THE DESIGN OF PHOTOGRAPHIC OBJECTIVES OF THE TRIPLET FAMILY

I. THE DESIGN OF THE TRIPLET TYPE 111 OBJECTIVE

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Summary

A general study has been made of the initial design of photographic objectives of the triplet family of objectives. A classification is suggested. The point of view is taken that it is convenient to regard the first two members of a triplet as constituting a compound corrector system to the rear positive member.

Part I. An account is given of the design of the type 111 triplet objective to which any triplet may be reduced. A simple algebraic solution is then possible for the arrangement of the initial thin component system having any selected corrector power, χ. It is shown that in triplets of unit power the only systems of any practical interest have corrector powers approximately in the range $-2<\chi<0\cdot4$. In this range χ specifies the distribution of power between the two positive members of the triplet and is the variable which determines the spherical aberration when all other primary aberrations have been adjusted to desired small values. The effect of the residual values of the Petzval curvature and the chromatic aberration on the initial solution is discussed. The method provides a starting point for the development of the design of any objective having a basic triplet structure.

I. INTRODUCTION

It is proposed to consider the general problem of the initial design of photographic objectives which are basically of a triplet construction. For this purpose it is convenient to regard these as constituting a family of lens systems logically derived from the simple triplet, the process of derivation consisting of the replacement of one or more members of the parent triplet by a group of lenses. The replacing group is commonly a cemented doublet or triplet. Some of the members of this family are very well known and are currently manufactured under various names such as the Tessar, the Heliar, the Sonnar, etc. In Figure 1 some of the possible constructions within the family are shown, the arrangement being determined by the complexity of the components. In this scheme it is convenient to assign to each construction a type number based on the number of elements in the three component groups. Thus the Tessar is specified as a type (1, 1, 2) triplet or, more briefly, type 112. On the same basis the Heliar is a type 212 triplet, the Sonnar is a type 133 triplet, and so on. The broad problem then is to establish a general pattern according to which the design of any member of this family of objectives may be developed from first principles.

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The process of designing an optical system falls broadly into three stages. In the first of these it is usual to determine an initial arrangement of thin lenses in which certain basic requirements such as power, Petzval sum, and paraxial achromatism are met. In the second stage the requisite axial thicknesses are introduced and the shapes of the lenses determined so that the primary (third order) aberrations of the system are under control. In the third stage the secondary (fifth order) and higher order aberrations are determined and some means of differential correction employed for the final adjustment of the system until the desired balance of aberrations is achieved. An early problem then is

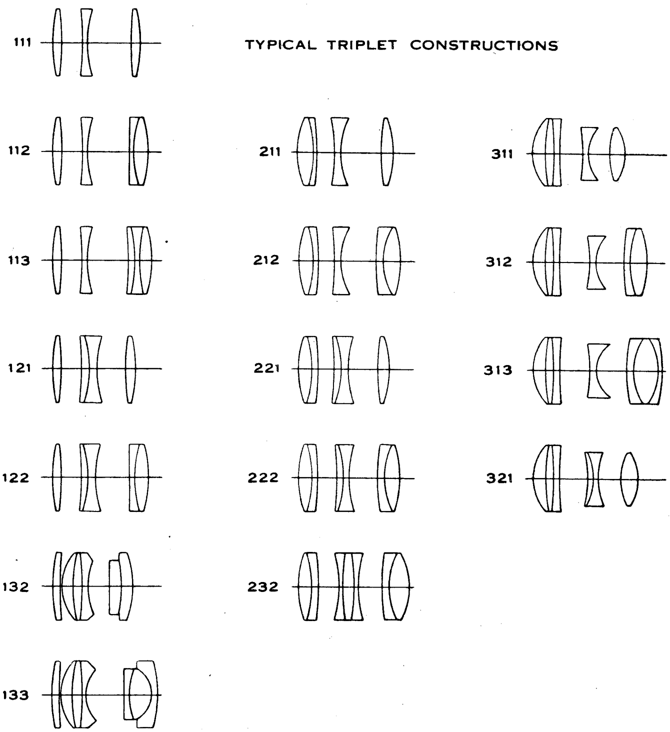

Fig. 1.—Some members of the triplet family of objectives.

to obtain the initial thin lens arrangement in the case of the parent triplet, type 111. This is discussed in detail in the opening sections of the present paper (Part I). This must be in a form capable of application to the more complex members of the family of objectives. Any other member of the family may then be considered as derived from a type 111 triplet, the derivation involving the replacement of one or more of the thin singlet components of the parent type by thin doublets or triplets.

In Part II (in preparation) this process of replacement is discussed. Consider the thin lens (φ, N, V) of power φ made from glass of refractive index N and V-number V. Suppose this is to be replaced by a doublet consisting of two thin lenses (φ_1, N_1, V_1), ($k\varphi_1$, N_2, V_2) in contact. It will be obvious that this can be done in such a way that the doublet has the same power and *either* the same Petzval sum *or* the same paraxial chromatic aberrations as the singlet lens which it replaces. Conversely, such a doublet constructed from some pair of glasses is equivalent as regards power, Petzval sum, and chromatic aberrations to a singlet lens having glass constants (N, V) which need not correspond with those of any known glass. These values (N, V) may be varied widely by intelligent choice of the two glasses of the doublet and the value of k. Again, when the replacement of a single lens by a cemented triplet is considered it will be seen that it is formally possible to design a thin cemented triplet which is equivalent as regards power, Petzval sum, and chromatic aberrations to a single thin lens having any desired value of N and, independently thereof, any desired value of V.

From the point of view of the first stage of design, then, the passage from the basic type 111 triplet to any other member of the family is equivalent simply to a change of glass. In other words, in considering the initial design of the basic type 111 the glass constants of the thin lens components may be treated as continuously variable within a certain range. Values which do not correspond to known glass types may be achieved by using compound components, and this leads to the multiplicity of types which constitute the family of objectives. The effect of the glass constants on the basic properties of the triplet becomes therefore a very fundamental problem. It is investigated in Part II. In Part III (in preparation) the special problem of the high aperture triplet is discussed.

II. The Design of the Triplet Type 111 Objective

Taylor (1893) arrived at the triplet construction by considering what happens when the components of an achromatic doublet are separated. The separation increases the power of the system without adding to the Petzval sum, and, in addition, if achromatism is to be maintained after separation, the power of the flint lens must be increased. Each of these effects provided a means for reducing the Petzval sum. The separated doublet, however, has considerable distortion and transverse chromatic aberration. Taylor met this situation by splitting the convex lens into two parts and placing these one on either side of the negative lens. From the method of its evolution, then, the triplet is to be regarded as a modified dialyte.

As already described (Cruickshank 1956) another view of the derivation of the triplet leads to more profitable procedures for its design. In 1812 Wollaston proposed an objective for the camera obscura consisting of a single meniscus positive lens placed behind a diaphragm. The triplet objective may be regarded logically as derived from the Wollaston lens by the addition of a compound correcting system comprised of a positive and a negative lens placed in front of the diaphragm. The function of this corrector system is to introduce aberrations which will compensate those of the single positive lens behind the diaphragm. In many triplets it is found that the positive and negative lenses in front of the diaphragm constitute a system of zero or very low power. Investigation shows, however, that it is not necessary to limit the corrector to zero or very low power.

(a) *The Initial Arrangement of the System*

Consider the important practical case of the initial design of a triplet photographic objective corrected for an object plane at infinity. Three different glasses will be used for the components. Disregarding the axial thickness of the lenses, five parameters are required to specify the system initially, namely, the powers φ_a, φ_b, φ_c, and the separations t_1, t_2, of the three co-axial thin lenses a, b, and c. The following five conditions may then be fulfilled: (i) the power of the system shall be unity; (ii) the power of the corrector system shall be χ; (iii) the system shall have a value R_4 for the Petzval sum; and for an object plane at infinity the system shall have (iv) a residual axial chromatic aberration, R_6, given in transverse measure, and (v) a residual transverse chromatic aberration R_7 for a pencil of obliquity u_a and a diaphragm coincident with the central lens b. Using well-known relations for systems of separated thin lenses in air these five conditions may be formulated analytically as follows:

$$1/y_{0a}\sum_{j=a}^{c}\varphi_j y_{0j}=1, \qquad (1)$$

$$1/y_{0a}\sum_{j=a}^{b}\varphi_j y_{0j}=\chi, \qquad (2)$$

$$\sum_{j=a}^{c}\varphi_j/N_j=R_4, \qquad (3)$$

$$1/u'_{0c}\sum_{j=a}^{c}\varphi_j y_{0j}^2/V_j=R_6, \qquad (4)$$

$$1/u'_{0c}\sum_{j=a}^{c}\varphi_j y_{0j} y_j/V_j=R_7, \qquad (5)$$

where y_j and y_{0j} are the incidence heights at the jth component of a principal paraxial ray of obliquity u_a and an axial paraxial ray respectively, and u'_{0j} is the inclination angle of the axial ray after refraction at the jth component.

Since the diaphragm is initially in coincidence with the second thin lens of the system the incidence heights of the principal paraxial ray are such that

$$y_a/y_c=-t_1/t_2, \quad y_b=0, \qquad (6)$$

while for the axial ray

$$\begin{aligned} y_{0b}&=y_{0a}-t_1u'_{0a}\\ &=y_{0a}(1-t_1\varphi_a), \end{aligned} \qquad (7)$$

and

$$y_{0c}=y_{0b}-t_2y_{0a}\chi. \qquad (8)$$

A substantial reduction in symbols specifying the glasses of the system is achieved by writing

$$V_a/V_b=\alpha, \quad V_a/V_c=\xi, \qquad (9)$$

$$N_a/N_b=\beta, \quad N_a/N_c=\gamma. \qquad (10)$$

In addition we will also write

$$y_{0b}/y_{0a}=\eta_{0b}, \quad y_{0c}/y_{0a}=\eta_{0c}, \qquad (11)$$

$$R_4N_a=P, \quad R_6V_a/y_{0a}=L, \quad R_7V_a/u_a=T \qquad (12)$$

With this notation equations (**1**)–(**8**) now become

$$\varphi_a+\eta_{0b}\varphi_b+\eta_{0c}\varphi_c=1, \quad \text{(13)}$$
$$\varphi_a+\eta_{0b}\varphi_b=\chi, \quad \text{(14)}$$
$$\varphi_a+\beta\varphi_b+\gamma\varphi_c=P, \quad \text{(15)}$$
$$\varphi_a+\alpha\eta_{0b}^2\varphi_b+\xi\eta_{0c}^2\varphi_c=L, \quad \text{(16)}$$
$$(1+T)t_1\varphi_a-t_2\xi\eta_{0c}\varphi_c=T, \quad \text{(17)}$$
$$1-t_1\varphi_a=\eta_{0b}, \quad \text{(18)}$$
$$\eta_{0b}-t_2\chi=\eta_{0c}. \quad \text{(19)}$$

The solution of these equations is quite simple. Equations (**13**) and (**14**) give at once

$$\eta_{0c}\varphi_c=1-\chi, \quad \text{(20)}$$

and, combining this with equations (**17**)–(**19**), we obtain

$$t_1=(1-\eta_{0b})/\varphi_a, \quad \text{(21)}$$
$$t_2=[1-(1+T)\eta_{0b}]/\xi(1-\chi). \quad \text{(22)}$$

Substitution in (**19**) gives

$$\varkappa\eta_{0b}-\xi(1-\chi)\eta_{0c}=\chi, \quad \text{(23)}$$

where

$$\varkappa=\xi+\chi(1-\xi+T). \quad \text{(24)}$$

Subtracting (**14**) from (**16**) yields

$$\varphi_b\eta_{0b}(\alpha\eta_{0b}-1)+\xi(1-\chi)\eta_{0c}=L-\chi,$$

which on combination with (**23**) leads to

$$\varphi_b=(\varkappa\eta_{0b}-L)/(1-\alpha\eta_{0b})\eta_{0b}. \quad \text{(25)}$$

Equations (**20**) and (**21**) give at once

$$\varphi_c=\xi(1-\chi)^2/(\varkappa\eta_{0b}-\chi), \quad \text{(26)}$$

while (**14**) and (**25**) give

$$\begin{aligned}\varphi_a&=\chi-\eta_{0b}\varphi_b\\&=[L+\chi-(\varkappa+\alpha\chi)\eta_{0b}]/(1-\alpha\eta_{0b}).\end{aligned} \quad \text{(27)}$$

Eliminating φ_a, φ_b, φ_c from equation (**15**) by means of (**25**)–(**27**) we obtain the cubic equation

$$G_3\eta_{0b}^3+G_2\eta_{0b}^2+G_1\eta_{0b}+G_0=0, \quad \text{(28)}$$

where

$$G_3=\varkappa^2+\alpha\varkappa(\chi-P), \quad \text{(29)}$$
$$G_2=\varkappa(P-L-2\chi-\beta\varkappa)+\alpha[\gamma\xi(1-\chi)^2-\chi(\chi-P)], \quad \text{(30)}$$
$$G_1=\beta\varkappa(\chi+L)-[\gamma\xi(1-\chi)^2-\chi(\chi-P)]+L\chi, \quad \text{(31)}$$
$$G_0=-L\beta\chi. \quad \text{(32)}$$

These coefficients are computed and the cubic equation (**28**) is easily solved either by, say, Cardan's method or by successive approximation. In general only one root of this equation gives a physically useful solution. There is an important exception, however, in one case in which a second real root leads to the possibility of the construction of another group of objectives of high aperture. These will be considered separately later. With the value of η_{0b} obtained from (**28**), the values of the powers and separations are calculated from the appropriate foregoing equations, and the initial arrangement is thus determined. If $L=0$, the coefficient G_0 vanishes and the cubic (**28**) reduces to a quadratic.

(b) Triplets from Two Glasses Only

It is very common practice to use two glasses only in the construction of the triplet, the two positive components a and c being of the same crown glass. In this case, then,

$$\gamma=1=\xi, \quad \varkappa=1+\chi T.$$

Inserting these values in the equations of the previous section we obtain in place of (**28**) the modified cubic

$$g_3\eta_{0b}^3+g_2\eta_{0b}^2+g_1\eta_{0b}+g_0=0, \quad \ldots\ldots\ldots\ldots \quad (\mathbf{33})$$

in which

$$g_3=\varkappa^2+\alpha\varkappa(\chi-P), \quad \ldots\ldots\ldots\ldots \quad (\mathbf{34})$$

$$g_2=\varkappa(P-L-2\chi-\beta\varkappa)+\alpha[1+\chi(P-2)], \quad \ldots\ldots \quad (\mathbf{35})$$

$$g_1=\beta\varkappa(\chi+L)+\chi(L-P+2)-1, \quad \ldots\ldots\ldots\ldots \quad (\mathbf{36})$$

$$g_0=-L\beta\chi. \quad \ldots\ldots\ldots\ldots \quad (\mathbf{37})$$

The expressions for the powers and separations become

$$\varphi_b=(\varkappa\eta_{0b}-L)/(1-\alpha\eta_{0b})\eta_{0b}, \quad \ldots\ldots\ldots\ldots \quad (\mathbf{38})$$

$$\varphi_c=(1-\chi)^2/(\varkappa\eta_{0b}-\chi), \quad \ldots\ldots\ldots\ldots \quad (\mathbf{39})$$

$$\varphi_a=\chi-\eta_{0b}\varphi_b, \quad \ldots\ldots\ldots\ldots \quad (\mathbf{40})$$

$$t_1=(1-\eta_{0b})/\varphi_a, \quad \ldots\ldots\ldots\ldots \quad (\mathbf{41})$$

$$t_2=[1-\eta_{0b}(1+T)]/(1-\chi). \quad \ldots\ldots\ldots\ldots \quad (\mathbf{42})$$

(c) Discussion of the Solution for a Given Set of Glasses

The general characteristics of the equations of the previous sections should now be considered. Suppose that a selection of three glasses is made for the system and a set of values chosen for the aberration residuals P, L, and T. The G coefficients in equation (**28**) depend thereafter only on the value of χ, so that in this situation η_{0b}, φ_a, φ_b, φ_c, t_1, and t_2 are effectively functions of χ only. It is necessary to consider then how these quantities vary with χ, i.e. how the initial arrangement of the triplet depends on the power of the corrector system. In Figures 2 and 3 the variation of each parameter with χ is shown for a typical triplet in which

$N_a=1\cdot6226$	$V_a=60\cdot2$	$P=0\cdot40$
$N_b=1\cdot61706$	$V_b=36\cdot53$	$L=0$
$N_c=1\cdot6226$	$V_c=60\cdot2$	$T=0$

It is clear that values of χ in the range $0 \cdot 4 < \chi < 1$ are to be excluded because the discontinuity which occurs here either provides no solution or one in which the powers of the components and the large rear airspace are totally unsuitable. For values of $\chi > 1$, φ_c and t_2 become negative. The system would then consist of two negative lenses with a strong positive lens between them, but the negative separation renders the arrangement impracticable, at least with simple lenses.

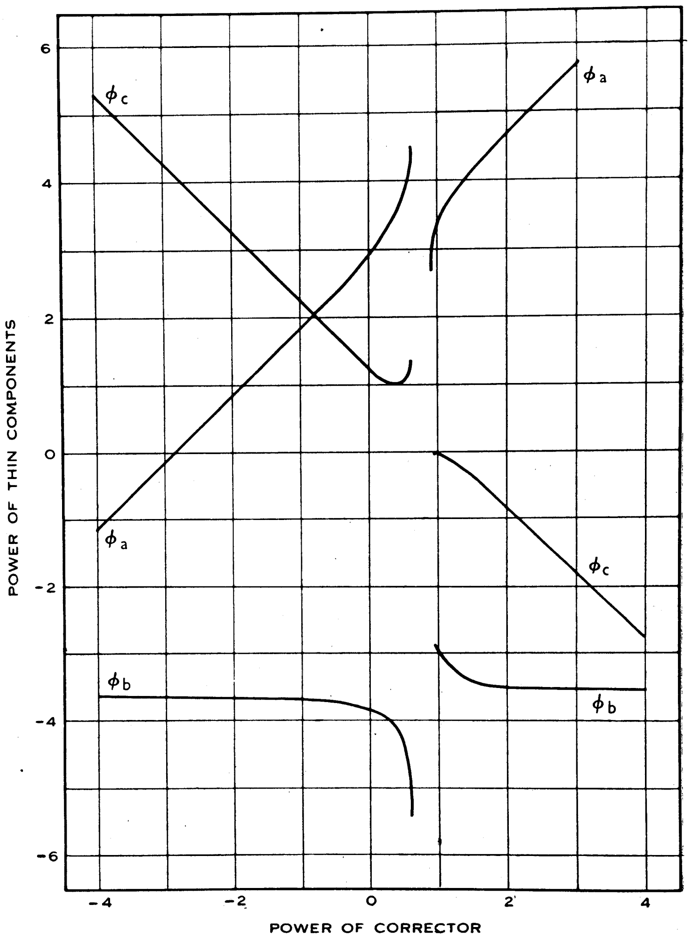

Fig. 2.—The curves show the dependence of the powers, φ_a, φ_b, φ_c, of the three thin components of a unit power triplet upon the parameter χ. The useful range of χ-values is seen to be approximately $-2 < \chi < 0 \cdot 4$.

Again, if $\chi < -2 \cdot 2$, the rapidly increasing front airspace renders the solution useless, in addition to which φ_a and t_1 soon become negative, providing again an impracticable arrangement. We are left then with the values $-2 < \chi < 0 \cdot 4$ as approximately defining the useful range of χ.

It will be noted that in this range φ_b changes very little with χ, while φ_a and φ_c vary rapidly and almost linearly with χ. Moreover, the slopes of these two lines are almost opposite. Within this practical range of χ-values, therefore, a change in χ leaves the negative lens almost unaffected but results in an exchange of power between the two positive lenses. In addition, as Figure 3 shows, there are changes in the airspaces. At one end of the range the front airspace is small and the back airspace large, while at the other end of the range the situation is reversed. Near the middle of the range the total thickness of the system is a minimum. *The essential point is that χ determines the distribution of power between the two positive components of the system.*

There now remains the consideration of the effect on the solutions of variations of the aberration residuals, P, L, and T. Typical changes in the powers and separations due to the variation of P alone are shown in Figures 4

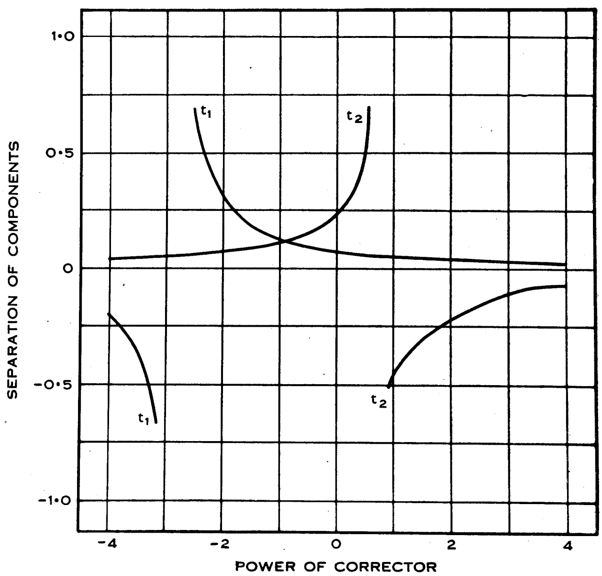

Fig. 3.—The curves show how the airspaces, t_1 and t_2, vary with the parameter χ in a unit power triplet employing the glasses specified in Section II (*c*).

and 5. As is to be expected from equation (**15**), an increasing positive value of P is accompanied by reduction in the power of all components, though the power of the back lens is affected least. This reduction in the curvatures throughout the system following relaxation of the Petzval condition is accompanied by increase of the front airspace and decrease of the rear airspace. In Figures 6 and 7 the effects of variation of the residual L are shown. If it is sought to adjust the longitudinal chromatic aberration of an objective to zero for the $0 \cdot 7$ zone of the aperture, as is frequently desired, an appropriate positive residual value for L will be required. The figures show that this will result in reductions in the powers of the first two lenses and a very slight increase in the power of the third lens. The accompanying increases in the airspace are quite substantial, however, and have to be considered carefully in the design of the triplet. There is little

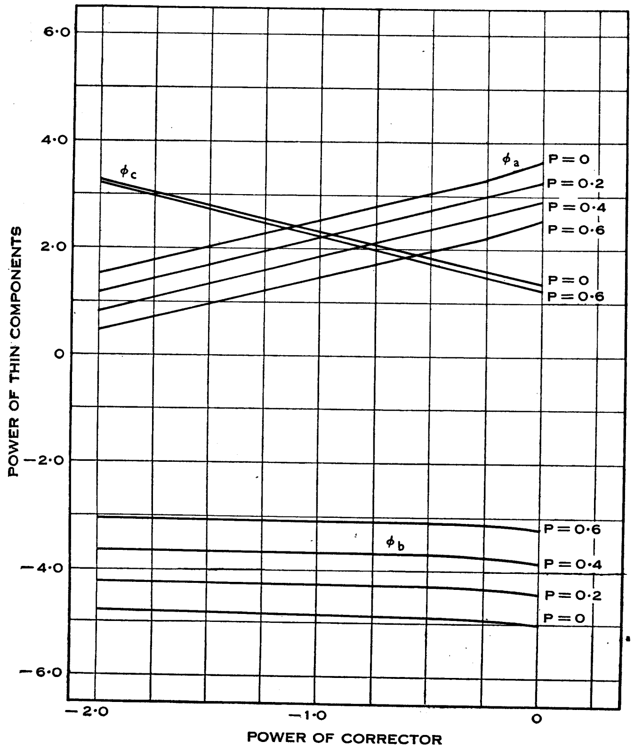

Fig. 4.—The graphs show the effect of the variation of the Petzval residual P upon the powers, φ_a, φ_b, φ_c, of the thin components of a unit power triplet. Increase of P reduces the absolute values of the powers of lenses a and b, but has little effect upon that of lens c.

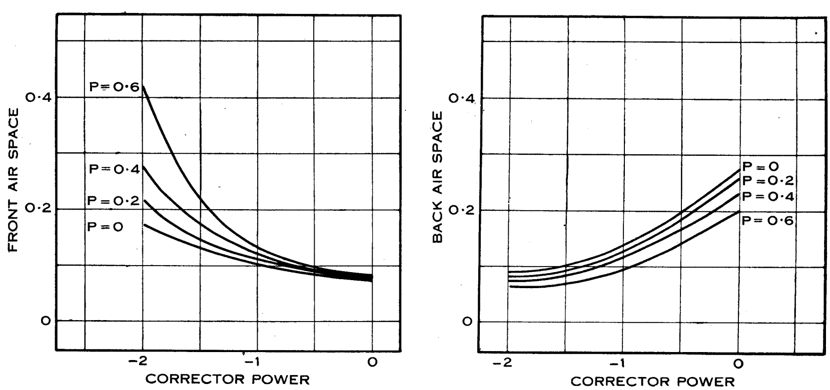

Fig. 5.—The corresponding effect upon the airspaces accompanying variation of the Petzval residual P is shown. An increase in the value of P enlarges the front airspace, particularly for large negative values of χ, and diminishes the back airspace. For values of χ near zero it is the rear airspace that is mainly affected.

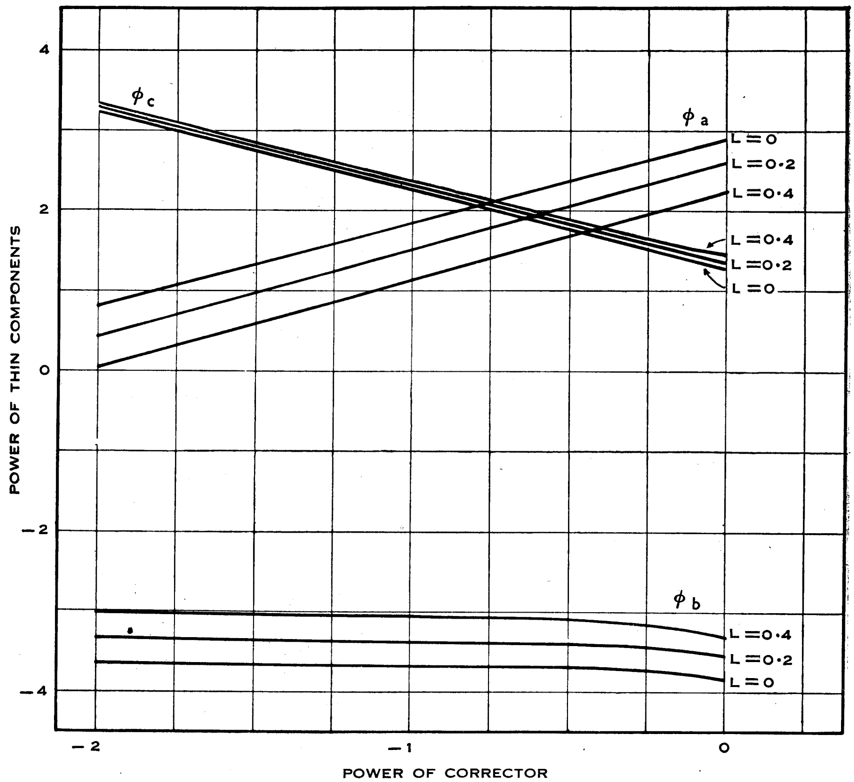

Fig. 6.—The effect of the variation of the axial chromatic aberration residual L upon the powers of the thin components is shown. A positive value of L, corresponding to a correction of longitudinal colour at an outer zone of the aperture, reduces the total curvatures in the first two components, but increases the power of the rear positive lens slightly.

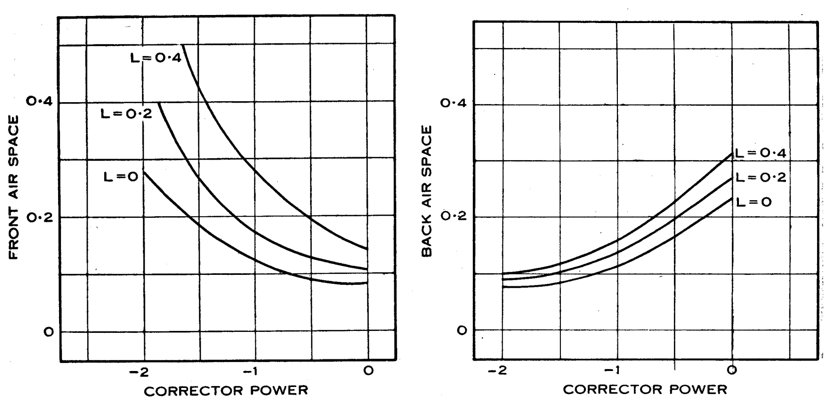

Fig. 7.—The effect on the airspaces of a variation of L is exhibited in these graphs. Both separations are enlarged by the introduction of a positive residual value for L.

effect from the variation of T over the small range of values which may be required to achieve a satisfactory compromise for the transverse chromatic aberration across the field. The changes in the powers and separations of the components are only of the order of 1 or 2 per cent. of their values and have not been plotted. This statement as to T may require modification later when consideration is given to the design of objectives with compound members.

(*d*) *The Adjustment of Primary Coma, Astigmatism, and Distortion*

For the adjustment of the primary astigmatism, distortion, and coma we use the degrees of freedom available in the selection of the shapes, S_a, S_b, S_c, of the three components. Either the shape function introduced by Coddington or that of Argentieri is suitable. Using the Coddington shape function defined by

$$S=(c_1+c_2)/(c_1-c_2),$$

where c_1 and c_2 are the curvatures of the two surfaces of the thin lens, it is well known that for a system of separated thin lenses the coefficients of the primary coma, astigmatism, and distortion are given respectively by equations of the form

$$\sigma_2=\Sigma(a_{21}S^2+a_{22}S+a_{23}), \qquad \text{(43)}$$

$$\sigma_3=\Sigma(a_{31}S^2+a_{32}S+a_{33}), \qquad \text{(44)}$$

$$\sigma_5=\Sigma(a_{51}S^2+a_{52}S+a_{53}). \qquad \text{(45)}$$

The partial coefficient of each lens in respect of each of these aberrations is thus a quadratic function of its shape and the summation is extended over all the lenses. If the diaphragm coincides with one of the thin lenses, then for that lens

$$a_{21}=a_{31}=a_{32}=a_{51}=a_{52}=a_{53}=0.$$

In the foregoing work the diaphragm of the triplet has been set initially in coincidence with lens *b*. If the desired values of σ_3 and σ_5 are set at zero or some other small residual, equations (**44**) and (**45**) provide a pair of simultaneous equations, quadratic in S_a and S_c, which can be solved for the shapes of these lenses. Substitution of the values so obtained in (**43**) reduces this equation to one linear in S_b, which may then be solved to give zero or any other desired value of the coma. This provides a straightforward means of determining the shapes of the lenses for any triplet arrangement which will control the primary astigmatism, distortion, and coma.

(*e*) *Adjustment of the Spherical Aberration*

There remains for consideration, finally, the adjustment of the primary spherical aberration. Taylor (1904) pointed out long ago that the means for this lies in the distribution of the total positive power between the two collective lenses. An essential feature of the present treatment is that a variable χ has been introduced at the outset which (i) permits the initial solution to be easily made and (ii) specifies the distribution of power between the collective lenses. It is therefore the variable which controls the primary spherical aberration. It is not easy to derive a useful exact expression for the spherical aberration as a function of χ. Some elementary approximate reasoning, however, leads to the result that within the range of values of practical interest we should have

$$\sigma_1 \simeq a_0 + a_1\chi + a_2\chi^2 + a_3\chi^3,$$

where the a's are constant and a_3 generally small. It might be expected then that the spherical aberration could be closely approximated by a quadratic function of χ. Investigation shows this to be the case, the actual curve being adequately fitted by a quadratic function of χ. The coefficients of the quadratic expression are then determined from the computed spherical aberration of three systems with different corrector powers.

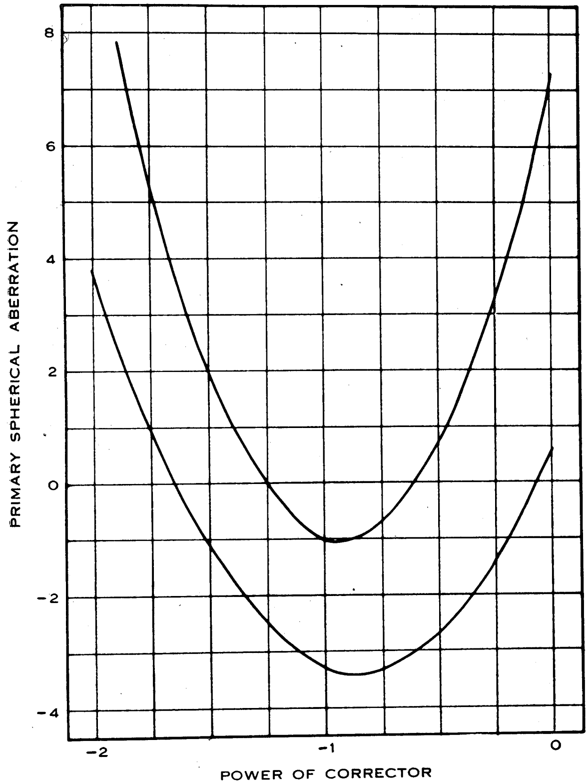

Fig. 8.—Curves showing the variation of the primary spherical aberration coefficient σ_1 with the parameter χ in unit power triplets from a given set of glasses. All other primary aberrations have been reduced to selected small values. The upper curve is for thin lenses and the lower curve for triplets with components of finite thicknesses.

In Figure 8 the upper curve shows the variation of the primary spherical aberration with χ in triplets from a given selection of glass and prescribed values of P, L, and T. All triplets represented in the curve are corrected for all other primary aberrations. As the curve intersects the axis in two points there are two values of χ for which the primary spherical aberration has some prescribed small value. Thus there are two possible types of solutions corresponding to quite different values of χ. Triplets quoted in the literature belong principally to the right-hand branch of the parabola, i.e. they belong to the type having the less negative value of χ. The other type with the more negative value of χ deserves to be investigated more because for certain purposes it is to be preferred.

(*f*) *The Thickening of the System*

It is now necessary to replace each thin component by one of finite axial thickness sufficient to provide the desired aperture for the component and the necessary edge thickness for easy manufacture. This process results, of course, in the introduction of residuals for all primary aberrations and these must be eliminated. A satisfactory procedure is to take the thin component system which has been computed and replace the thin components by lenses of appropriate axial thickness and compute the new primary aberrations. The changes in chromatic aberration and Petzval curvature are usually negligible. Returning to equations (**44**), (**45**), the derivatives $\partial\sigma_3/\partial S_a$, $\partial\sigma_3/\partial S_c$, $\partial\sigma_5/\partial S_a$, and $\partial\sigma_5/\partial S_c$ are calculated and hence the changes in the shapes, S_a and S_c, of the thin system necessary to give astigmatism and distortion residuals equal in amount, say, but opposite in sign to those introduced by the previous thickening. The system with the new shapes is then thickened and the new primary aberrations determined. Graphs may then be drawn of the astigmatism and distortion of the thin system against the astigmatism and distortion respectively of the thickened system, assuming the relations to be linear. The thin system residual corresponding to the desired thick system residual is read off from the graph and the final shapes computed. Similarly the coma of the system is adjusted by changing slightly the value of S_b. When this system is thickened it will be found to have primary aberration values very close to those desired.

In this way three triplets of different corrector powers are obtained having thickened components and all primary aberrations controlled except spherical aberration. The lower curve in Figure 8 represents the variation of spherical aberration with χ in thickened triplets in which all other primary aberrations have been adjusted to desired small values. This curve is fitted by a quadratic function of χ and so can be drawn from three calculated points. From it may be read the value of χ necessary to achieve any desired residual of spherical aberration. Normally a small positive residual of spherical aberration is required to offset the negative secondary and tertiary terms. The two values of χ which will provide these residuals are determined and the solutions of these triplets developed.

To complete the design it is necessary to obtain a good balance between the primary and higher order aberrations. Using the new and very effective means introduced by Buchdahl (1954) the secondary aberrations may then be calculated and the balance inspected. In a particular case it is generally a fairly straightforward matter to see how the balance of aberrations could be improved, and the primary design is accordingly altered to secure this.

(*g*) *Finite Object Distance*

In the cases considered so far the object has been confined to a plane at infinity. If this restriction is removed some of the basic equations of Section (*b*) become more complex and the writer has only obtained a general solution of these for the special case for which $\chi=0=L=T$.

There is, however, another simple way to deal with this problem. Of the five initial conditions the first three are independent of the position of the object, while the remaining two conditions, those for achromatism, do involve the object position. The problem may therefore be solved initially for an infinitely distant object position and the chosen values of L and T. The change in the chromatic aberration of the thin component system may then be calculated when the object plane is moved to the desired position at a finite distance from the lens. The initial solution may then be repeated with the values of L and T altered by amounts depending on the chromatic changes introduced by the shift of the object plane.

(*h*) *Recapitulation*

The method of design just described may be summarized as follows:

(i) A set of glasses is chosen and residual values of the Petzval sum and the chromatic aberrations are prescribed.

(ii) With these constants the initial equations are solved for three triplets with different values of χ, say $\chi=0$, $-0\cdot5$, $-1\cdot0$.

(iii) The shapes are determined in each of these so that astigmatism, distortion, and coma are controlled as desired.

(iv) The systems are thickened and the shapes readjusted to maintain correction of the three oblique aberrations.

(v) The curve of primary spherical aberration against χ is plotted and the values of χ selected which will give the desired primary spherical aberration.

(vi) Either or both of these solutions are then developed.

It may be thought that this is a very long procedure. What is done, of course, is to survey the complete possibilities with one set of glasses and residuals, which is more than the development of the design of one triplet. At every stage there is complete control of all the factors affecting the design and a clear understanding of the effect of the variation of each parameter on the primary aberrations. After the method has been used a little it will be realized that there are a number of short cuts which can be taken due to the experience acquired, which reduce the work considerably.

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