METHODS FOR NUMERICAL CALCULATIONS WITH THE TYPE I COUNTER

By P. G. Guest*

[*Manuscript received January* 20, 1958]

Summary

Detailed schemes have been prepared for performing numerical calculations of frequency, expectation, and variance. Tables are given of the deviations of the asymptotic formulae from the exact formulae when the time of observation is small.

I. Introduction

A number of papers (Jost 1947 ; Feller 1948 ; Elmore 1950 ; Ramakrishnan and Mathews 1953 ; Hull and Wolfe 1954 ; Campbell 1956 ; Takacs 1956) dealing with the Type I counter have appeared in recent years. The present paper differs from these in being concerned primarily with the calculation of numerical magnitudes rather than with the derivation of general formulae.

A Type I counter is one in which each count is followed by a fixed dead time τ during which the counter is insensitive, the counter returning to the sensitive condition at the end of the period τ. Any event occurring in the dead time has no effect. It will be assumed that the events which are being counted are distributed in time according to a Poisson distribution with parameter μ. The quantities which are of interest are the frequency function $F(t)$ and the expected number $N(t)$ of counts in time t. $F(t)\mathrm{d}t$ is the probability that a count occurs in the interval $(t,t+\mathrm{d}t)$ and

$$N(t)=\int_0^t F(x)\mathrm{d}x.$$

The counting will be assumed to start at $t=0$, the counter being sensitive at that instant. $F(t)$ has the form shown in Figure 1, which corresponds to the special case $\mu\tau=1$. At $t=0$, $F(t)$ equals μ, and $F(t)$ oscillates with decreasing amplitude about the value $\mu/(1+\mu\tau)$ as t increases. When t is large $F(t)$ may be assumed to have this value and $N(t)$ may be taken as $\mu t/(1+\mu\tau)$. If the events occur in short bursts, as will happen when particles produced by a pulsed source are being counted, then the behaviour of $F(t)$ and $N(t)$ must be investigated for small values of t.

Instead of supposing the counter to be sensitive at $t=0$, it is sometimes convenient to consider the case where a count occurs at $t=0$. Clearly, in this case the counter will be sensitive at $t=\tau$, and the frequency, expectation, and variance of subsequent counts will be given by the same formulae as for the case where the counter is sensitive at $t=0$ if t in these formulae is replaced by $t-\tau$.

* Physics Department, University of Sydney.

II. Exact Values of Frequency and Expectation

The function $F(t)$ is given by the expression

$$F(t)=\mu \sum_{k=0}^{p} e^{-\mu(t-k\tau)} \frac{\mu^k(t-k\tau)^k}{k!}, \quad \ldots\ldots\ldots\ldots\ldots \quad (1)$$

where

$$t=p\tau+\alpha, \quad 0\leqslant\alpha<\tau. \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

The kth term of the sum gives, when multiplied by dt, the probability that the $(k+1)$th count occurs in $(t,t+\mathrm{d}t)$. The term differs from the corresponding term for a Poisson distribution in that the dead times following each of the first k counts are subtracted from t.

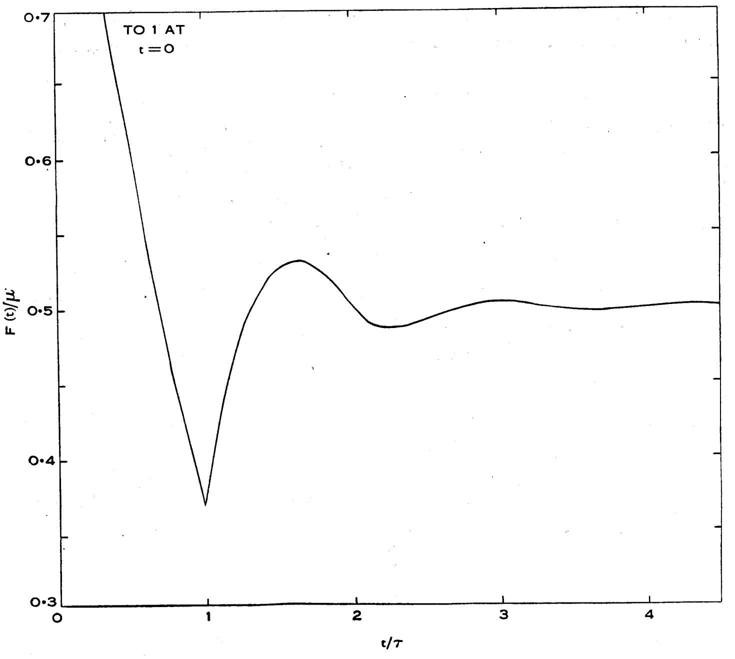

Fig. 1.—Graph of $F(t)/\mu$ against t/τ for a Type I counter with $\mu\tau=1$.

$N(t)$ may be found by integrating $F(t)$, giving

$$N(t)=\sum_{k=0}^{p} \Gamma(k+1\,;\ \mu t-k\mu\tau)/\Gamma(k+1), \quad \ldots\ldots\ldots \quad (3)$$

the terms $\Gamma(k+1\,;\ x)$ being the incomplete gamma functions. Alternatively, the equation

$$F(t)=\mu\left\{1-\int_{t-\tau}^{t} F(x)\mathrm{d}x\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

may be used, the term in brackets being the probability that the counter is sensitive at the instant t (i.e. that no count has occurred in $(t-\tau,t)$). (4) can be written

$$F(t)=\mu\{1-N(t)+N(t-\tau)\},$$

giving the recurrence relation

$$N(t)=1+N(t-\tau)-\mu^{-1}F(t),$$

and so

$$N(t)=p+1-\mu^{-1}\sum_{j=0}^{p} F(t-j\tau),$$

or, using (1),

$$N(t)=p+1-\sum_{k=0}^{p} e^{-\mu(t-k\tau)}\exp_k \mu(t-k\tau), \qquad (5)$$

where

$$\exp_k t=\sum_{j=0}^{k} t^j/j! \qquad (6)$$

Formulae more suitable for numerical calculations are obtained in terms of the variable

$$\beta=\mu(t-p\tau)=\mu\alpha. \qquad (7)$$

It is found that

$$F_p(\beta)=\mu e^{-\beta}\sum_{k=0}^{p} a_{p-k}\beta^k/k! \qquad (8)$$

and

$$N_p(\beta)=p+1-e^{-\beta}\sum_{k=0}^{p} b_{p-k}\beta^k/k! \qquad (9)$$

the coefficients a_r and b_r being functions of $\mu\tau$ given by the expressions

$$a_r=\sum_{j=0}^{r} (j\mu\tau)^{r-j}e^{-j\mu\tau}/(r-j)! \qquad (10)$$

and

$$b_r=\sum_{s=0}^{r} a_s=b_{r-1}+a_r. \qquad (11)$$

TABLE 1

SCHEME FOR THE CALCULATION OF a_r AND b_r

A / r	1	$e^{-\mu\tau}$	$e^{-2\mu\tau}$	$e^{-3\mu\tau}$	$e^{-4\mu\tau}$	a_r	b_r
0	1					a_0	$b_0=a_0$
1		1				a_1	$b_1=b_0+a_1$
2		$\mu\tau$	1			a_2	$b_2=b_1+a_2$
3		$\frac{1}{2}(\mu\tau)^2$	$2\mu\tau$	1		a_3	$b_3=b_2+a_3$
4		$\frac{1}{6}(\mu\tau)^3$	$\frac{1}{2}(2\mu\tau)^2$	$3\mu\tau$	1	a_4	$b_4=b_3+a_4$

A scheme for the calculation of a_r and b_r is shown in Table 1. The entries in each column are the successive terms of $e^{j\mu\tau}$. a_r is the sum of the products of corresponding elements in row r and row A. The sums in (8) and (9) can be evaluated by successive multiplications and additions. For example, when p is 3,

$$\Sigma a_{p-k}\beta^k/k!=\{(0\cdot 1\dot{6}\beta+0\cdot 5a_1)\beta+a_2\}\beta+a_3.$$

Example 1

What is the value of $N(t)$ when t equals 450 μsec, if τ is 134 μsec and μ is 4070 events per second?

Here $\mu\tau$ is 0·545380 and μt is 1·831500, and so p is 3 and β is 0·195360. The values a_r and b_r are calculated in Table 2. From formula (9)

$$N_p(\beta)=4-3\cdot346455e^{-\beta}=1\cdot247411.$$

TABLE 2

CALCULATION OF a_r AND b_r FOR EXAMPLE 1

A / r	1	0·579622	0·335962	0·194731	a_r	b_r
0	1				1	1
1		1			0·579622	1·579622
2		0·545380	1		0·652076	2·231698
3		0·148720	1·090760	1	0·647386	2·879084
Sum, S	1	1·694100	2·090760	1	Check, $S\times A$	2·879085

For values of p less than 2 the formulae for $F(t)$ and $N(t)$ are:

$$p=0:\quad \mu^{-1}F(t)=e^{-\beta},\qquad N(t)=1-e^{-\beta},$$
$$p=1:\quad \mu^{-1}F(t)=e^{-\beta}(\beta+e^{-\mu\tau}),\qquad N(t)=2-e^{-\beta}(1+\beta+e^{-\mu\tau}).$$

For higher values of p it is best to use the scheme of Table 1.

III. Asymptotic Formulae

Equation (4) can be integrated to give

$$N(t)=\mu t-\mu\int_{t-\tau}^{t} N(x)\mathrm{d}x. \qquad (12)$$

If the asymptotic formula, valid for large t, is written

$$n(t)=\gamma(\mu t+\psi), \qquad (13)$$

γ and ψ can be found by substituting this expression in (12) and equating terms in t on each side and constant terms on each side. This gives

$$\gamma=1/(1+\mu\tau),\quad \psi=\tfrac{1}{2}\gamma(\mu\tau)^2. \qquad (14)$$

The asymptotic value $f(t)$ of $F(t)$ is of course equal to γ.

For Example 1,

$$\gamma=0\cdot647090,\quad \gamma\psi=0\cdot062273,\quad \mu t=1\cdot831500,$$

and

$$n(t)=1\cdot185145+0\cdot062273=1\cdot247418.$$

This is very close to the exact value. Since the asymptotic values are easily calculated, it is of interest to determine by how much the asymptotic values differ from the exact values when μt is small. This question has been investigated by determining the maximum values of the fractional divergences

$$|F(t)-f(t)|/F(t) \text{ and } |N(t)-n(t)|/N(t)$$

for values of t greater than $p\tau$, $p=1(1)4$. The maximum divergences (expressed as percentages) of $f(t)$ are shown in Table 3 and of $n(t)$ in Table 4 for values of $\mu\tau$ up to 2. From these tables it is possible to determine in any particular problem whether the asymptotic formula is sufficiently accurate.

TABLE 3

MAXIMUM PERCENTAGE DEVIATION OF ASYMPTOTIC VALUE $f(t)$ FROM EXACT VALUE $F(t)$ WHEN $t \geqslant p\tau$

p / $\mu\tau$	1	2	3	4
0·0125	0·01			
0·025	0·03			
0·05	0·12			
0·075	0·27	0·01		
0·1	0·5	0·01		
0·15	1·0	0·04		
0·2	1·8	0·09		
0·3	3·8	0·24		
0·4	7	0·4	0·02	
0·5	10	0·7	0·05	0·01
0·6	14	0·9	0·11	0·02
0·7	18	1·0	0·21	0·04
0·8	24	1·2	0·37	0·05
0·9	29	1·8	0·6	0·07
1·0	36	2·5	0·9	0·10
1·2	51	4	1·6	0·27
1·4	69	7	2·4	0·6
1·6	91	11	3·1	1·0
1·8	116	15	3·6	1·7
2·0	146	21	5	2·6

For Example 1, $\mu\tau$ is 0·55 and p is 3, and so $n(t)$ should differ from $N(t)$ by an amount not exceeding 0·01 per cent.

It is shown in Section IV that the standard deviation is approximately equal to $\surd(\gamma^3\mu t)$, and so the ratio of the constant term $\gamma\psi$ in $n(t)$ to the standard deviation is $\psi/\surd(\gamma\mu t)$, which will be considerably less than unity unless t is very small. This term can then often be omitted and $n(t)$ taken as simply $\gamma\mu t$. However, when the experiment consists of a large number of repeated determinations over the time t it is advisable to retain the constant term to avoid bias.

TABLE 4

MAXIMUM PERCENTAGE DEVIATION OF ASYMPTOTIC VALUE $n(t)$ FROM EXACT VALUE $N(t)$ WHEN $t \geqslant p\tau$

p / $\mu\tau$	1	2	3	4
0·025	0·01			
0·05	0·04			
0·075	0·08			
0·1	0·13			
0·15	0·25			
0·2	0·39	0·01		
0·3	0·7	0·01		
0·4	1·0	0·02		
0·5	1·2	0·04	0·01	
0·6	1·3	0·07	0·01	
0·7	1·4	0·12	0·02	
0·8	1·4	0·18	0·02	
1·0	1·4	0·35	0·03	0·01
1·2	2·1	0·6	0·06	0·02
1·4	2·8	0·8	0·10	0·04
1·6	3·7	1·0	0·17	0·05
1·8	4·6	1·2	0·26	0·06
2·0	5·6	1·3	0·37	0·08

IV. VARIANCE

The variance of the observed number of counts is given by the formula

$$V(t) = \sum_{k=0}^{p} \{2(p-k)+1\} e^{-\mu(t-k\tau)} \exp_k \mu(t-k\tau) - \left\{ \sum_{k=0}^{p} e^{-\mu(t-k\tau)} \exp_k \mu(t-k\tau) \right\}^2. \quad \ldots\ldots\ldots\ldots (15)$$

For purposes of numerical calculation it is convenient to introduce the variable β defined by equation (7), and to write

$$V_p(\beta) = e^{-\beta} \sum_{j=0}^{p} g_{p-j} \beta^j/j! - \{e^{-\beta} \sum_{j=0}^{p} b_{p-j} \beta^j/j!\}^2, \quad \ldots\ldots\ldots (16)$$

where

$$g_r = \sum_{s=0}^{r} f_s \text{ and } f_r = \sum_{j=0}^{r} (2j+1) e^{-j\mu\tau} (j\mu\tau)^{r-j}/(r-j)! \quad \ldots\ldots (17)$$

It will be observed that the terms of f_r differ from those of a_r only by the factor $(2j+1)$, and so the scheme of Table 1 can be used to calculate f_r and g_r if the row A is replaced by a row F whose elements are $(2j+1)e^{-j\mu\tau}$.

Table 5 shows how the calculations given in Table 2 for Example 1 may be extended to include the variance calculations. Using (**16**), with $\beta=0\cdot195360$,

$$V_3(\beta)=9\cdot923019e^{-\beta}-(3\cdot346455e^{-\beta})^2=0\cdot585318.$$

For values of p less than 2 the formulae for $V(t)$ are:

$$p=0:\ \ e^{-\beta}-e^{-2\beta},$$
$$p=1:\ \ e^{-\beta}(1+\beta+3e^{-\mu\tau})-\{e^{-\beta}(1+\beta+e^{-\mu\tau})\}^2.$$

For higher values of p it is best to use the scheme of Table 5.

Table 5

EXTENSION OF TABLE 2 TO PERMIT THE CALCULATION OF VARIANCE

F	1	1·738866	1·679810	1·363117				
A	1	0·579622	0·335962	0·194731				
r					a_r	b_r	f_r	g_r
0	1				1	1	1	1
1		1			0·579622	1·579622	1·738866	2·738866
2		0·545380	1		0·652076	2·231698	2·628153	5·367019
3		0·148720	1·090760	1	0·647386	2·879084	3·453991	8·821010
Sum, S	1	1·694100	2·090760	1	Check, $A\times S$	2·879085	Check, $F\times S$	8·821009

The asymptotic formula $v(t)$ for the variance is

$$v(t)=\gamma^3(\mu t+\chi), \qquad (18)$$

where

$$\gamma=1/(1+\mu\tau),\quad \chi=\frac{1}{12}\gamma(\mu\tau)^2\{18+4\mu\tau+(\mu\tau)^2\}. \qquad (19)$$

For Example 1,

$$\gamma=0\cdot647090,\ \gamma^3=0\cdot270953,\ \chi=0\cdot328465,\ \mu t=1\cdot831500,$$

and so

$$v(t)=0\cdot585249.$$

This is very close to the value given by the exact formula. Calculations have been made of the divergence of the asymptotic value $v(t)$ from the exact value $V(t)$ when μt is small. Since the variance is seldom required very accurately, a detailed table similar to Table 4 will not be given. It is found that the percentage difference between the two values is less than 10 per cent. for all $t\geqslant\tau$ when $\mu\tau<0\cdot89$, and for all $t\geqslant 2\tau$ when $\mu\tau<1\cdot78$.

For small values of μt the standard deviation (the square root of the variance) will be of the same order as the expectation. However, if the average number of counts is taken over a large number k of intervals of duration t, the standard deviation will be divided by the factor $\sqrt{k}$.

V. Estimation of μ

In many investigations an experimental determination n of the counts in time t is made and an estimate of the parameter μ is required. The value μ_0 derived from the asymptotic formulae (**13**) and (**14**) is

$$\mu_0 = \frac{\sqrt{(t^2+2n\tau^2)}-(t-2n\tau)}{2t\tau-(2n-1)\tau^2}. \qquad \ldots\ldots\ldots\ldots (\mathbf{20})$$

Example 2

For a counter with dead time 134 μsec the average number of counts in 350 μsec is $1 \cdot 276$. What is the value of μ?

From (**20**),

$$\mu_0 = \frac{410 \cdot 27 - 8 \cdot 03}{(93800-27868)\times 10^{-6}} = 6100 \cdot 8.$$

Since p is 2 and $\mu_0\tau$ is $0 \cdot 82$, the error through use of the asymptotic formula is, from Table 4, less than $0 \cdot 2$ per cent.

Table 6

Calculation of value of $N(t)$ corresponding to μ_0 (Example 2)

$\mu_0\tau = 0 \cdot 817507$, $\mu_0 t = 2 \cdot 135280$, $p=2$, $\beta = 0 \cdot 500266$

C	0	0·441531	0·389900			
A	1	0·441531	0·194950			
r				a_r	b_r	c_r
0	1			1	1	0
1		1		0·441531	1·441531	0·441531
2		0·817507	1	0·555905	1·997436	0·750855

$N(t) = 3 - 2 \cdot 843718 e^{-\beta} = 1 \cdot 275656$

As a check, the value of $N(t)$ corresponding to μ_0 is calculated in Table **6**. This differs from n by $0 \cdot 000344$, which is negligible in the present case. However, if it is desired to obtain the value μ for which $N(t)$ is exactly equal to n, the formula

$$\mu\frac{dN(t)}{d\mu} = e^{-\beta}(\beta\Sigma a_{p-j}\beta^j/j! + \mu\tau\Sigma c_{p-j}\beta^j/j!) \quad \ldots\ldots\ldots\ldots (\mathbf{21})$$

may be used. The quantities c_r are calculated in the same way as the quantities a_r, with $e^{-j\mu\tau}$ replaced by $je^{-j\mu\tau}$. In Table 6 the c_r are calculated for Example 2 by intermultiplying row C and row r. From (**21**), the change $d\mu$ required to increase $N(t)$ by $0 \cdot 000344$ is given by

$$\begin{aligned} 0 \cdot 000344 &= (d\mu/\mu_0)e^{-\beta}(\beta\Sigma a_{p-j}\beta^j/j! + \mu\tau\Sigma c_{p-j}\beta^j/j!) \\ &= (d\mu/\mu_0)0 \cdot 606370(0 \cdot 451200 + 0 \cdot 794403), \end{aligned}$$

and so

$$\mu/\mu_0 = (\mu_0 + d\mu)/\mu_0 = 1 \cdot 000455,$$

and

$$\mu = 6103 \cdot 58.$$

A recalculation of Table 6 with this new value of μ does in fact give $N(t)=1\cdot276000$.

By differentiation of (**20**) it is found that

$$\frac{\mathrm{d}\mu_0}{\mathrm{d}n}=\frac{1}{t}\left\{\frac{1+\mu_0\tau+\tau/2\sqrt{(t^2+2n\tau^2)}}{1-(2n-1)\tau/2t}\right\}, \qquad (\mathbf{22})$$

and an estimate of the variance of μ_0 may be obtained from

$$\mathrm{var}\ \mu_0=(\mathrm{d}\mu_0/\mathrm{d}n)^2\ \mathrm{var}\ n. \qquad (\mathbf{23})$$

For Example 2, $\mathrm{d}\mu_0/\mathrm{d}n$ is 8052. To obtain an estimate of var n, the approximate value μ_0 may be substituted in equations (**18**) and (**19**), leading to the value $0\cdot467622$. The standard deviation of μ_0 is then 5506. If the estimate n is the average of k determinations, this quantity should be divided by $\sqrt{k}$.

VI. Scaling Circuits

The probability $Q(k+1,t)$ that $k+1$ or more counts occur in time t is given by the kth term of the expansion (**5**) of $N(t)$,

$$Q(k+1,t)=1-\mathrm{e}^{-\mu(t-k\tau)}\ \exp_k\ \mu(t-k\tau). \qquad (\mathbf{24})$$

This differs from the corresponding formula for the Poisson distribution in that t is replaced by $t-k\tau$. Molina (1942) in his Table II tabulates a sum $P(c,a)$ which may be used to obtain $Q(k,t)$, the relation being

$$Q(k,t)=P(c=k\ ;\ \ a=\mu t-(k-1)\mu\tau). \qquad (\mathbf{25})$$

The square root approximation to the significance limits of the Poisson distribution (Blom 1954) leads to the formula

$$\sqrt{k}-\sqrt{\{\mu t-(k-1)\mu\tau\}}=\tfrac{1}{2}X, \qquad (\mathbf{26})$$

where X is the value of the standardized normal variate corresponding to the significance level Q.

Since mechanical recorders have dead times τ^* of the order of $0\cdot1$ sec, it is often necessary to interpose a scaling circuit between the counter and the recorder to reduce the number of counts reaching the recorder by a factor m. If more than m counts occur in the time $\tau^*-\tau$ during which the counter has become sensitive again following a recorded count while the recorder is still insensitive, there will be a counting loss in the recorder. It is customary to choose the scaling factor m so that this loss can be neglected. This means choosing m so that $Q(m,\tau^*-\tau)$ is negligible, using either Molina's table with

$$Q(m,\tau^*-\tau)=P(m,\mu\tau^*-m\mu\tau), \qquad (\mathbf{27})$$

or the square root approximation

$$\sqrt{m}-\sqrt{(\mu\tau^*-m\mu\tau)}=\tfrac{1}{2}X. \qquad (\mathbf{28})$$

When m is small the estimate from (**28**) will be slightly higher than the exact estimate from (**27**).

Example 3

If τ is 134 μsec and μ does not exceed 4000 events per second, what scaling factor is required in front of a fast mechanical recorder of dead time $0 \cdot 02$ sec if the scaling loss is to be less than $0 \cdot 1$ per cent.?

The values $\mu\tau^*$ and $\mu\tau$ are 80 and $0 \cdot 536$. Molina's table gives

$$P(66,44 \cdot 6)=0 \cdot 0017, \quad P(67,44 \cdot 1)=0 \cdot 0008,$$

and so m must exceed 66 if the scaling losses are to be less than $0 \cdot 1$ per cent. For the square root approximation, X is $3 \cdot 09$ for the $0 \cdot 1$ per cent. level, while

$$\sqrt{66}-\sqrt{44 \cdot 6}=1 \cdot 45, \quad \sqrt{67}-\sqrt{44 \cdot 1}=1 \cdot 54, \quad \sqrt{68}-\sqrt{43 \cdot 6}=1 \cdot 64,$$

which would imply that m must be greater than 67.

VII. Two Counters in Series

If the functions for the second counter are distinguished by an asterisk,

$$F^*(t)=F(t)-\int_\tau^{\tau^*} F^*(t-x)F(x-\tau)\mathrm{d}x, \quad \ldots\ldots\ldots\ldots \quad (\mathbf{29})$$

and

$$N^*(t)=N(t)-\int_\tau^{\tau^*} N^*(t-x)F(x-\tau)\mathrm{d}x. \quad \ldots\ldots\ldots\ldots \quad (\mathbf{30})$$

The exact expressions for $F^*(t)$ and $N^*(t)$ will be quite complicated, but the asymptotic forms can be found fairly easily. It will be observed first that, when $\tau^* \leqslant \tau$, $N^*(t)$ is equal to $N(t)$ and the second counter has no effect on the counting rate. This condition should clearly be aimed at in the design of the equipment.

To find the asymptotic formula $n^*(t)$ for $N^*(t)$ when $\tau^*>\tau$,

$$n^*(t)=\gamma^*(\mu t+\psi^*) \text{ and } n(t)=\gamma(\mu t+\psi)$$

are substituted in (**30**), and terms in t equated and constant terms equated. This gives

$$\gamma^*=\gamma\{1+N(\tau^*-\tau)\}^{-1}, \quad \psi^*=\psi+(\gamma^*/\gamma)\int_\tau^{\tau^*} x\mu F(x-\tau)\mathrm{d}x. \quad \ldots \quad (\mathbf{31})$$

The integral can be put in the form

$$\int_\tau^{\tau^*} x\mu F(x-\tau)\mathrm{d}x=\tfrac{1}{2}q(q+1)(1+\mu\tau)-\sum_{p=0}^{q-1}(\mu\tau^*+q-p)\mu^{-1}F_p(\beta^*), \quad \ldots \quad (\mathbf{32})$$

where

$$\mu\tau^*=q\mu\tau+\beta^*.$$

Example 4

A Geiger counter with a dead time of 100 μsec is operating a recording circuit whose dead time is 240 μsec. If the rate of arrival of the particles being counted is 1000 per second, find an expression for the average number of counts in time t.

In this case

$$\mu\tau=0\cdot1,\quad \mu\tau^*=0\cdot24,\quad q=2,\quad \beta^*=0\cdot04,$$
$$N(\tau^*-\tau)=N_1(\beta^*)=0\cdot131422,$$
$$\mu^{-1}F_0(\beta^*)=0\cdot960789,\quad \mu^{-1}F_1(\beta^*)=0\cdot907789,$$
$$\gamma=0\cdot909091,\qquad \psi=0\cdot004545.$$

Hence, from (**31**) and (**32**),

$$\gamma^*=0\cdot803494,\quad \psi^*=0\cdot004545+0\cdot022174\gamma^*/\gamma=0\cdot024143,$$

and the average number $n^*(t)$ of particles counted in time t will be given by

$$n^*(t)=0\cdot803494(1000t+0\cdot024143).$$

No detailed investigation of the deviations of the asymptotic values from the exact values has been made for the case of two counters in series, but it seems probable that the percentage deviations are similar to those for a single counter of dead time τ^*. The variance of the number of counts is quite close to $\gamma^{*3}\mu t$ when μt is large; in fact,

$$v^*(t)=\gamma^{*3}(\varkappa\mu t+\chi^*), \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (\mathbf{33})$$

where

$$\varkappa=\{2\gamma^*(1+\psi^*)-1\}/\gamma^{*2}$$

is very close to unity. The expression for the constant term χ^* is very complicated.

VIII. References

Blom, G. (1954).—Transformations of the binomial, negative binomial, Poisson and χ^2 distributions. *Biometrika* **41**: 302.

Campbell, L. L. (1956).—Standard deviation of dead time correction in counters. *Canad. J. Phys.* **34**: 929.

Elmore, W. C. (1950).—Statistics of counting. *Nucleonics* **6**: 26.

Feller, W. (1948).—On probability problems in the theory of counters. Courant Anniversary Volume, p. 105. (Interscience Publishers: New York.)

Hull, T. E., and Wolfe, W. A. (1954).—On inverting Laplace transforms. *Canad. J. Phys.* **32**: 72.

Jost, R. (1947).—Bemerkungen zur mathematischen Theorie der Zähler. *Helv. Phys. Acta* **20**: 173.

Molina, E. C. (1942).—"Poisson's Exponential Binomial Limit." (D. van Nostrand Co.: New York.)

Ramakrishnan, A., and Mathews, P. M. (1953).—A stochastic problem relating to counters. *Phil. Mag.* **44**: 1122.

Takacs, L. (1956).—On a probability problem arising in the theory of counters. *Proc. Camb. Phil. Soc.* **52**: 488.