicated limits of S and with the flux

TABLE 6

NGC 1275 AS STANDARD (SHAKESHAFT ET AL.)

Category	S	N	$\log N$	$(S_s/S)^{\frac{1}{2}}$	$3\log\ (S_s/S)^{\frac{1}{2}}$	$\frac{3}{E}\delta_s(S_s/S)^{\frac{1}{2}}$
(*a*)	7×10^{-26}	1601	3·20	4·80	2·043	0·116
(*b*)	20	1108	3·04	2·84	1·359	0·069
(*c*)	40	257	2·41	2·01	0·907	0·049
(*d*)	80	29	1·46	1·42	0·456	0·034
(*e*)	160	5	0·70	1·00	0·004	0·024

density $S_s=161\times10^{-26}$ W m^{-2} (c/s)$^{-1}$ obtained with the Cambridge instrument for NGC 1275. The red-shift of this standard source is still, of course, $\delta_s=0\cdot018$. The equations of condition for A and b_1, assuming that the data in Table 6 can be represented by (**4.01**), are

$$\begin{aligned}
&(a)\quad A-0\cdot1162b_1=1\cdot2780,\\
&(b)\quad A-0\cdot0687b_1=1\cdot7545,\\
&(c)\quad A-0\cdot0486b_1=1\cdot5514,\\
&(d)\quad A-0\cdot0344b_1=1\cdot0412,\\
&(e)\quad A-0\cdot0243b_1=0\cdot7192.
\end{aligned}$$

A least squares solution gives

$$A=0\cdot97,\quad b_1=-5\cdot06, \qquad \text{............} \quad (\mathbf{4.10})$$

and therefore (**4.01**) becomes

$$\log N=0\cdot97-\frac{3}{2}\log\ (S/S_s)+0\cdot098(S_s/S)^{\frac{1}{2}}, \qquad \text{......} \quad (\mathbf{4.11})$$

the graph of which is curve I in Figure 3. The observational points for the five categories of Table 6 are also shown on this figure. Curve II of Figure 3 was obtained by inspection, ignoring the observational point for category (*a*); it has

$$A=0\cdot62,\quad b_1=-12\cdot46, \qquad \text{............} \quad (\mathbf{4.12})$$

and its equation is

$$\log N = 0 \cdot 62 - \frac{3}{2} \log (S/S_s) + 0 \cdot 277 (S_s/S)^{\frac{1}{2}}. \quad \ldots\ldots \quad (\mathbf{4.13})$$

The enormous negative values of b_1 in (**4.10**) and (**4.12**) show that the Cambridge data imply, on the present interpretation, that the strength of the radio sources at emission greatly exceeds that of NGC 1275. If it is permissible to use (**3.20**) to calculate the red shift with

$$b_1 = -5 \cdot 06, \quad q_0 = 2 \cdot 3, \quad \delta_s = 0 \cdot 018,$$

then δ for category (*a*) again turns out to be of the order of $0 \cdot 1$ and we thus have even more rapid secular variations in radiative properties than were found in

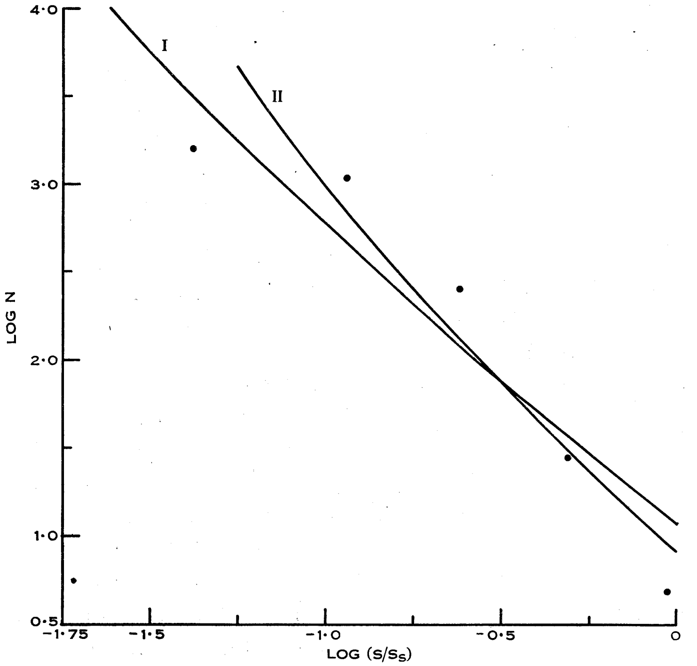

Fig. 3.—Plot of log N against log (S/S_s) : Cambridge catalogue with NGC 1275 as standard source.

Section IV (*a*) above. It hardly seems necessary to discuss the Cambridge data with Cygnus A as standard source, since still smaller values of b_1 would be expected to arise.

The upshot of this discussion of the Sydney and the Cambridge data is that, if a first-order log N model is used together with the hypothesis that the radio sources were radiating more powerfully in the past than they are now doing, the former can be made to fit the " −3/2 law ". If the standard source is no more powerful than is NGC 1275 at present, the hypothesis of intensified radiation in the past can perhaps be discarded (curve II, Fig. 1). But the hypothesis must be retained in an acute form for the Cambridge data, even if

NGC 1275 is the standard source. Indeed it is better to conclude that the present theory is hardly applicable to the Cambridge results at all.

It is also worth remarking that, when $b_1 < -1$, equation (**3.30**) indicates that the slope of the $\log N$ versus $\log S$ curve decreases from the value $-3/2$ as soon as $(S_s/S)^{\frac{1}{2}}$ becomes sufficiently large. This effect is barely noticeable for the curves in Figure 3 and is much less pronounced than for the curve drawn by Ryle and Scheuer (1955) through the observational points of Figure 3. This curve has a slope of about $-1\cdot 5$ for the strongest, and of $-2\cdot 5$ for the weakest, sources.

V. Alternative Treatments

In the present paper the observational fact that the counts of radio sources follow a " $-3/2$ law " has been interpreted by assuming that the space-density parameter α of (**2.08**) is constant but that secular changes in the radiative properties of the sources are taking place. The following variations on the treatment suggest themselves.

(*a*) The space-density of radio sources is determined by the parameter α in (**2.08**). This must be distinguished from the gravitational density of matter which, for a model universe (**2.01**), is given by the quantity ρ of (**2.02**). The latter is a function of the time t; the former has been assumed to be a constant. But it would be possible to suppose that α also was a function of t and to include $\alpha(t)$ under the sign of integration in (**2.08**). If this were done, each value of t in the integrand of (**2.08**) would be related to a corresponding r by the null-geodesic equation (**2.04**). Using the method of expansions for the function $\alpha(t)$ in the same way as has been done for $R(t)$ (GRC Section 9.1), and then integrating the right-hand side of (**2.08**), a series expansion for N analogous to (**3.17**) would be obtained. The functions C and p in (**3.01**) could be regarded either as constant or as variable with time. A time-varying α would thus include the possibility that collisions between galaxies—assuming that the radio sources are indeed colliding galaxies—were more frequent in the past than they are at present.

(*b*) Assuming again that α is independent of t, it would be desirable to take into account the dispersion in absolute flux density between the radio sources. The analogy here is with the dispersion in the absolute magnitudes, M, of galaxies. Since the absolute flux density of a sufficient number of radio sources is at present unknown, the analogue of the optical luminosity function is not available. However, some idea of what would happen if it were might be reached in the following way. Equation (**3.15**) can also be written in the form

$$\log N = A' + 3\log y - \frac{3}{E}\{(1+b_1)(h_1 y/c) - b_3(h_1 y/c)^2 + \ldots\}, \quad \ldots \quad (\mathbf{5.01})$$

where

$$A' = \log(4\pi\alpha/3QR_0^3),$$

and y is given by (**3.11**). Now suppose that the number of radio sources is not very large, as in the Mills and Slee catalogue, for example, in which there are

311. Assigning the numbers 1 to 311 to the sources it would be possible with the aid of a table of random numbers to arrange them in a random sequence, e.g. *123*, 233, 32, 111, *301*, 79, 302, 221, *17*, . . . As an illustration, suppose we assert arbitrarily that there are three times as many NGC 1275 sources as there are Cygnus A sources. Then we could regard sources 123, 301, 17, . . . as the Cygnus A sources whilst sources 233, 32, 111, 79, 302, 221, . . . would be NGC 1275 sources. In each class, the y for each source could be calculated by (**3.11**) assuming that I_s and D_s for the standard source of the class were known, and that its δ_s were small enough to be neglected in (**3.11**). Counting cumulative totals of sources of *both classes* to suitable limits of y, the resulting data could be interpreted in terms of (**5.01**). The process could be repeated for various proportions of types of sources and with more than two standard sources.

A weakness in a treatment of this kind lies in the fact that the luminosity-distances D_s of the standard sources must be known. Whereas positive statements regarding the distances of, for instance, NGC 1275 and Cygnus A are to be found in the literature (e.g. Shakeshaft 1954), these values do not stand up to critical analysis (McVittie 1957). We may regard ourselves as fortunate if the uncertainty in the distances is less than a factor of two. The same objection applies to the determination of the proportions of standard sources of different types by estimates depending on the number of such sources per cubic parsec. Not only are the distances uncertain but the nature of space is also unknown, a spherical space containing a lesser volume for a given luminosity-distance than does a hyperbolic.

Though methods (*a*) and (*b*) could be applied to the present data if the labour of the algebraic and numerical computations involved was thought to be worth while, such extensive work seems to be premature. An overriding preliminary is the harmonizing of the Sydney and Cambridge catalogues which, as the results of Section IV have shown, contain discrepancies so important that the two catalogues cannot be used together. Until this question is settled it seems profitless to embark on further elaborations of the theory given in the present paper.

VI. Acknowledgments

I should like to express my thanks to Dr. J. L. Pawsey and to Dr. M. Ryle for the valuable comments and criticisms which they sent me regarding the original draft of this paper.

VII. References

Adgie, R., and Smith, F. G. (1956).—*Observatory* **76** : 181.

Baldwin, J. E., and Dewhirst, D. W. (1954).—*Nature* **173** : 164.

Baum, W. A. (1957).—*Astr. J.* **62** : 6.

Bondi, H. (1952).—" Cosmology." (Cambridge Univ. Press.)

Dingle, H. (1950).—" The Special Theory of Relativity." 3rd Ed. p. 50. (Methuen : London.)

Hoyle, F., and Sandage, A. R. (1956).—*Publ. Astr. Soc. Pacif.* **68** : 301.

Humason, M. L., Mayall, N. U., and Sandage, A. R. (1956).—*Astr. J.* **61** : 97.

Lilley, A. E., and McClain, E. F. (1956).—*Astrophys. J.* **123** : 172.

McVittie, G. C. (1956).—" General Relativity and Cosmology." (Chapman & Hall : London.)

McVittie, G. C. (1957).—"Handbuch der Physik." Vol. 53. (Springer-Verlag: Berlin-Gottingen-Heidelberg.) (In press.)

Mills, B. Y. (1952).—*Aust. J. Sci. Res.* A **5**: 266.

Mills, B. Y., and Slee, O. B. (1957).—*Aust. J. Phys.* **10**: 162.

Minkowski, R., and Wilson, O. C. (1956).—*Astrophys. J.* **123**: 373.

Pawsey, J. L. (1955).—Jodrell Bank Symposium, No. 360.

Ryle, M. (1955).—*Observatory* **75**: 137.

Ryle, M., and Scheuer, P. A. G. (1955).—*Proc. Roy. Soc.* A **230**: 448.

Shakeshaft, J. R. (1954).—*Phil. Mag.* **45**: 1136.

Shakeshaft, J. R., Ryle, M., Baldwin, J. E., Elsmore, B., and Thomson, J. H. (1955).—*Mem. R. Astr. Soc.* **47**: 97.