THE CRAB NEBULA AND THE ORIGIN OF INTERSTELLAR MAGNETIC FIELDS

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[*Manuscript received June* 11, 1957]

Summary

Electromagnetic effects in the Crab nebula have been investigated with the following conclusions :

(1) No appreciable proportion of the present central magnetic field could have passed in or out through the surrounding ionized supernova shell during its 900 years life.

(2) A limitation of the generality of the virial theorem of Chandrasekhar and Fermi (1953) is mentioned and some consequences : the upper limit of magnetic field strength in a star must, in general, be reduced below their estimate. The theorem does not appear relevant and should not be used in the case of fields which fall off slowly beyond the photospheric surface. Force-free fields seem improbable in nature, particularly in relation to the Crab nebula.

(3) The nebular magnetic field was not formed from the general interstellar field, nor by turbulence in the supernova shell, nor was it in existence at the time of the explosion.

(4) The main field has been created during the expansion as complete loops of magnetic force, within the shell but linking neither the shell nor the central star. The average rate of generation of magnetic energy is some 10^4 times the total output of solar energy and must derive more or less directly from nuclear energy.

(5) A significant part of the galactic magnetic field may have originated inside supernova shells.

(6) The mass of the expanding supernova shell is probably about one solar mass.

(7) Irregular motions seen in the nebula are hydromagnetic waves causing a compression of the magnetic field with consequently enhanced light emission.

(8) The cosmic rays in the nebula are accelerated by hydromagnetic waves according to the process of statistical acceleration. A significant fraction of all galactic cosmic rays may originate in this way ; not with the supernova explosion itself but subsequently within the space between the star and the shell.

(9) The nebular magnetic field may be generated by the central star spinning, the energy and angular momentum being supplied by fast particles ejected from the star. A somewhat similar process may be responsible for stellar and other cosmic magnetic fields.

I. Introduction

The Crab nebula (optical object NGC 1952, radio object IAU 05N2A) is a unique visible object, the only example of what may be a class of especially powerful (Type 1) supernovae. As such it merits interest, not only in its own right, but also as a possible key to three outstanding astrophysical problems : the origin of interstellar magnetic fields, of cosmic rays, and of cosmic (non-thermal) radio emission.

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The Crab nebula has long interested astronomers who encountered difficulty in trying to explain its peculiar spectral features (Baade 1942 ; Minkowski 1942 ; Minkowski and Greenstein 1953). Its radio spectrum also showed unusual features. It was first suggested by Shklovsky (1953) that all peculiarities might be explained if neither the visible nor the radio emission were due to the usual atomic processes but to cosmic ray electrons spiralling in a magnetic field. The same process had previously been suggested as a source of radio emission from " radio stars " (Alfvén and Herlofson 1950). Shklovsky's suggestion offered the possible advantage of resolving both optical and radio anomalies simultaneously.

A crucial test of the suggestion was that the light emission should show plane polarization. This has been demonstrated by Dombrovsky (1954), Oort and Walraven (1956), and others, and it is now fairly certain that most of the light and radio emission from the amorphous central region of the nebula is by the synchrotron process as suggested.

The optical and radio emission intensities allow some very important inferences to be made about conditions in the nebula. Oort and Walraven (1956) find that a magnetic field of strength about 10^{-3} G (they believe it is limited to the small range 10^{-3}–3×10^{-3} G) is present and also a cosmic ray electron gas of about the same energy density, 4×10^{-8} erg cm^{-3}. The total energy density of about 10^{-7} erg cm^{-3} must be regarded as a minimum because, if the magnetic field has a value other than 10^{-3} G, then the total energy necessary to explain the observed emission is greater.* The magnetic field and cosmic ray gas both exert pressure, the total being about 10^{-7} dyn cm^{-2} or more, perhaps higher than in any other observed region of interstellar space.

The main objectives of the present paper are to determine likely origins of the magnetic field and cosmic ray gas of the nebula, and to estimate its mass. The most significant conclusion reached is that magnetic field was, and perhaps still is, being created within the nebula at a rate corresponding to about 10^4 times the *total* power output of the Sun. Such a rate of generation of magnetic field implies a process of more or less direct conversion of nuclear to magnetic energy. It is suggestive of a possible origin of interstellar magnetic fields in general.

In Section III a digression is necessary for a rather general discussion of the possible strength of magnetic fields associated with stars and in particular with force-free fields.

II. Electromagnetic Properties of the Nebula

The most likely strength of the magnetic field is 10^{-3} G in the central regions and less near the shell. The radius of the whole is about 1 parsec, and, in estimating the total magnetic energy and flux, a simple model is adopted in the

* *Note added in Proof.*—After the completion of this work I have seen a paper by Pikelner (1956) in which a lower estimate of the magnetic field ($\sim3\times10^{-4}$ G) was obtained. This results in a lower estimate of the mass of the shell, 0·1 solar mass instead of 1 solar mass. Pikelner also concluded that the magnetic field of the nebula may have been created by reinforcement of the galactic field. We reach a contrary conclusion, which is not affected by the possibility of a lower value of magnetic field strength and lower shell mass.

form of a sphere of radius $\frac{1}{2}$ parsec containing a field of uniform strength 10^{-3} G. The total volume is $1\cdot5\times10^{55}$ cm^3 and the total magnetic energy $6\cdot1\times10^{47}$ ergs. The maximum flux, if the field were toroidal in form, would be that through an appropriate semicircular section, $3\cdot8\times10^{33}$ G cm^2. Fields of other forms, provided they were reasonably uniform, would give comparable values of flux.

The general form of the magnetic field is seen in a sketch by Oort and Walraven (1956, Fig. 15). It is more or less uniform in direction over an extended region near the centre and they estimate that the flux through this region is 10^{33} G cm^2. It is possible that the total flux through a suitably chosen, more extended region would be several times that value, perhaps as high as the above estimate for the simple model. The significance of this large value of flux is discussed in Section IV.

For the present discussion, an important property of the nebula is the degree to which magnetic field could have penetrated the ionized gaseous shell thrown out at the time of the explosion and now surrounding the central magnetic field. The time taken for a magnetic field to diffuse into or out of a cloud of conducting gas of radius D is σ_3D^2, where σ_3 is the " effective " electrical conductivity (see, for example, Cowling 1956 ; Piddington 1957). The same time is needed for an internal or external field to effectively permeate a confining shell of thickness D The time T is then given by

$$T=\sigma_3D^2=\rho(\rho+\rho')D^2/\rho'\tau_0H^2, \qquad \ldots\ldots\ldots\ldots (1)$$

where ρ and ρ' are the mass densities of the ion plasma and neutral atom gas, τ_0 is a certain collision period of a neutral atom, and H is the magnetic field strength.* For a given value of shell mass per unit area, $(\rho+\rho')D$, we have $\tau_0\propto D$ so that T is independent of D but is proportional to mass per unit area and to ρ/ρ'.

The value of σ_3 for a mixture of ionized hydrogen (density 10 cm^{-3}) and neutral helium (1 cm^{-3}) in a field of 5×10^{-6} G has been calculated (Piddington 1957) as $1\cdot2\times10^{-20}$ e.m.u. The field at the inner surface of the nebular shell might be taken as 5×10^{-4} G giving $\sigma_3=1\cdot2\times10^{-24}$ e.m.u. for the same gas. The radius of the shell is 1 parsec and its mass about 1 solar mass (see Section V below) so that its mass per unit area is $1\cdot7\times10^{-5}$ g cm^{-2}. If it were composed of the same gas mixture its thickness would be about $0\cdot23$ parsec† and so $T=6\cdot0\times10^{11}$ sec or about 2×10^4 years. The spectrum of the shell shows it to be highly ionized and to emit lines from He II as well as from He I (Minkowski 1942). It is probable, therefore, that its degree of ionization and hence its conductivity is at least as high as for the above mixture and the diffusion time at least 2×10^4 years. When the shell had a smaller radius R and greater density the value of ($T\propto\rho+\rho'\propto R^{-2}$) would be still larger.

The volume of the conducting shell is only a few ($\sim$6) per cent. of the volume it encloses. Thus, even if T were as low as 900 years (the life of the nebula),

* This formula may be derived from Cowling's equation (**30**) by neglecting the small terms $1/\sigma_0$ and K_e and remembering that τ_0 is the average time to accelerate a neutral atom to the velocity of the ion plasma. It is derived by different methods by Piddington.

† The actual thickness may be nearer $0\cdot02$ parsec but T has been shown to be independent of D and the present assumption simplifies the calculation.

only an insignificant proportion of the central field could have diffused from one side of the shell to the other. This conclusion is not invalidated by turbulent motion of the shell. Such motion may move two originally adjacent particles a distance apart comparable with D but not with R, for otherwise the shell would have lost its identity. Thus magnetic field which has diffused into the shell may possibly be carried to the other side in a relatively short time. However, when the shell is saturated no more field diffuses in and an inappreciable amount diffuses out the other side.*

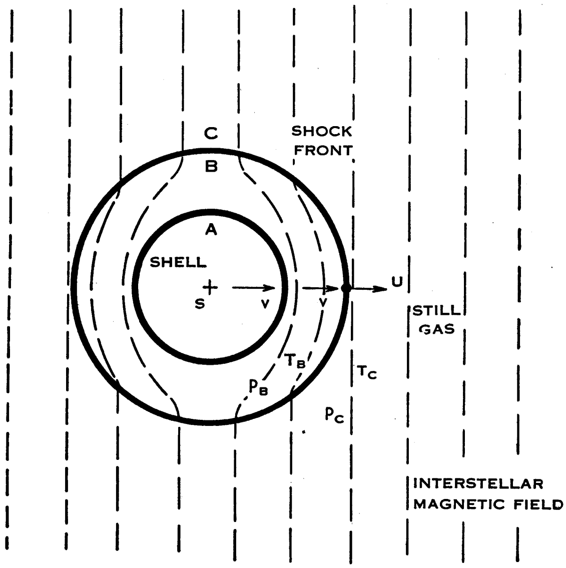

Fig. 1.—A slowly exploding nebula (sphere A) surrounded by a shock wave (shell B) immersed in still gas (C).

It is desirable also to consider the magnetic field which may exist outside the nebula. It is shown in Section III that the magnetic flux associated with the shell at the time of the explosion must be small and would result in a negligible field inside or outside the present shell. The external field will then be the normal interstellar field, perhaps as strong as 10^{-5} G, distorted in the manner shown in Figure 1. Outside the nebula A, a shock front is formed between compressed gas B moving with the nebula and the undisturbed interstellar gas C. In Section V it is shown that the density in region B is about four times that in C, so that the field strength is 4×10^{-5} G, still small compared with the internal field.

* Because of the large dimensions and short lifetime of the shell it may be considered essentially a perfect conductor. A theorem of Bondi and Gold (1950) may then be used to support this argument. It states that no hydrodynamic motion of a body of perfectly conducting liquid of finite dimensions in an otherwise empty space can generate an external field, provided the body is simply connected.

Using equation (**1**) again, it is found that the time taken for this compressed external magnetic field to penetrate a distance D into the shell is about 3×10^6 years. Again it is concluded that negligible penetration has occurred.

III. Magnetic Fields Associated with Stars

It has been suggested that the magnetic field of the Crab nebula was associated with the central star prior to the explosion. Before discussing this possibility some general properties of stellar magnetic fields are considered—in particular their effects on gravitational stability.

Chandrasekhar and Fermi (1953) have made a most useful extension of the virial theorem to include the energy of a magnetic field. However, the theorem was derived and used in a way which implies that all the magnetic field concerned was contained within the star (or other " configuration " in question). The difficulty has been resolved in personal communications with Professor S. Chandrasekhar who had already noted (Chandrasekhar and Limber 1954) that " for a star with a prevailing magnetic field, the effective boundary may have to be placed quite outside the conventional photospheric surface ". It seems desirable here, before using the theorem, to make clear the limitation on its generality implicit in this statement.

The limitation concerns the surface integrals of Chandrasekhar and Fermi's equations (**8**) and (**10**) which are assumed zero " since the magnetic pressure vanishes on the boundary of the configuration ". In general the magnetic field only vanishes at infinity, and if the integral is to be zero the boundary must be taken to infinity, the field strength H falling off in such a way that $H^2r^3\rightarrow0$ as $r\rightarrow\infty$, where r is the distance from the centre of the configuration. As will be shown, this limitation does not greatly reduce the value of the theorem but it does draw attention to the importance of the distribution of the field and its associated current system.

Chandrasekhar and Fermi give a relationship (**18**), a condition for the stability of a star or other configuration, relating the mean square magnetic field strength with the mass M and radius R (solar units) of a spherical configuration. To allow for the proportion η of magnetic energy outside the star this equation should be modified to

$$\sqrt{(H^2)_{\text{av.}}}<2\cdot0\times10^8(M/R^2)(1-\eta)^{\frac{1}{2}}, \quad \ldots\ldots\ldots\ldots \quad (2)$$

where $(H^2)_{\text{av.}}$ still refers to the field within the star. The value of η depends on the distribution of electric currents inside and outside the star. For example, if the currents flow entirely within a star which is uniformly magnetized, then $\eta=\frac{1}{3}$. If the currents flow deeper within the star, so that the magnetic field is stronger at lower levels, then η is smaller than $\frac{1}{3}$. However, if currents also flow outside the star, a case which is also covered by the theorem, then η is larger and may approach unity, in which case $(H^2)_{\text{av.}}$ must be vanishingly small.

Let us now consider the difference in the significance of magnetic fields due to currents flowing inside and outside a star. It will be shown that the virial theorem may have greater power and also greater safety when applied to the former alone.

If current, of density **j**, flows only inside a star then the field on the outside is given by curl $\mathbf{H}=4\pi\mathbf{j}=0$ and so may be written $\mathbf{H}=-\text{grad}\,\varphi$, φ being a scalar potential function. Beyond several stellar radii $\varphi \propto r^{-2}$ approximately, so that $H \propto r^{-3}$, which easily satisfies the new criterion of the applicability of the virial theorem. Now consider fields which fall off as $H \propto r^{-1\cdot 5-\delta}$ where $0<\delta<1\cdot 5$; these satisfy the criterion but fall off more slowly than those due to internal currents. At distances of many stellar radii from the star these fields must be due almost entirely to currents which are themselves distant many stellar radii. The proportion of magnetic flux through the star resulting from these currents is small because of the relatively small cross section of the star. Since only lines of force which link with the star may contribute to the disruptive force $\mathbf{j}\times\mathbf{H}$, the presence of the external currents and their fields may often be neglected, thus strengthening the theorem. On the other hand, the external field will, unless force-free, tend to disrupt the gas cloud in which the current flows. The gravitational force due to the star on this cloud may be negligible and instability may occur, even though the virial theorem indicates possible stability of the system as a whole. Thus neglect, or separate consideration, of the external current system is a safeguard against misuse of the theorem as evidence of the possible existence of a large-scale, external-current field.

We now consider the class of fields which do not satisfy the criterion $H^2r^3\to 0$ as $r\to\infty$. As has been pointed out by Chandrasekhar (personal communication) these include all the force-free fields so far investigated (the class of axisymmetric fields derived by Chandrasekhar (1956), which include the case of Lüst and Schlüter (1954)) and must include all force-free fields, otherwise the theorem is invalidated because such a field, no matter how powerful, cannot disrupt a star. The most noteworthy feature of these fields is their low rate of decrease with distance from the star and the corresponding inference that they are caused mainly by an external and widespread current system. For example, if the field at the surface of the star is due equally to internal and external currents, then at 10 radii an average of about 97 per cent. or more of the field is due to the external current system.

These extensive fields may be described by the relation $H^2r^3=\alpha$ where α may vary but never falls below α_m, which is finite. The total energy of the field is given by a volume integral

$$E=\int_V \frac{H^2}{8\pi}\mathrm{d}V > \frac{1}{8\pi}\int_V \frac{\alpha_m}{r^3}\mathrm{d}V$$
$$=\frac{\alpha_m}{2}\int_0^\infty \frac{\mathrm{d}r}{r}.$$

However small α_m, this expression is infinitely large. The force-free fields investigated by Chandrasekhar (1956, equations (**17**) and (**18**)) fall off even more slowly and also have infinitely great magnetic energy. This appears to constitute an objection to the existence of such fields in nature. The objection may be stated in an alternative way: Prendergast (1956) has shown that, if the magnetic field due to currents in a model configuration is force-free ($k=0$ in his equation

(**20**)), then there can be no field ($k=0$ in his equation (**45**) and so $H=0$). This means, as far as his model is concerned, that a force-free field cannot exist within any configuration, no matter how large, unless there are further currents flowing outside the configuration. It would seem unlikely then that significant force-free fields could occur in nature.

The possibility of a force-free field being associated with the Crab nebula is important in the arguments of Sections IV and V and may warrant further brief discussion. Lüst and Schlüter (1954) have stressed the possible importance of cosmic force-free fields on the grounds that the mechanical force $\mathbf{j}\times\mathbf{H}$ tends to destroy other fields and might leave a residue of force-free field. The reverse of this argument may be nearer the truth : it is generally accepted that cosmic fields are created by a dynamo action involving the electric induction field $\mathbf{v}\times\mathbf{H}$ due to the movement $\mathbf{v}$ of the gas. This field is perpendicular to $\mathbf{H}$ and gives rise to currents flowing perpendicular to $\mathbf{H}$, whereas the current in a force-free field flows only parallel to $\mathbf{H}$. These latter currents might possibly tend to flow as perturbations in the induced currents, due to build-up of charge on boundaries. However, whereas the original field could be built up rapidly by stretching the lines of force, the force-free field would take a time greater than $L^2\sigma_3$ where L is the size of the field and σ_3 the effective conductivity. For a field of radius 1 parsec this would be very large, thus raising a doubt as to the possible development of significant force-free fields.

IV. The Origin of the Magnetic Field

At its present rate of penetration the external (compressed interstellar) magnetic field would provide a negligible proportion of the internal field of the Crab nebula in a period of a few million years. The rate of penetration in the past was even lower and as the life of the nebula is only 900 years the external field must have the form shown approximately in Figure 1. Unless a field of other origin were present the nebula would be a magnetic vacuum. Thus the observed internal field must either have been present in association with the original star or have been created within the supernova shell after the explosion.

In considering the first of these two possibilities three types of magnetic field must be considered :

(*a*) Fields due to currents flowing in the star.

(*b*) Fields due mainly to currents outside the star but for which the virial theorem criterion ($r^3H^2\rightarrow 0$) holds.

(*c*) Fields for which this criterion does not hold.

Possibility (*a*) may be excluded by the following argument. Assume for the moment that the star was uniformly magnetized and had mass and radius equal to those of the Sun. Inequality (**2**) then gives a maximum magnetic field strength and total flux of less than $1\cdot 3\times 10^8$ G and $2\cdot 1\times 10^{30}$ G cm^2. If the lines of force are bent in any way, or unevenly spaced, then the maximum flux (through any plane intersecting the star) must be reduced. This follows because inequality (**2**) limits $(H^2)_{\text{av.}}$ and any irregularities cause an increase in $(H^2)_{\text{av.}}$ for a fixed maximum flux. The limit of $2\cdot 1\times 10^{30}$ must itself be much too high because near the surface of the star the magnetic pressure is about

7×10^{14} dyn cm^{-2} which corresponds to the gas pressure in the Sun at a depth of about half a solar radius and so would cause disruption of the outer layers of the star. The total permissible flux would be increased by assuming M larger. However, the theory of Type 1 supernovae sets a fairly definite upper limit of less than two solar masses. A decrease in R would result in an increase in average field strength but not of total flux because the area is correspondingly reduced. Inequality (2) is based on an assumption of uniform stellar density. Suppose instead that the density obeyed the law $\rho = \rho_0 r^{-s}$, where $0 < s < 2$ and the stellar radius remained R. The flux is increased by a factor $\{(3-s)/(5-2s)\}^{\frac{1}{2}}$ or, at most, $1 \cdot 3$. Furthermore, the difficulty mentioned above is accentuated, that, although the virial theorem may be satisfied, nevertheless the outer layers of the star are torn away.

It was seen in Section II above that the maximum flux through a region "where the field is more or less uniform in direction" is at least 10^{33} G cm^2. The form of this field is not known; it may be poloidal, or toroidal, or more complex. Within a factor of about 2 the shape of the field does not appear to matter when considering its flux. If the field were originally compressed within a star, then a flux of at least 10^{33} G cm^2 would have to be accounted for. It is concluded that this possibility, (*a*), may be excluded.

Chandrasekhar (reported by Oort and Walraven 1956, footnote, p. 304) applied the virial theorem to a star of solar dimensions and obtained a permissible magnetic field of total energy about $7 \cdot 6 \times 10^{48}$ ergs which, if "diluted" in a sphere of radius 1 parsec, provides a field strength of $1 \cdot 2 \times 10^{-3}$ G, a value close to that required by Oort and Walraven. This suggestion might take either of two forms depending on the meaning of "diluted". It might mean that the original field was contained mainly within the star, the dilution taking place after the explosion. The objections to this hypothesis, (*a*), are listed above. The error implicit is in comparing the energies of the field before and after expansion (instead of the total flux). If expansion is uniform, then the total energy falls by an amount proportional to the radius of the configuration, that is, by a factor of about 4×10^7 and so fails by a factor of about 10^7 to explain the observed energy.

The alternative interpretation is that the original field had dimensions of about 1 parsec; this corresponds to possibility (*b*) above. The use of the virial theorem in this way is not justified, since it implies stability of the whole configuration due to the gravitational attraction of the central star. The field and current system, both 1 parsec from the star, would not be appreciably affected by it and would explode. Furthermore, even if the required field had existed at the time of the explosion it would have been mainly *outside* the supernova shell and could not, as shown in Section II, have contributed appreciably to the flux now found inside the shell. The latter must have linked with the star at the time of the explosion.

Possibility (*c*) must include all types of force-free fields and, since these exert no force, the flux through the star is not limited by considerations of stability. However, there are other objections to such fields given in Section III in terms of the infinite extent and energy of the field. Required conditions within the star are equally improbable : to provide the necessary flux of more than 10^{33} G cm^2 a root-mean-square magnetic field greater than 10^{11} G is required and internal magnetic energy of more than 10^{54} ergs. The field is some 10^7 times stronger than any stellar field yet observed, the energy more than 10^5 times the turbulent kinetic energy if all the stellar material had a velocity of 1000 km sec^{-1} and more than 10^5 times the gravitational energy available as the star contracted from infinity.

It is concluded that possibility (*c*), together with (*a*) and (*b*), must be rejected : thus the field must have been created *within* the expanding shell, *outside* the stellar remnant, and *after* the supernova explosion.

Oort and Walraven (1956) have stated that the magnetic field " probably has its seat in the expanding shell of filaments " and consider that the rough correspondence in size of the shell and amorphous centre supports the inference that the field is connected with the filaments.

At the time of the explosion the maximum magnetic flux through the star was less than 10^{-3} of the present flux in the central part of the nebula. Since the shell was part of the star the flux through the shell at the time of the explosion was even less. If the present magnetic field had its origin in the shell, then the original field must have been increased by a factor greater than 10^3 (probably greater than 10^4). There are several reasons why this is not likely. The shell, of thickness less than 1/10 and perhaps 1/50 of its radius, has a well-defined identity. This means that internal motions of the shell have separated originally adjacent particles by less than 1/10 and perhaps 1/50 parsec. Any magnetic field created by this turbulence would have a scale of 1/10 parsec or less. Furthermore it would be confined mainly to the interior of the shell (Bondi and Gold 1950) except for the leakage discussed in Section II above due to finite conductivity. It would certainly be strongest in and near the shell and would fall off both outside and inside the shell. The nebular field, on the other hand, is known to have quite different characteristics, being more or less uniform over distances of about $\frac{1}{2}$–1 parsec and being strongest near the centre and weakest in and beyond the shell. To create such a field, pieces of the shell on opposite sides of the nebula would have to pass back and forth many times in a very ordered manner. It might be supposed that large-scale electric currents could be caused to flow in the shell by a process not yet understood in hydromagnetic theory. Apart from the lack of a known mechanism, objections to this hypothesis are met in the large self-induction of the system and the shape of the consequent magnetic field. The time of decay of a field of dimensions equal to the thickness of the shell was found to be at least 2×10^4 years. The corresponding time of decay of the whole field associated with the nebula would be much greater. Correspondingly, improbably powerful electric fields would be necessary to cause a build-up of currents and field in the 900 years available. Also such fields would again tend to be strongest in and near the shell. A final objection to the field originating in the shell in any way is met in energy considerations. The mass of the shell cannot be substantially more than one solar mass (see Section V) and its velocity is 1100 km sec^{-1}, so that its total kinetic energy is about 10^{49} ergs. There is evidence, discussed below, that not only has the shell not decelerated but that it has probably accelerated long after the explosion. Hence only the turbulent energy could have been turned into magnetic energy. The turbulent energy is and was considerably less than 10^{47} ergs and so could not have given rise to the field of energy about 10^{48} ergs.

The connexion observed by Oort and Walraven between the shell and the field does not require an interpretation in terms of the origin of the field. It would result, in any case, from the outward pressure of the field and cosmic ray gas and the retention of this " explosion " by the more massive shell. This is discussed below.

Summing up : it has been shown that the main magnetic flux could not have originally (or at any time) linked with the central star nor with the shell. It could not have been created by turbulent motion of the shell nor from the original interstellar field. Therefore *the lines of force are complete within the space between*

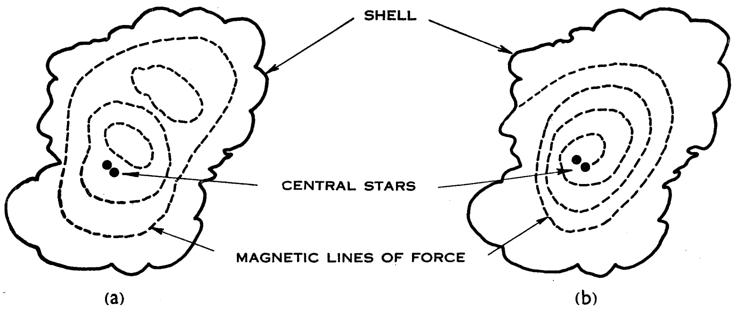

Fig. 2.—Schematic representation of possible loops of magnetic field within the Crab nebula. (*a*) Complete loops, some enclosing and some not enclosing the central star. (*b*) A spiral loop emerging from the region of the central star.

the star and the shell, as shown schematically in Figure 2 (a) *and were created within this space.* Further-more, as seen in the next section, there is evidence that some of the field was created recently and that the process may be still in operation.

One may speculate that this process contributes significantly to the whole galactic field. If a field of 10^{-3} G extending throughout a sphere of radius 1 parsec is expanded equally in all directions until its strength falls to a few times 10^{-6} G (the strength of the galactic field), then it occupies about 10^{-8} the volume of the galactic field (assumed a sphere of radius 10^4 parsecs). There may have been 10^7 supernovae and they may have contributed much more than assumed, so that the possibility is not unreasonable.

V. The Mass of the Nebula

A tentative value of the mass of the Crab nebula adopted by Shklovsky (1953) and Oort and Walraven is 10^{-2} solar mass. The new data now available permit a more definite estimate.

The pressure of the electron cosmic ray gas in the nebula is about 4×10^{-8} dyn cm^{-2}. The " magnetic pressure " $H^2/8\pi$ has a similar value.

The magnetic field will exert no outward force if the field is force-free, but such a situation has been shown to be most improbable. In any case the *total* pressure of the gas and field will be about 10^{-7} dyn cm^{-2} in some directions. The fact that the nebula is to a first approximation spherical indicates that either the internal pressure is reasonably uniform or it has not a first-order effect on the shell velocity.*

The total mass inside the shell is small compared to that of the shell and the pressure is large. It will be shown that the central region would " explode " with a velocity much greater than that observed were it not for the restraint offered by the shell inertia and perhaps also the effect of interstellar gas outside the shell. If the latter effect can be estimated then a determination of the mass of the shell is possible.

As the shell expands into the interstellar gas it creates a shock front as shown schematically in Figure 1, where A is the nebula surrounded by its shell, B is a mantle of gas moving with the shell, and C the undisturbed interstellar gas. The boundary between B and C is a shock front. The pressures, number densities, and temperatures of the gas (assumed hydrogen) in regions B and C are p_B, n_B, T_B and p_C, n_C, T_C respectively. The velocities of the gas B and the shock front are v and U, the former being equal to that of the shell, $1\cdot1\times10^8$ cm sec^{-1}.

It is assumed, for reasons given earlier, that the nebular magnetic field is retained within the shell. The most likely value of the original interstellar field in region C is 5×10^{-6} G, the corresponding magnetic pressure $H_C^2/8\pi$ being 10^{-12} dyn cm^{-2}.

Strictly speaking the shock phenomenon is hydromagnetic but approximates the simpler hydrodynamic shock provided the magnetic pressure ahead of the shock is much less than the force necessary to overcome the inertia of the still gas, that is, provided

$$H_C^2/8\pi \ll \tfrac{1}{2} n_C M v^2,$$

where M is the mass of a hydrogen atom, that is, provided $n_C \gg 10^{-4}$, which is fairly certain to be the case since the average interstellar density is 1 cm^{-3}. The shock will be analysed using the hydrodynamic equations of conservation of matter and momentum :

$$n_B(U-v)=n_C U, \quad \ldots\ldots\ldots\ldots \quad (3)$$

$$p_B+n_B M(U-v)^2=p_C+n_C M U^2. \quad \ldots\ldots\ldots\ldots \quad (4)$$

These equations may be solved, together with the corresponding energy equation (see, for example, Burgers (1951), equation (**141**)) to give

$$U=\frac{2}{3}v+\left(\frac{4}{9}v^2+u^2\right)^{\frac{1}{2}}, \quad \ldots\ldots\ldots\ldots \quad (5)$$

* In pesonal discussions Professor J. H. Oort has described tangential movements of parts of the shell. These are presumably due to magnetic field pressure and indicate that it is capable of second-order effects on the shell velocity. The somewhat irregular shape of the nebula may also be partly or wholly due to magnetic pressure variations.

where u is the velocity of sound in the region C and would not be greater than about 16 km sec^{-1} and so does not appreciably affect the value of U which is $1\cdot5\times10^8$ cm sec^{-1}. The pressure in the shock region is found from equations (3) and (4) and the fact that p_C may be neglected, so that

$$p_B=n_C MUv=2\cdot8\times10^{-8}n_C \text{ dyn cm}^{-2},$$

also from equation (3)

$$n_B=4n_C.$$

Since there is no sign of an interstellar gas cloud around the Crab nebula, the most likely value of n_C (the average intercloud value) is $0\cdot1$ cm^{-3}. The corresponding value of p_B is 3×10^{-9} dyn cm^{-2}, which means that the interstellar gas provides no significant force opposing the internal pressure of 10^{-7} dyn cm^{-2}. Only in the unlikely (but possible) event of the interstellar gas density attaining a value of $3\cdot6$ cm^{-3} would the internal pressure be neutralized by the external. It may be noted that in this case the mass of matter swept up between the shock front and the shell would be about two solar masses.

We now consider the (probable) case where the rate of expansion of the shell is controlled mainly by its own mass.

Baade (1942) has considered the relationship between the known diameter, rate of expansion, and age of the Crab nebula. He finds that, at the present rate of expansion, the observed size would be attained in about 760 years, whereas the age of the nebula is about 900 years. This means that the shell must have suffered substantial acceleration, not only at the initial explosion but during the later period of expansion. Baade arbitrarily assumed a constant rate of acceleration* and so found that the observational data fitted an initial velocity of 798 km sec^{-1} and a steady acceleration of $0\cdot0011$ cm sec^{-2}. Neglecting the mass of the gas inside the shell, the shell density ρ is connected with the internal pressure p and rate of acceleration A by the formula $p=\rho A$ and inserting appropriate values we find $\rho=9\cdot1\times10^{-5}$ g cm^{-2} corresponding to a shell mass of about $2\cdot3$ solar masses if the radius is $\frac{2}{3}$ parsec. It is possible to reduce this estimate by assuming that the acceleration was greater in the later stages of expansion. If we make the somewhat unlikely assumption that no acceleration took place for 750 years and uniform acceleration in the last 150 years, this results in a reduction of mass to $0\cdot5$ solar mass. Thus a mass of less than one solar mass seems unlikely.

Supernova theory provides strong evidence against a shell mass as high as two solar masses, although one solar mass is acceptable. The best estimate of the shell mass would seem to be one solar mass, provided, as seems likely, that the interstellar gas density is not greater than about 2 atoms cm^{-3}. In any case the total mass of the nebula and mantle would be between one and two solar masses.

* In the absence of any apparent suitable accelerating agency Baade was forced, at the conclusion of his analysis, to retract this assumption and conclude that the measured rate of angular expansion was too large. With the internal pressure now known to exist and to provide an accelerating agency, the analysis and results may stand.

VI. Irregular Motions in the Nebula

Observations of irregular motions within the central amorphous part of the nebula provide evidence of the continued creation of magnetic field.

Oort and Walraven describe " light-ripples " or drifting bright wisps observed by Baade and others near the centre of the nebula. These wisps appear about halfway between the central pair of stars and a more or less stationary and permanent wisp distant about 0·05 parsec from the stars. They drift with a velocity of about 3×10^9 cm sec^{-1} towards the stationary wisp with which they eventually merge. A typical wisp may emit about 1/1500th of the total light of the nebula and one may appear every three months or so. There are also light and dark patches in the outer parts of the (not completely) amorphous part of the nebula which appear to move with velocities of about 10^9 cm sec^{-1} or a little less. The rapidly moving wisps near the centre are polarized and so the radiation must be due to spiralling electrons.

There are only two possible explanations for these results : a mass drift of clouds of relativistic electrons, or some kind of wave motion. There are a number of objections to the first possibility. Some recent measurements of polarization* indicate that the light wisps appear to move directly across the magnetic field ; this effect can also be noted by comparing Oort and Walraven's Figures 10 and 15. Such motion is not possible, since the transverse motion is confined to relatively small circles (about 10^{-7} parsec for 10^{11} eV electrons in a field of 10^{-3} G). On the other hand the movement of electrons along the field is not inhibited and a cloud in a fairly uniform field (as observed) would expand with approximately the velocity of light. Such expansion is not observed, in fact the outer wisp, although only about $\frac{1}{5}$ light-year in size, has been a feature for about 30 years. It is concluded that the wisps are a wave phenomenon.

Of four possible wave types (Piddington 1955*a*) the two shear hydromagnetic waves have suitable properties to explain the light wisps. Both phase and group velocities are $V=H_0(4\pi\rho)^{-\frac{1}{2}}$ where ρ is the gas mass density. Any wave originating near the central star will travel out as a " wave packet " comprising an additional portion of magnetic energy and flux introduced into the otherwise uniform field. The flow of energy is given by the Poynting vector†

$$P=H_0H_pv/4\pi, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (6)$$

where H_0 and H_p are the steady and perturbation fields and v the gas velocity given by

$$v=VH_p/H_0. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (7)$$

Thus we have the proportionality

$$P\propto H_0H_p^2\rho^{-\frac{1}{2}}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (8)$$

Since the gas in the region concerned must be fully ionized, these waves will suffer negligible damping (Piddington 1957) and P will decrease with an inverse distance-

* Kindly shown to me by Professor J. H. Oort ahead of publication.

† These formulae (Piddington 1955*b*) have been derived for weak waves but they will be assumed here to hold approximately for perturbation fields as strong as the steady field.

squared law (assuming spherical waves). Over short distances P is nearly constant and so $H_p \propto \rho^{\frac{1}{2}}$; when the wave encounters some denser gas it slows down and the peak magnetic field ($\mathbf{H}_0+\mathbf{H}_p$) increases.

The stronger magnetic field in the wave packet accounts for the additional visual emission (the so-called light wisps). According to Oort and Walraven, the average emission for their derived cosmic ray spectrum is proportional to $H^{2 \cdot 15}$, where H is the total magnetic field. Thus if $\mathbf{H}_p=\mathbf{H}_0$ the emission in the wave increases by a factor of about 4·4; actually the factor would be larger because the 2·15 law refers to a single electron and does not take account of the compression of the cosmic ray gas. The appearance and disappearance of the wisps results from the wave travelling through regions of greater and less density (that is, density of the " cold " gas, not the cosmic ray gas, although the latter could also cause variations); when ρ increases, H_p increases, and so the emission. The density of gas where the wisps are visible is found from the known values of $\mathbf{H}_0=10^{-3}$ G and $V=3\times10^9$ cm sec^{-1}; it is about 10^{-26} g cm^{-3} or a proton density of 6×10^{-3} cm^{-3}, which, although surprisingly small, is about 5000 times greater than the cosmic ray density. The corresponding mass of the central regions (volume 10^{55} cm^3) is about 1/2000 solar mass and so negligible compared with that of the shell.

The semi-permanent wisp may be explained in terms of a region of higher gas density which so slows down the succession of wave packets that it is continually illuminated. Its long, thin form might result from diffusion of the gas, which can only take place along the magnetic field. If its temperature were 10^6 °K the time to diffuse to the observed length of 1/20 parsec would be several hundred years, so that its semi-permanent existence (for 30 years) is easily explained.

The average flow of magnetic energy into the nebula has been estimated at about 3×10^{37} erg sec^{-1}. If in equation (**6**) we put $H_p=H_0=10^{-3}$ G and $v=V=3\times10^9$ cm sec^{-1} the flow found is 240 erg cm^{-2} sec^{-1}. Through a sphere of radius 1/20 parsec the total flow is $7\cdot2\times10^{37}$ erg sec^{-1}. Such close agreement with the required flow is no doubt fortuitous, but shows that the theory of wave packets is consistent with the energy requirements found otherwise.

VII. The Origin of the Cosmic Rays

The energy of the cosmic ray electrons is depleted in providing the observed radiation and must be replenished every 200 years or so at a rate determined by Oort and Walraven as a few times 10^{37} erg sec^{-1} or some 10^4 times the total solar power output. This presents an outstanding astrophysical problem which they answered with the suggestion that fresh cosmic rays were continually squirted out from the central star. This emission, they believe, would also account for the light wisps.

This problem might now be replaced by another: the origin of about the same amount of energy in the form of magnetic wave packets. These are needed to explain the origin of the nebular magnetic field; they might also explain the light wisps and will now be considered in connexion with the origin of cosmic rays.

The total energy flux over a sphere of radius 1/20 parsec of hydromagnetic waves of strength $H_p = H_0 = 10^{-3}$ G and velocity 3×10^9 cm sec^{-1} is about 10^{38} erg sec^{-1} and so is of the required order of magnitude to provide the cosmic ray energy. There exists a well-known mechanism of conversion of electromagnetic to cosmic ray energy by statistical acceleration. Originally due to Fermi (1949) this has been extended to include the betatron and other possible effects. It appears that with the short length and high velocity of the waves, inferred from the movements in the amorphous region, the mechanism is extremely efficient.

The gain of energy δE when a cosmic ray electron " collides " with a magnetic field is $\delta E = E(v/c)^2$. If the irregularities in the magnetic field are hydromagnetic waves of frequency ν and velocity V then the collision rate is approximately $c\nu/\sqrt{3}V$, and since $v = H_p V/H_0$ we have

$$\frac{1}{E}\frac{\mathrm{d}E}{\mathrm{d}t} = \frac{\nu V}{\sqrt{3}c}\left(\frac{H_p}{H_0}\right)^2.$$

This agrees with Thompson's (1955) formulation provided $v \sim 0 \cdot 4V$ ($H_p \sim 0 \cdot 4H_0$), a condition which is likely to hold in the Crab nebula. If the waves have a length $0 \cdot 01$ parsec (the size of the light ripples), strength (H_p/H_0) of $0 \cdot 4$, and velocity 3×10^9 cm sec^{-1}, then the time taken for the energy of the particle to increase by a factor exp 1 is about 36 years. The nebula is full of cosmic ray electrons which decay, due to radiation by the synchrotron process, in a period of about 200 years. If a fraction of the nebula is filled with hydromagnetic waves capable of returning the lost energy in a period of 36 years then the supply of high-energy electrons could be maintained at the observed level.

The theory of hydromagnetic wave packets provides a single explanation for the magnetic field of the nebula, for the light wisps, and for the cosmic rays. It also removes a difficulty met by Ginzburg, Pikelner, and Shklovsky (1955), who found that, while supernovae seemed to provide concentrations of cosmic rays, the statistical process during expansion would be expected to *decelerate* particles and so cause the removal of cosmic rays. In turn this may remove an objection to the widely known theory that most galactic cosmic rays have their origin in nova and supernova explosions. On the present interpretation of conditions in the Crab nebula the theory would be changed somewhat: the acceleration process continues for long after the explosion in the region between the star and its receding shell. If as many cosmic ray protons as electrons were created, their total energy would now be about 10^{49} ergs and might be much more in the future. If this were average for all supernovae it might provide a fair proportion of all galactic cosmic rays.

One difficulty remains that the Fermi acceleration greatly favours protons above electrons. If the initial velocities are not relativistic then the protons gain energy some 2000 times faster than the electrons. Furthermore, the protons do not lose so much energy by radiation. The difficulty might be overcome by an injection mechanism which gave the electrons more kinetic energy than the protons.

VIII. The Generation of the Magnetic Field

If the conclusions of Section IV are accepted, then magnetic lines of force, complete within the interior of the nebula, are being (or have been during much of the life of the nebula) created in the form shown schematically in Figure 2 (*a*). The total magnetic energy is about 10^{48} ergs and the average rate of generation therefore about 3×10^{37} erg sec^{-1}. This is more than 10^4 times the energy output of the Sun and corresponds to one average nova explosion (10^{45} ergs) every year or so. Such an enormous quantity of energy must have its origin in nuclear processes, presumably within the central star. Thus, in addition to the initial explosion, the star would have experienced a more or less " steady explosion " during at least a part of its lifetime. This phenomenon is probably occurring at the present time, the evidence being the light wisps and the continuing supply of radiating cosmic ray electrons throughout the nebula. As shown above, the light ripples seen may indicate a considerable flow of magnetic energy from the central region.

The introduction of fresh complete loops of magnetic field into the central region seems possible in only one way: by a continual stretching and spinning of existing loops. The process is illustrated, in part, in Figure 2 (*b*), where the central region is assumed to rotate in a clockwise direction. Since div $\mathbf{H}=0$ everywhere, the line of force shown must be completed somehow. Its path will be through the shell and the star and then either through another spiral wound in the opposite direction or by a more direct path between star and shell. Two opposed spirals placed together will start to annihilate one another but the process is soon stopped by the development of a layer of gas between them which is more or less impervious to the field and separates its two components. In its simplest form the whole model would comprise an initial, relatively weak, axially symmetric poloidal field (say a dipole field) in the star before it exploded. After the explosion, lines of force would link both shell and remnant and differential rotation would cause two toroidal fields to be formed.

Without a source of energy providing a torque to maintain rotation the spinning central configuration would soon slow down. The energy must be provided by nuclear reactions which result in particles being ejected along more or less radial lines of magnetic force. A suitable twist in the magnetic field allows this energy to be utilized and also to provide the required momentum to counteract the tension of the magnetic lines of force. This problem has been solved in a two-dimensional case and will be discussed elsewhere. The total magnetic energy is about 10^{48} erg and the total material within the nebula is perhaps 10^{29} g. If all this (and no other) material had been used in the above manner to create magnetic from kinetic energy, and if the efficiency of the process had been, say, 50 per cent., then each particle emitted must have had energy of about 2×10^7 eV. This does not seem unreasonable and the corresponding energy after the process (1×10^7 eV) would alleviate the difficulty mentioned in the previous section, that protons and other heavy ions would be accelerated too efficiently to cosmic ray energies. The speed of rotation of the central configuration may be roughly estimated on the assumption of an initial field of say 1000 G at the surface of a star the size of the Sun. The field needs amplifying by a factor 10^8, needing 10^8 revolutions in 900 years or one revolution in 5 min.*

IX. Acknowledgments

The author is grateful to Professor S. Chandrasekhar of Yerkes Observatory; to Drs. W. Baade, A. Deutch, and R. Minkowski of the Mount Wilson Observatory; and to Professor J. H. Oort of Leiden Observatory for helpful discussions.

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* *Note added in Proof.*—The recent demonstration of non-conservation of parity raises an interesting possibility. Unstable nuclei, aligned in a cosmic magnetic field, could create fresh field as a disintegration product. If particular ions were ejected in preferred directions this would result in fresh electric current and magnetic field.