GALACTIC TURBULENCE AND THE ORIGINS OF COSMIC RAYS AND THE GALACTIC MAGNETIC FIELD

By J. H. Piddington*

[*Manuscript received April* 4, 1957]

Summary

A new absorption mechanism is discussed in relation to the turbulence of the interstellar gas in the presence of a magnetic field. It is evaluated and compared with ordinary viscous absorption in different types of interstellar gas. Wherever there is a proportion, even though very small, of neutral atoms (helium being most likely) the new mechanism predominates for waves of length comparable with one parsec or more.

The fact that hydromagnetic shear waves, either travelling or standing waves, of length about one parsec are heavily damped provides evidence against Fermi's theory of cosmic rays. The new difficulty must be faced for any of the recent formulations of the theory, only being absent when the gas is fully ionized.

The mechanism also raises additional difficulties in explaining the observed irregular motion of the H I gas clouds.

The rate of dissipation of magnetic energy is found the same for hydromagnetic waves and for non-oscillatory distortions of the field ; even for a field in a solid conductor. The dissipation time depends only on the conductivity and the size of the irregularities in the field.

Theories of spontaneous growth of magnetic fields are discussed critically and a minimum criterion of growth is suggested.

I. Introduction

Interstellar space is filled with tenuous, irregularly distributed and partially ionized gas. Although the density is usually less than one particle per cm^3, nevertheless its presence and its motion are highly significant in many ways, particularly in connexion with a galactic magnetic field, with cosmic rays, and with cosmic radio noise.

The whole Galaxy is also permeated by a magnetic field whose strength is not known precisely but is probably a few times 10^{-6} G. This field is not confined to the disk-shaped region occupied by most of the gas, but extends into a more or less spherical system of diameter about 20,000 parsecs or more. The whole of this region is filled with cosmic rays which are retained by the magnetic field. The system of gas, magnetic field, and cosmic rays is in a state of irregular motion which is referred to as " turbulence " and whose nature is discussed in Section II below.

Some of the outstanding problems connected with our Galaxy (apart from those directly involving stars) are the following :

(*a*) The origin of cosmic rays ; the currently most promising theory being that of Fermi.

(*b*) The origin of the galactic magnetic field.

* Division of Radiophysics, C.S.I.R.O., University Grounds, Chippendale, N.S.W.

(*c*) The transfer of kinetic energy of mass motion to and from the gas clouds.

(*d*) Processes leading to heating, excitation, ionization, and cooling of the interstellar gas.

The solutions of these problems depend largely on a knowledge of the rates of decay of the galactic hydromagnetic waves and galactic magnetic fields in general.

In the present paper a new absorption mechanism is discussed. It is shown that even a small proportion of neutral atoms present in the gas may greatly increase the rate of absorption. The quantitative results appear to be significant in connexion with the above investigations.

II. Turbulence and Wave Motion

Two types of turbulence will, in general, coexist in a compressible medium which is free from a magnetic field: " compression turbulence " and " shear turbulence " (see, for example, Burgers 1955). The former constitutes a system of sound waves, the latter a system of *non-oscillatory* vortex and shearing motions.* Hence only a fraction of the turbulent energy may be regarded as associated with wave or oscillatory motions.

When the medium is ionized gas, permeated by a magnetic field, the situation is much more complex. Four types of wave or oscillatory motion are now possible (Piddington 1955*a*). One is a space-charge electric wave or compression wave in the electron gas. Its range of frequencies is above the " plasma " frequency, that is, above about 10^3 c/s or more in the interstellar gas. The possible effects of this wave will not be considered here. The second wave is a longitudinal hydromagnetic wave or modified sound wave having an associated electromagnetic field (Piddington 1955*b*). The " compression turbulence " of the gas may now be considered as a system of such hydromagnetic waves instead of pure sound waves.

It is with the remaining two possible waves that we are mainly concerned, since it is these which are invoked in Fermi's explanation of cosmic ray acceleration and other theories discussed below. These waves are the *shear* type hydromagnetic waves (Piddington 1955*b*) sometimes called Alfvén waves. They may be conveniently referred to as the " ordinary " (O) and " extraordinary " (E) waves since at radio frequencies they degenerate into the simpler (no heavy ion motion) O and E waves of the well-known magneto-ionic theory. The phase and group velocities of these waves are $V_0 \cos \psi$ and V_0 respectively where $V_0 = H_0/(4\pi\rho_0)^{\frac{1}{2}}$, H_0 being the strength of the steady magnetic field and ρ_0 the mass density of the gas, and ψ is the angle between the wave normal and the direction of the magnetic field. The gas velocity due to the passage of a wave has the maximum value $v_0 = V_0(H_p/H_0) = H_p/(4\pi\rho_0)^{\frac{1}{2}}$, where H_p is the perturbation magnetic field.

Thus, in the presence of a magnetic field, it is possible that most or all of the " turbulent " motion of the Galaxy could be described as a combination of

* Some might like to describe these as a system of " shear " or " viscosity " waves, but as such " waves " are damped out within a fraction of a wavelength (Lamb 1945) it seems preferable not to associate them with the various other travelling waves discussed here.

three sets of hydromagnetic waves. In this case the kinetic energy of the gas turbulence is periodically transferred to and from the magnetic field. It is also possible that some of the turbulence is *not* part of a set of waves, the kinetic energy of the gas remaining as such until the magnetic field is so distorted that a recovery does not occur. There is no sharp division between the two cases; even when $H_p \gg H_0$ the elastic properties of the medium may predominate and return much of the magnetic energy to the gas.

It seems convenient to differentiate between the two cases by the criterion $H_p \gtrless H_0$. Strictly speaking the wave theory used only applies when $H_p \ll H_0$, but is probably reasonably applicable for conditions when H_p equals or even exceeds H_0 (see Piddington 1954*a*). There are observational data which suggest that the galactic magnetic field is reasonably uniform over distances of many parsecs. This would mean that in the corresponding wavelength range the observed motion is oscillatory. Also the best estimates of the value of H_0 lead to wave velocities of the order 10 km sec^{-1} which agree with observed random velocities of the gas clouds within a region of say 100 parsecs or so. Once again oscillatory motion is indicated with $v_0 \sim V_0$.

It seems that within limited regions of the Galaxy at least a substantial proportion of the turbulent motion may be oscillatory. The rates of absorption of these oscillations are determined in Sections III–VI.

Some of the motion in these regions and perhaps the greater part of the differential motion in regions of some kiloparsecs in extent may be non-oscillatory. The rate of dissipation or growth of the magnetic field under these conditions is discussed in Sections VIII and IX.

III. Absorption of Hydromagnetic Waves

In a fully ionized gas, absorption of hydromagnetic waves is caused by Joule loss and viscosity. The absorption coefficients* due to these effects in weak waves are given by (van de Hulst 1951; Piddington 1955*b*)

$$\varkappa_1 = \omega^2 (8\pi\sigma_0 S^3 V_0^3)^{-1} \text{ cm}^{-1}, \quad \ldots\ldots\ldots\ldots\ldots \quad (1)$$

$$\varkappa_2 = \omega^2 \mu (2 S^3 V_0^3)^{-1} \text{ cm}^{-1} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

respectively, where ω is the wave angular frequency, σ_0 the electrical conductivity in the absence of a magnetic field, μ the kinematic viscosity, and SV_0 the wave velocity, and c.g.s. electromagnetic units are used. The value of S is $\cos\psi$ or unity for the O and E waves respectively.

In interstellar space $\varkappa_2 \gg \varkappa_1$ (van de Hulst 1951; Elsasser 1954), so that $\varkappa_1$ will not concern us further here.

When the gas is not fully ionized but contains a proportion of neutral atoms the effective electrical conductivity may be greatly reduced (Piddington 1954*b*, p. 658; Cowling 1956; Piddington 1956*a*, 1956*b*) and the absorption of shear hydromagnetic waves correspondingly increased.

In determining the effects of the neutral atoms we will consider the gas, electrically, as a *fully ionized gas* coexisting with a neutral atom gas. The only

* The strength of the wave magnetic vector decays with distance x as $\exp(-\varkappa x)$.

effect of the latter is to provide frictional resistance to movement of the ion plasma under the influence of the electromagnetic field. This approach is permissible provided the electrons collide with ions much more frequently than with neutral atoms, a condition which is generally satisfied in interstellar gas. The equations of momentum of the two separate gases are used without the terms involving a pressure gradient, this being permissible when dealing with the shear waves concerned (Piddington 1955*b*).

Let ρ, ρ', $\mathbf{v}$, and $\mathbf{v}'$ be the mass densities and velocities of the plasma and neutral atom gases respectively, so that $\rho_0=\rho+\rho'$. The neutral atom gas is accelerated by a frictional force proportional to the velocity difference $(\mathbf{v}-\mathbf{v}')$ so that

$$\frac{\partial \mathbf{v}'}{\partial t}-(\mathbf{v}-\mathbf{v}')\frac{1}{\tau}=0, \qquad (3)$$

where τ is a constant whose physical significance and numerical values are discussed in Appendix I. The equation of motion of the plasma contains a similar term, the constant being determined from the fact that the force *per unit volume* on one gas is equal and opposite that on the other. The plasma also experiences an electromagnetic force so that we have

$$\frac{\partial \mathbf{v}}{\partial t}+(\mathbf{v}-\mathbf{v}')\frac{\eta}{\tau}+\frac{V^2}{H_0^2}(\mathbf{H}_0\times \operatorname{curl}\mathbf{H})=0, \qquad (4)$$

where $\eta=\rho'/\rho$ and $V^2=H_0^2/4\pi\rho$ (that is, V is the hydromagnetic velocity in the plasma alone). These equations are combined with the equation of the magnetic field in a moving, fully ionized gas (Piddington 1954*a*, equation (**17**)) and solved as before (Piddington 1955*b*).

The absorption coefficient $\varkappa_3$ due to neutral atom damping is given by

$$\varkappa_3=\omega X/SV, \qquad (5)$$

where X is the imaginary part of $-\{1+\eta/(1+i\omega\tau)\}^{\frac{1}{2}}$.

In the particular case when the wave period is substantially greater than the collision period ($\omega\tau\ll 1$), a sufficiently close approximation is given by

$$\varkappa_3=\omega^2\tau\eta/\{2SV(1+\eta)^{\frac{3}{2}}\}.$$

This may be written in an alternative form by replacing τ and V by τ_0 and V_0 where $\tau=(1+\eta)\tau_0$, τ_0 being defined in Appendix I. We now have

$$\varkappa_3=\omega^2\tau_0\eta/2SV_0. \qquad (6)$$

IV. The Interstellar Gas

The available evidence indicates that the average density of the interstellar gas is about 1 atom cm^{-3} in the spiral arms and much less between them (see, for example, Spitzer and Savedoff 1950). The gas in the spiral arms comprises clouds of density a few atoms cm^{-3} up to 10^3 atoms cm^{-3} or more. These occupy some 5 per cent. of the space, the remainder having an average density of perhaps 0·1 atom cm^{-3}. The gas may be mainly ionized (H II) or mainly neutral (H I). Taking the average gas density in the clouds as 10 atoms cm^{-3} the relevant data are summed up in Table 1. The values of collision period τ and kinematic viscosity μ given here are calculated in Appendix I. The parameter σ_3 is discussed in Section VII.

Chemically the gas is mainly hydrogen with about 10 per cent. of helium and smaller proportions of other elements. The quantitative results of the present investigation depend on the proportion of neutral atoms present, and in H II regions helium is the only element likely to supply such atoms. The central part of H II clouds will also be He II regions beyond which there will be a layer

TABLE 1

DATA ON GAS DENSITIES AND OTHER PROPERTIES IN INTERSTELLAR SPACE

Region	H II Intercloud	H II Cloud	H I Cloud
Proportion of space (%)	95	$0\cdot5$	5
H atoms cm^{-3}	0	0	10
He atoms cm^{-3}	0 to $0\cdot01$	0 to $1\cdot0$	$1\cdot0$
Protons cm^{-3}	$0\cdot1$	10	0
Electrons cm^{-3}	$0\cdot1$	10	5×10^{-3}
Other ions* cm^{-3}	—	—	5×10^{-3}
Temperature (°K)	10^4	10^4	100
Kinematic viscosity, μ (cm^2 sec^{-1})	$1\cdot1\times10^{17}$ to $7\cdot5\times10^{20}$	$1\cdot1\times10^{15}$ to $7\cdot5\times10^{18}$	$2\cdot4\times10^{18}$
Mean collision period, τ (sec)	$2\cdot6\times10^{10}$	$2\cdot6\times10^{8}$	$2\cdot7\times10^{11}$
Mass density ratio, η	0 to $0\cdot4$	0 to $0\cdot4$	56
σ_3 (e.m.u.)	6×10^{-9} to $1\cdot2\times10^{-24}$	6×10^{-9} to $1\cdot2\times10^{-20}$	$3\cdot5\times10^{-24}$

* Heavy ions of low ionization potential. Their average atomic weight is taken as 50.

of He I with H II. In the intercloud region most of the hydrogen will be ionized by starlight so we only discuss H II intercloud regions. There is no conclusive evidence of the proportion of this space occupied by He I. Dr. J. L. Greenstein (personal communication) considers that the theoretical evidence is in favour of a predominance of He II.

In Table 1 the kinematic viscosity is given for the two extremes : all of the helium ionized and all the helium neutral. In calculating the neutral atom absorption the latter extreme is adopted and for smaller proportions of neutral helium the value of $\varkappa_3$ will be reduced proportionally.

V. WAVE ATTENUATION IN H II REGIONS

The rates of attenuation of shear hydromagnetic waves due to ordinary viscosity and neutral atom damping may now be compared. The assumed value of the galactic magnetic field is 5×10^{-6} G.

In Figure 1 values of X are plotted against values of $\omega\tau$ for $\eta=0\cdot4$ and $100>\omega\tau>0\cdot01$. The value of $\eta=0\cdot4$ corresponds to all the helium neutral; for lower values of η we have X proportional to η so that values of $\varkappa_3$ for any degree of ionization of the helium may be readily estimated. If $\omega\tau$ lies outside the range plotted, the corresponding value of X may be found by extrapolation along the straight lines.

The relative values of $\varkappa_3$ and $\varkappa_2$ are found by comparing equations (**5**) and (**2**); S is taken as unity, so neglecting the case of the O wave travelling at a substantial angle to the magnetic field.

In the intercloud region $V_0=29$ km sec^{-1} and using the data of Table 1 we find $\varkappa_3/\varkappa_2=690X/\omega\tau$. An inspection of Figure 1 shows that $\varkappa_3>\varkappa_2$ for all values of $\omega\tau$ less than about 10, that is, for all waves of length greater than about $0 \cdot 015$ parsec. An inspection of the expression for the viscosity (see Appendix I) reveals that nearly all the viscosity, except about one part in 10^4, is due to the neutral helium atoms. If the proportion of these is reduced then $\varkappa_3/\varkappa_2$ does not change until the proportion approaches 10^{-4}. As we are concerned mainly with waves of length about 1 parsec or more, only $\varkappa_3$ need be considered for any significant proportion of neutral atoms.

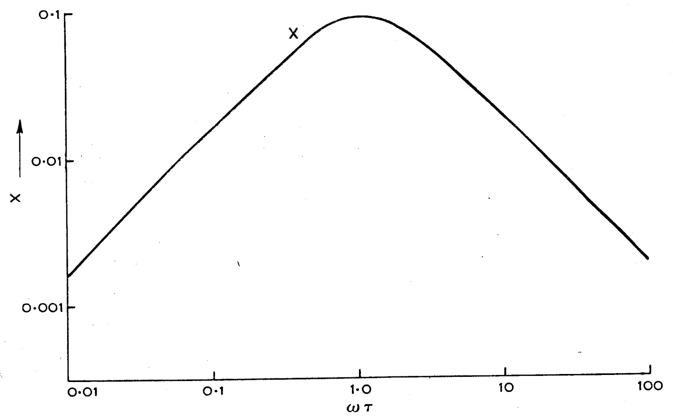

Fig. 1.—A plot of the variable X against $\omega\tau$ for $\eta=0 \cdot 4$.

The values of $\varkappa_3$ in H II intercloud regions with all the helium neutral and for waves of length 1 and 10 parsecs are $4 \cdot 5 \times 10^{-20}$ and $4 \cdot 5 \times 10^{-22}$ respectively. This means that the waves are attenuated by a factor exp 1 in distances of $7 \cdot 2$ and 720 parsecs respectively. The attenuation distance (due to ordinary viscosity) of a wave of length 1 parsec in intercloud gas in which all the helium is ionized is $4 \cdot 1 \times 10^6$ parsecs, which indicates the overwhelming importance of neutral atoms in their effect, not only on $\varkappa_3$ but also on $\varkappa_2$.

In H II clouds $V_0=2 \cdot 9$ km sec^{-1} and using the data of Table 1 we find $\varkappa_3/\varkappa_2=6 \cdot 9X/\omega\tau$. An inspection of Figure 1 shows that $\varkappa_3>\varkappa_2$ for values of $\omega\tau$ less than about $0 \cdot 8$, that is, for waves of length greater than about $1 \cdot 9 \times 10^{-4}$ parsec. The value of $\varkappa_3$ in an H II cloud with all the helium neutral for a wave of length 1 parsec is $4 \cdot 5 \times 10^{-23}$, so that the wave is absorbed in a distance of about 7000 parsecs. Since the size of the H II clouds is generally comparable with 100 parsecs or less the waves will pass freely across the clouds, being reflected at or transmitted through the boundaries depending on the gas density gradient (Piddington 1956*b*).

VI. Attenuation in H I Regions

In the typical H I region described in Table 1 there is a small proportion of ions, corresponding to elements of low ionization potential. These suffice to render the gas electrically conducting and so capable of transmitting hydromagnetic waves. Although the neutral atoms outnumber the ions by a factor of 2000 their collision cross section is so much smaller (by a factor greater than 10^4) that the condition stated in Section III is satisfied, namely, that electrons collide with ions much more frequently than with atoms.

The wave velocity V_0 has a value $2\cdot 9$ km sec^{-1} so that a wave of length 1 parsec has $\omega=5\cdot 9\times 10^{-13}$ and $\omega\tau=0\cdot 16$. The corresponding value of X is $0\cdot 58$ and a comparison of equations (2) and (5) gives $\varkappa_3/\varkappa_2=5\cdot 1\times 10^5$. Again only neutral atom electromagnetic damping need be considered. Equation (5) gives the damping for waves of length one parsec as $\varkappa_3=1\cdot 6\times 10^{-19}$ so that the wave is absorbed in a distance of $2\cdot 0$ parsecs and a time of about 10^6 years.

This result may be compared with that of Parker (1955, p. 248) who calculated the rate of absorption of hydromagnetic waves of length 2 light years in H I gas of density 1 cm^{-3}. He considered only the effects of viscosity and found that the waves were absorbed in a distance of $1\cdot 6\times 10^3$ parsecs. When neutral atom absorption is considered (the gas having the composition given in Table 1) the absorption distance is found to be about $0\cdot 5$ parsec.

VII. The Origin of Cosmic Rays

It is probable that most cosmic rays receive their energy from a series of " collisions " with elements of a magnetic field. In its initial form (Fermi 1949) this theory envisaged encounters with more or less separate fields, each embedded in a drifting interstellar gas cloud. A quantitative examination led to the rejection of this particular mechanism and its replacement by several alternatives, in which the magnetic field elements were parts of hydromagnetic waves. The galactic magnetic field was assumed fairly regular in form, perhaps directed along the spiral arms and perturbed by travelling hydromagnetic waves (Fermi 1954 ; Morrison, Olbert, and Rossi 1954) or by standing hydromagnetic waves (Davis 1956). The latter might be regarded as combinations of two travelling waves, moving in opposite directions so that in a linear theory, as used here, the rate of absorption would be the same as for the waves discussed in Section III above.

The efficiency of the statistical acceleration process decreases with increasing wavelength and it appears that strong waves ($H_p \gtrsim H_0$) of length about one parsec or less would be needed to provide the observed cosmic ray spectrum. As suggested in Section II such a system of hydromagnetic waves might constitute part of the general galactic turbulence. It is necessary, however, to examine the energy balance of the system as part of the more general problem of transfer of energy from the original source to the form of cosmic ray energy. A discussion of this problem has already been given by Parker (1955) who unfortunately neglected the neutral atom damping effect which may be the most important factor.

Most of galactic space is intercloud H II region so that it is in this type of gas that the Fermi waves must be maintained. As we have seen, waves of length 1 and 10 parsecs are absorbed in distances 7·2 and 720 parsecs, provided the helium is neutral. The energy flow associated with these waves is given by the Poynting flux $P=H_0H_p^2/8\pi^{3/2}\rho_0^{\frac{1}{2}}$ and the rate of loss of wave energy per unit volume is $P\varkappa_3$. When $H_p=H_0$ the energy losses for the above waves are $2\cdot6\times10^{-25}$ erg cm^{-3} sec^{-1} and $2\cdot6\times10^{-27}$ erg cm^{-3} sec^{-1} respectively. The energy required to maintain the existing supply of cosmic rays is about 3×10^{-27} erg cm^{-3} sec^{-1} and this is also approximately the energy available as random motion of gas clouds due to their disruption by early type stars (Oort and Spitzer 1955). Any energy requirement substantially greater than this value poses a difficult problem (Parker 1955). Thus, if the Fermi theory requires waves of length 1 parsec or less, the energy balance problem is very serious. If waves of length 10 parsecs or more suffice, the difficulty is largely removed.

It may well be that most of the intercloud helium is ionized, in which case the hydromagnetic waves travel more or less freely between clouds. However, waves travelling in or near the plane of the Galaxy will encounter H I clouds at intervals of a kiloparsec or so and will then be absorbed in a few parsecs. This effect would require detailed consideration in relation to any particular form of the Fermi theory but appears to raise a potential difficulty.

One region in which the Fermi theory may find application is in the galactic corona whose presence is inferred from radio observations. Spitzer (1956) has suggested a density and temperature of 5×10^{-4} cm^{-3} and 10^6 °K, in which case the helium would be almost entirely ionized and neutral atom damping absent. However, at the high temperature assumed, the viscosity is high and the absorption distances for waves of length 1 and 10 parsecs are 3·1 and 310 parsecs respectively. Thus, for conditions as assumed by Spitzer, absorption in the corona would seem to pose a serious problem.

All of these considerations appear to provide some general evidence in support of the hypothesis that cosmic rays are created, perhaps by the Fermi process, in regions of very limited extent. The regions may be around supernovae (see, for example, Ginzburg 1953). If a region has a diameter of say 1 parsec and the hydromagnetic wavelength is a small fraction of this then the efficiency of the Fermi process becomes very high and the problem of energy balance less acute. In the next paper (Piddington 1957) the process is discussed in relation to the Crab Nebula which has recently been investigated by Oort and Walraven (1955).

VIII. The Decay of Non-oscillatory Magnetic Fields

Galactic turbulence may be regarded as a combination of oscillatory and non-oscillatory motions of the gas and magnetic field. The rates of dissipation of energy in the former case have been discussed and we now consider those in the latter.

It may help to discuss the different types of motion in terms of the simple schematic diagrams in Figure 2. In Figure 2 (*a*) the magnetic field $\mathbf{H}_0$ is perturbed by the sinusoidally varying field $\mathbf{H}_p$ and the medium oscillates. There is equipartition between the perturbation kinetic and magnetic energy and both are dissipated as heat by the processes already discussed. In Figure 2 (*b*) a blob of gas has so much momentum that it distorts the magnetic field from the position shown by the dashed line to that shown by the full line; it may continue to distort it further but we will consider the situation when the magnetic field is as shown, roughly in the form of an arc of radius R.

We must first estimate the rate of dissipation of energy of the magnetic field shown in Figure 2 (*b*). In general the time of decay of the magnetic field in a sphere of radius R is of order $\sigma_3 R^2$ (Piddington 1954*a*, Section 7; Cowling 1956) where $\sigma_3 \equiv \sigma_1 + \sigma_2^2/\sigma_1$ is the " effective " conductivity, depending on the direct conductivity (σ_1) and the Hall conductivity (σ_2), expressed in e.m.u.

The value of σ_3 may be found from a comparison of equation (**1**), in a form appropriate to *partially* ionized gas, and equation (**6**). Consider a plane hydromagnetic wave propagated along the field, so that $S=1$. The appropriate

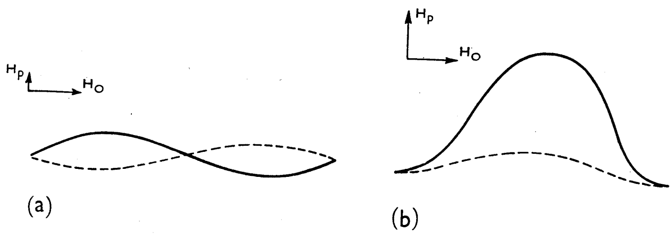

Fig. 2.—Schematic diagram of magnetic lines of force distorted by (*a*) a passing or stationary hydromagnetic wave and (*b*) a non-oscillatory motion of the gas in the direction of H_p.

electromagnetic field equations (Piddington 1954*a*, equations (**14**) to (**18**)) show that the appropriate conductivity is always σ_3. Thus the general form of our equation (**1**) is

$$\varkappa_3 = \omega^2 (8\pi\sigma_3 V_0^3)^{-1}. \quad \dots\dots\dots\dots\dots\dots \quad (7)$$

If the gas is fully ionized, then $\sigma_3 = \sigma_0$ and we have the particular case given by equation (**1**). The $\varkappa_3$ of equations (**6**) and (**7**) may now be identified* and, replacing $4\pi V_0^2$ by $H_0^2/(\rho+\rho')$, we find

$$\sigma_3 = \rho(\rho+\rho')/\rho'\tau_0 H_0^2. \quad \dots\dots\dots\dots\dots\dots \quad (8)$$

An expression for σ_3 has also been derived by Cowling (1956, equation (**30**)). When the approximations used in determining equation (**8**) are adopted (the effects of finite plasma conductivity and of collisions of electrons with neutral atoms are neglected) the two expressions are identical.

The time of decay (by a factor of exp 1) of a magnetic field of " size " R is now determined as

$$T = \rho(\rho+\rho')R^2/\rho'\tau_0 H_0^2.$$

* The full solution of the equations of Section III include a dissipation term $\varkappa_1$ due to the finite conductivity σ_0 of the ion plasma alone. This, as we have seen, is negligible and so is neglected. Also the value of $\varkappa_3$ given in equation (**5**), which represents a more general solution, cannot be used here because the concept of " conductivity " may only be introduced when a large number of all relevant microscopic processes occur in one wave period, that is, when $\omega\tau \ll 1$.

This may be compared with the decay time of a wave of length λ, given by $1/\varkappa_3 V_0$. Replacing ω^2 by $4\pi^2 V_0^2/\lambda^2$ we find

$$T'=\frac{\rho(\rho+\rho')}{\rho'\tau_0 H_0^2}\cdot\frac{2\lambda^2}{\pi},$$

which is identical with the time of non-oscillatory decay provided

$$\lambda=(\tfrac{1}{2}\pi)^{\frac{1}{2}}R.$$

It is satisfactory to note that the rates of decay of a twist or kink in a magnetic field as determined by two methods are in agreement. It does not matter whether the distortion is part of a wave or of a non-oscillatory motion or, for that matter, of a semi-permanent irregularity in a solid conductor ; the dissipation period depends only on the effective conductivity σ_3 and the size of the irregularity.

There remain two further factors requiring consideration : first, the possible increase in the rate of dissipation of magnetic energy due to turbulence (Sweet 1950 ; Elsasser 1955). This problem is discussed elsewhere. The second problem, closely allied to the first, concerns the *increase* of magnetic energy at the expense of the kinetic energy of turbulence. This is discussed briefly in the following section.

IX. The Growth of Magnetic Fields

Some current theories of the origin of the galactic magnetic field (see Cowling 1955) depend on its growth from a " seed " field due to turbulence of the gas. Obviously hydromagnetic damping is relevant.

Batchelor (1950) and Chandrasekhar (1950) find an analogy between the magnetic field and the vorticity vectors. They argue that equipartition of energy between the field and the turbulence is only likely at the shorter " wavelengths " of the turbulence " spectrum " where vorticity is more important compared with the velocity. Batchelor starts with the equation of the magnetic field in a moving isotropically conducting (conductivity σ) medium

$$\nabla^2\mathbf{H}=4\pi\sigma\left\{\frac{\partial\mathbf{H}}{\partial t}-\mathrm{curl}\,(\mathbf{v}\times\mathbf{H})\right\},$$

and concludes that the field continues to grow as long as $4\pi\sigma\mu>1$. This theory has been criticized in rather general terms by Cowling (1955) and Elsasser (1955).

The correct field equation for ionized gas (Piddington 1954*a*, equation (**14**) et seq.) differs from the above in that σ is replaced by σ_3 and there are two additional terms. These terms correspond to elliptical orbits of the gas when disturbed and to currents flowing along the magnetic field. They do not appear to affect the rate of dissipation, so that Batchelor's criterion of growth might be replaced by $4\pi\sigma_3\mu>1$. In H I clouds $\sigma_3=3\cdot5\times10^{-24}$ so that, for our assumed value of H_0, $4\pi\sigma_3\mu=1\cdot1\times10^{-4}$ and, if growth had stopped when $4\pi\sigma_3\mu=1$, the corresponding value of H_0 would be $5\cdot2\times10^{-8}$ G. Thus the theory does not seem capable of explaining the currently accepted value of the galactic magnetic field in terms of turbulence within gas clouds (these are mainly H I regions).

An alternative approach by Elsasser (1954) resulted in two equations of motion in which the mass velocity of the gas $\mathbf{v}_0$ was replaced by the variables $\mathbf{v}_0 \pm \mathbf{H}_0/(4\pi\rho_0)^{\frac{1}{2}}$ and the kinematic viscosity μ by $\mu \pm (4\pi\sigma)^{-1}$. This indicates an analogy between v_0 and $H_0/(4\pi\rho_0)^{\frac{1}{2}}$ or between the magnetic energy density $H_0^2/8\pi$ and the kinetic energy density $\frac{1}{2}\rho_0 v_0^2$. The analogy suggests the possibility of at least a tendency to equipartition. However, since $H_0/(4\pi\rho_0)^{\frac{1}{2}} = V_0$, the hydromagnetic wave velocity (both phase and group), the analogy is between mass velocity v_0 and wave velocity V_0 and so may be neither surprising nor significant. Also the analogy between μ and $(4\pi\sigma)^{-1}$ corresponds to that between the absorption due to viscosity and magnetic field decay ; the ratio of the rates of dissipation is $4\pi\sigma\mu$, which is identical with the ratio $\varkappa_2/\varkappa_1$ obtained from equations (**1**) and (**2**) above.

There seems to be general agreement that when the energy density of the magnetic field is much smaller than that of the turbulent motion there will be a tendency, on the whole, for the magnetic lines of force to be stretched and the total magnetic energy to be increased. There does not, however, seem to be any clear evidence that the process should continue until equipartition is reached. The problem must be considered for each small " wavelength " interval in the turbulence " spectrum ". Some of the kinetic energy of the motion will be dissipated by viscosity and some transformed into magnetic energy at a rate depending on the relative total energy densities and probably also on the wave-length. The magnetic energy will in turn be dissipated as discussed above. The unknown quantity is the efficiency of transfer of kinetic to magnetic energy ; until this can be determined a satisfactory criterion for magnetic field growth cannot be given. However, by adopting a simple model a crude order-of-magnitude criterion may be found.

Consider the magnetic field of Figure 2 (b) as it changes from strength H_0 to $(H_0 + H_p)$ in time δt under the influence of the gas moving with velocity v_0. The ratio H_p/H_0 is given approximately by $v_0 \delta t/R$ and is taken as order unity or less. Thus we have the increase in magnetic energy density.

$$\begin{aligned}\frac{1}{8\pi}\{(H_0+H_p)^2 - H_0^2\} &\sim \frac{2H_0H_p}{8\pi}\\ &= \frac{H_0^2}{8\pi}\cdot\frac{2H_p}{H_0}\\ &\sim \frac{H_0^2}{8\pi}\cdot\frac{2v_0\delta t}{R}.\end{aligned}$$

In the same period the decay of magnetic energy density is of the order $(H_0^2/8\pi)(\delta t/\sigma_3 R^2)$ so that the criterion for growth of the field is

$$2\sigma_3 R v_0 > 1.$$

It is unlikely that the gas would everywhere move in such a way as to create magnetic energy at the (maximum) rate assumed above so that the criterion might be written

$$q\sigma_3 R v_0 > 1,$$

where $q<2$ but might be of order unity. As equipartition is approached $v_0 \to V_0$ and the criterion becomes

$$q\sigma_3 R V_0 > 1.$$

A similar result is obtained when a wave of length λ is considered. The time of transfer of wave energy from the kinetic to the magnetic state is a quarter period or $\pi/2\omega$ and the time of decay $1/\varkappa_3 V_0$, where $\varkappa_3$ is given by equation (7). The condition that energy is transferred before it decays is

$$(8/\pi)\sigma_3 \lambda V_0 > 1.$$

In the H I regions defined in Table 1 the smallest scale of turbulence capable of amplifying a field up to 5×10^{-6} G, assuming the gas velocity to be 5 km sec^{-1}, is about $0\cdot2/q$ parsecs. It should be noted that the appropriate value of v_0 is the differential velocity over the distance R.

It is of interest to compare our criterion with that of Batchelor ($4\pi\sigma_3\mu>1$). It is possible to satisfy either one without the other according to the conditions

$$4\pi\mu/q \gtrless Rv_0,$$

so that one or other of the criteria must be at fault. The appearance of the kinematic viscosity μ in Batchelor's criterion implies that it plays a role beyond that of simple absorption. In fact, near equipartition it states that the dissipation of energy by viscosity is greater than the magnetic dissipation (that is, $\varkappa_2/\varkappa_3>1$). It is difficult to see why a large viscous dissipation should be a criterion of magnetic field growth.

X. Effects of Hydromagnetic Waves on the Interstellar Medium

We have considered the interchange of energy between the galactic magnetic field and the mass motion of the interstellar gas and also the more or less gradual dissipation of both forms of energy into heat. It is of interest to estimate the possible rate of heating of the gas and to consider the possibility of direct excitation and ionization of the gas constituents by the hydromagnetic waves.

A " strong " wave ($H_p \sim H_0$) in the intercloud region has a maximum gas velocity of 29 km sec^{-1}. The most interesting effects are likely to occur when $\omega\tau \gtrsim 1$ and so we will consider a wave for which $\omega\tau=1$ so that $\omega=3\cdot8\times10^{-11}$ and $\lambda=0\cdot15$ parsec. The absorption coefficient is $1\cdot3\times10^{-18}$ so that such waves would be absorbed in a distance of about $0\cdot25$ parsec when all the helium is neutral. The rate of dissipation of energy is $7\cdot5\times10^{-24}$ erg cm^{-3} sec^{-1}. A comparison of this with other sources and sinks of energy (Savedoff 1955) suggests that the hydromagnetic wave energy would be a major factor.

A possibly more interesting effect of the wave considered is that it could excite and ionize the gas particles and so change the " excitation temperature " of the gas. The maximum kinetic energy of an ion of atomic weight about 10, moving under the influence of the above wave, is about 45 eV. When the wave period is comparable with the collision period of a neutral (He) atom a substantial proportion of this energy is available to excite or ionize the atom or ion. This means that if a hydromagnetic wave of sufficiently short period and of sufficient strength passes through ionized gas it may drastically change the "excitation temperature" of the gas. Not only does it heat the gas generally as discussed in the earlier sections but electromagnetic energy is transformed directly to energy of excitation and ionization and subsequently emitted as light quanta. It is interesting to speculate on the possible effects which could occur by this process when waves are reflected at the boundaries of gas clouds. Standing waves would be formed leading perhaps to striation effects.

XI. Acknowledgments

The author wishes to express his thanks to Professor J. L. Greenstein and Dr. Leverett Davis Jr. for helpful discussions.

XII. References

Batchelor, G. K. (1950).—*Proc. Roy. Soc.* A **201**: 405.

Burgers, J. M. (1955).—"Gas Dynamics of Cosmic Clouds." I.A.U. Symposium No. 2, Cambridge 1953, p. 228. (North Holland Publishing Co.: Amsterdam.)

Chandrasekhar, S. (1950).—*Proc. Roy. Soc.* A **204**: 435.

Chapman, S., and Cowling, T. G. (1952).—"The Mathematical Theory of Non-uniform Gases." (Cambridge Univ. Press.)

Cowling, T. G. (1955).—"Vistas in Astronomy." (Ed. A. Beer.) Vol. 1. (Pergamon Press: London.)

Cowling, T. G. (1956).—*Mon. Not. R. Astr. Soc.* **116**: 114.

Davis, L. (1956).—*Phys. Rev.* **101**: 351.

Elsasser, W. M. (1954).—*Phys. Rev.* **95**: 1.

Elsasser, W. M. (1955).—*Amer. J. Phys.* **23**: 590.

Fermi, E. (1949).—*Phys. Rev.* **75**: 1169.

Fermi, E. (1954).—*Astrophys. J.* **119**: 1.

Ginzburg, V. L. (1953).—*Proc. Phys. Sci.* (*U.S.S.R.*) **51**: 343.

van de Hulst, H. C. (1951).—"Problems of Cosmical Aerodynamics." I.U.T.A.M. and I.A.U. Symposium, Paris 1949, p. 45. (Central Air Documents Office: Dayton, Ohio.)

Lamb, H. (1945).—"Hydrodynamics." 6th Ed. p. 620. (Dover: New York.)

Morrison, P., Olbert, S., and Rossi, B. (1954).—*Phys. Rev.* **94**: 440.

Oort, J. H., and Spitzer, L. (1955).—*Astrophys. J.* **121**: 6.

Oort, J. H., and Walraven, T. (1955).—*Bull. Astr. Insts. Netherl.* **12**: No. 462.

Parker, E. N. (1955).—*Phys. Rev.* **99**: 241.

Piddington, J. H. (1954*a*).—*Mon. Not. R. Astr. Soc.* **114**: 638.

Piddington, J. H. (1954*b*).—*Mon. Not. R. Astr. Soc.* **114**: 651.

Piddington, J. H. (1955*a*).—*Phil. Mag.* **46**: 1037.

Piddington, J. H. (1955*b*).—*Mon. Not. R. Astr. Soc.* **115**: 671.

Piddington, J. H. (1956*a*).—*Observatory* **76**: 21.

Piddington, J. H. (1956*b*).—*Mon. Not. R. Astr. Soc.* **116**: 314.

Piddington, J. H. (1957).—*Aust. J. Phys.* **10**: 530.

Savedoff, M. P. (1955).—"Gas Dynamics of Cosmic Clouds." I.A.U. Symposium No. 2, Cambridge 1953, p. 55. (North Holland Publishing Co.: Amsterdam.)

Spitzer, L. (1956).—*Astrophys. J.* **124**: 20.

Spitzer, L., and Savedoff, M. P. (1950).—*Astrophys. J.* **111**: 593.

Sweet, P. A. (1950).—*Mon. Not. R. Astr. Soc.* **110**: 69.

Tyndall, A. M. (1938).—"The Mobility of Positive Ions in Gases." (Cambridge Univ. Press.)

Westfold, K. C. (1953).—*Phil. Mag.* **44**: 711.

Appendix I

Collision Periods and Viscosity

In the gas with which we were concerned in Section III it is seen that after a "collision" between an ion and neutral atom the former is immediately accelerated to its former drift velocity by powerful electromagnetic forces. It is convenient, therefore, to define the neutral atom collision period τ as the period taken to change its drift velocity from its initial value to that of the ion plasma.

In the absence of the moving magnetic field a more usual procedure is to define a different neutral atom collision period τ_0 being the time taken for the atom velocity to change from its initial value to that of the *whole gas*. Equation (**3**) then becomes

$$\partial \mathbf{v}'/\partial t+(\mathbf{v}'-\mathbf{v}_0)/\tau_0=0, \qquad (9)$$

where $\mathbf{v}_0$ is the velocity of the whole gas defined by

$$(\rho+\rho')\mathbf{v}_0=\rho\mathbf{v}+\rho'\mathbf{v}'.$$

It is then easy to show that

$$(\mathbf{v}'-\mathbf{v})=(1+\eta)(\mathbf{v}'-\mathbf{v}_0),$$

so that, comparing equations (**3**) and (**9**), we have

$$\tau=(1+\eta)\tau_0. \qquad (10)$$

When a neutral atom-gas drifts relative to a plasma, viscous drag results from collisions with both heavy ions and electrons. It has been shown (Piddington 1956*b*), however, that the greater mass of the former renders the electron contribution negligible so that we need only consider collisions between atoms and heavy ions. An equation for τ_0 has been given by Chapman and Cowling (1952, p. 335), and Westfold (1953) has shown that this is appropriate to the type of problem under discussion. Using equation (**10**) we have an expression for τ,

$$\tau=\frac{3}{8\rho\sigma_{12}^2}\left\{\frac{m_1m_2(m_1+m_2)}{2\pi kT}\right\}^{\frac{1}{2}}.$$

The variables are as defined by Chapman and Cowling except ρ, which is now the mass density of the *plasma only*. This formula has been used to determine the values of τ listed in Table 1. The values of collision distance σ_{12} are inferred from the data given by Chapman and Cowling (Chapters 12–14) and Tyndall (1938). The distances for helium atoms and hydrogen ions at 10^4 °K is taken as $1\cdot7\times10^{-8}$ cm and for hydrogen atoms and metallic ions at 100 °K as $3\cdot5\times10^{-8}$ cm.

An expression for the viscosity of a simple gas is given by Chapman and Cowling (p. 169). This is used to calculate the viscosity in H I regions, the collision distance of the hydrogen atoms being taken as $2\cdot2\times10^{-8}$ cm.

When a gas is fully ionized the effective collision distances of the particles are determined by electrostatic forces and, in the gases with which we a concerned, the distances are relatively very large. The appropriate formula for the viscosity is given by Chapman and Cowling (p. 179) and the corresponding values of kinematic viscosity are given in Table 1 (cases of no neutral helium). The highly significant effect of electrostatic forces was apparently overlooked by Parker (1955, p. 248), who found a value of kinematic viscosity too large by a factor of about 10^4.

When neutral helium is introduced into the H II region the position is greatly complicated. Even though its proportion may be small, the appropriate collision cross section is less than that of ions by a factor of about 10^4 and so it may raise the viscosity by a large factor. The appropriate formula is given by Chapman and Cowling (p. 167). It will be seen from Table 1 that the presence of 10 per cent. of neutral helium raises the kinematic viscosity by a factor of about 7×10^3.