THE WILSON EFFECT IN SUNSPOTS

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Summary

New observations of the Wilson effect in a small regular sunspot have demonstrated the reality of the phenomenon. The measurements are based on 38 photographs selected from some 24,000 obtained during the passage of the spot from the east to the west limb.

It is pointed out that the changing appearance of a sunspot during its passage across the disk is essentially a problem in the theory of radiative transfer. On the basis of a simple model a qualitative explanation is found for the Wilson effect and for the anomalous foreshortening of sunspot areas.

I. INTRODUCTION

The question of the structure in depth of sunspots is of considerable interest both in connexion with the properties of the photosphere and the mode of formation of the spots themselves (Cowling 1953). The changing appearance of a spot during its passage across the disk must depend on the absorption and emission coefficients of the spot material and of the photosphere, and on the variation of these quantities with depth. In particular, observations of the foreshortening effect as the spot approaches the limb of the Sun may be expected to provide some elucidation of this structure.

Many solar observers have noticed that, as a spot approaches the limb, the width of the penumbra on the inner side decreases at a greater rate than that on the side of the spot nearer the limb (Abetti 1957, cf. p. 65). This phenomenon was first described by A. Wilson in 1769 and is termed the Wilson effect. The work of subsequent observers has in general confirmed the reality of the effect, although the observed magnitude has decreased with the application of more modern techniques. Discrepancies among the results of the early observers can be attributed mainly to the smallness of the quantities to be measured. Near the limb the width of the highly foreshortened penumbra may be reduced to 2 sec of arc or less ; this can be measured only with the aid of a high-resolution instrument under conditions of good atmospheric seeing. In addition, by confining their measures to the limb, many early workers failed to take account of any possible asymmetry in the spot. To avoid systematic error due to this cause (cf. Abbot 1912) measurements should be made at both limbs and at the centre, where the foreshortening is negligible.

In recent times Unsöld (1955) and Waldmeier (1955, cf. p. 169) have even inclined to the view that the effect has a psychological origin.

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In the present paper observations are described of a small, fairly regular, stable sunspot which was photographed on all but one day of its passage across the disk with a 5 in. photoheliograph. The 38 photographs upon which the measurements are based were selected from some 24,000 obtained during this period. The interpretation of the observations is briefly considered in the final section.

II. Observations

The spot selected for this study was first seen at the east limb on May 19 and last seen at the west limb on May 30. During this period it was photographed repeatedly (except on May 27, when cloud intervened) with the 5 in. photoheliograph (Loughhead and Burgess 1958) of the C.S.I.R.O. Division of Physics Solar Observatory. This instrument is designed to photograph any selected region of the solar disk on 35 mm film at 5-sec intervals; the diameter of the solar image is 20 cm. The effective wavelength is 5400 Å with a bandwidth of 800 Å.

Plate 1 shows the changing appearance of the spot during its passage across the disk. Each photograph is orientated so that the direction of greatest foreshortening is horizontal. It will be noticed that, although the spot is fairly regular when seen near the central meridian, at both the east and west limbs the width of the penumbra on the side of the spot nearer the limb is significantly greater than that on the other side. This illustration leaves no doubt as to the reality of the Wilson effect for this particular spot; it remains to determine its magnitude.

III. Reduction

The quantities required are the apparent widths of the penumbra AN and $A'N'$ (Fig. 1) measured along the line of greatest foreshortening on the sides of the spot directed towards and away from the solar limb respectively. This line lies along the arc of the great circle whose plane contains the solar radius and the line-of-sight to the centre of the spot.

The ratio

$$f=AN/A'N', \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

which varies with the heliocentric angle θ of the spot, is a measure of the Wilson effect. A minor difficulty arises in that the spot, as seen from the Earth, appears to rotate about the direction of greatest foreshortening as it moves across the disk. Measurements made on successive days along this line therefore refer to different lines in the spot itself. However, for the 3 days during which the spot is close to the limb, the apparent rotation is so small ($\sim 2°$) that it may safely be ignored. Measurements of f were first made for the 3 days during which the spot was close to the east limb, and the results compared with measurements for the two days during which the spot was near the central meridian, where the foreshortening is negligible. The line in the spot corresponding to the limb measurements was transferred to the centre photographs with the aid of Stonyhurst disks. All measurements were therefore made along the same line in the spot. A similar set of measurements was made for the west limb and the results also compared with the centre observations. For the reasons mentioned above the east and west limb measurements were made along different lines in the spot, the angle between the lines being approximately 7°.

The 38 photographs upon which the measurements are based were selected from some 24,000 contained in the 35 mm records. This selection provided five good photographs for each day, except May 19, when only three of sufficient quality were obtained. By choosing from such a large number of photographs it

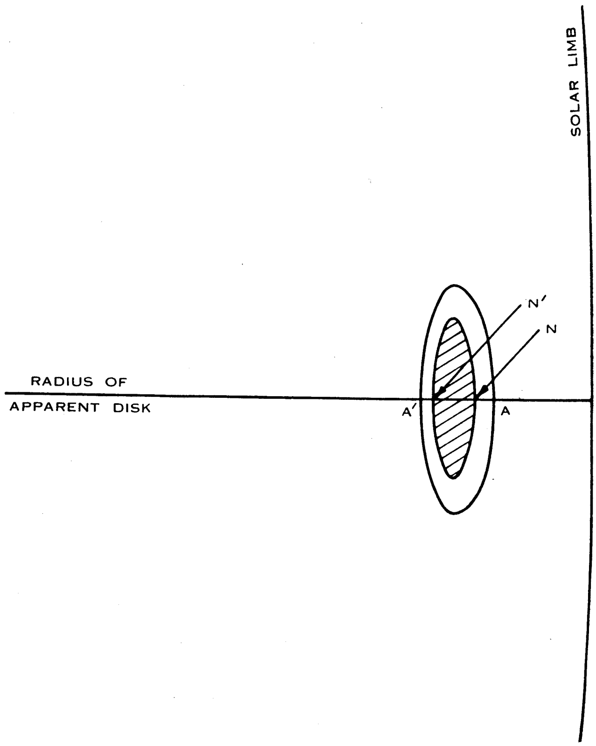

Fig. 1.—AA' is the arc intercepted by the spot on the great circle whose plane contains the solar radius and the line-of-sight to the centre of the spot. AN and $A'N'$ are the apparent widths of the penumbra on the sides of the spot towards and remote from the solar limb respectively.

is usually possible to obtain several of high quality even on days of relatively poor seeing. Enlargements were made of the 38 photographs on such a scale that 1 mm corresponded to 0·9 sec of arc. Kodak waterproof bromide paper was used to avoid possible distortion during processing.

The technique of measurement consisted of tracing the outlines of the umbra and penumbra from the enlargements onto transparent paper. In tracing the penumbra irregularities in the outer boundary were averaged out and similarly, in tracing the umbra, small dark projections into the penumbra were ignored. No difficulty was experienced except at the extreme limb where foreshortening reduces the apparent width of the penumbra to a few seconds of arc. In addition, at the extreme limb, the boundary between the umbra and penumbra, while quite sharp on the side nearer the limb, appears rather diffuse on the other side (cf. Plate 1). This phenomenon has been noticed in other sunspots near the limb and merits further study. To avoid either over- or underestimating the width an attempt was made to trace upper and lower limits to the boundary, the mean then being taken. For the limb photographs the penumbral widths were measured along the line of greatest foreshortening. This line in the spot was

TABLE 1

PENUMBRAL MEASUREMENTS*

(*a*) *East Limb*

	East Limb			Centre	
Date	19.v.57	20.v.57	21.v.57	24.v.57	25.v.57
θ	73·1°	59·3°	45·2°	8·6°	12·1°
AN	4·1±0·2″	5·6±0·3″	6·4±0·3″	9·3±0·3″	7·9±0·6″
$A'N'$	2·2±0·2″	3·5±0·3″	5·4±0·2″	8·1±0·3″	8·6±0·3″
f	1·88±0·15	1·57±0·15	1·18±0·07	1·15±0·06	0·91±0·08

(*b*) *West Limb*

	Centre		West Limb		
Date	24.v.57	25.v.57	28.v.57	29.v.57	30.v.57
θ	8·6°	12·1°	52·2°	64·2°	76·7°
AN	7·9±0·2″	9·0±0·2″	6·1±0·2″	4·8±0·1″	2·7±0·1″
$A'N'$	9·7±0·4″	7·4±0·2″	5·5±0·2″	3·5±0·1″	1·8±0·1″
f	0·82±0·04	1·22±0·04	1·10±0·06	1·38±0·07	1·51±0·11

* θ is the heliocentric angle of the spot; AN, $A'N'$ are the apparent penumbral widths on the sides of the spot directed towards and away from the solar limb respectively, and $f = AN/A'N'$. It should be noted that the AN of Table 1 (*a*) corresponds to the $A'N'$ of Table 1 (*b*), and vice versa. The r.m.s. errors of the mean values AN and $A'N'$ are calculated from the formula $s = \sqrt{[\Sigma\ (\text{residuals})^2/\{n(n-1)\}]}$, where n is the number of determinations; the corresponding errors in f are derived from these errors in the usual way. The east and west limb measurements were made along different lines in the spot, the angle between the lines being approximately 7°.

then transferred to the centre photographs and the measurements repeated. Errors due to seeing have been reduced by taking the mean of the five measures for each day (three for May 19). As experience has shown, even on a photograph of good quality, seeing can appreciably distort structures many times larger than the smallest detail resolvable on the photograph.

Tables 1(*a*) and 1(*b*) give the results of the measurements (expressed in seconds of arc) for the east and west limbs respectively, as well as the corresponding values of the ratio f defined by equation (**1**). The r.m.s. errors of the mean values are also given. However, these only represent the internal consistency of the measurements and systematic error may also be present. It will be noticed that the values of f steadily increase towards either limb by amounts much greater than the r.m.s. errors.

IV. Discussion

The reality of the Wilson effect is demonstrated by Plate 1.

The photographs for May 19, 20, and 21 show the appearance of the spot near the east limb. On May 19 it is seen that the width of the penumbra on the limb side is nearly double that on the other side, while the same disparity is present, though to a lesser degree, on May 20.* By May 21 the ratio of the penumbral widths is approaching unity and thereafter, until May 26, the spot displays a fairly regular structure as it moves across the central portion of the solar disk. The photographs for May 28, 29, and 30 show the appearance of the spot near the west limb. On May 29 the width of the penumbra on the limb side is again greater than that on the other side and this disparity is even more marked on May 30. However, as shown by a comparison of the results in Tables 1(a) and 1(b), the effect here is less marked than at the east limb. This reduction in the value of f may reflect the fact that the penumbral widths in the two cases are measured along somewhat different lines in the spot which, however, differ only by about 7° (cf. Section III). On the other hand it may be connected with the decay of the spot which occurs during its passage across the disk.

The fact that the Wilson effect is observed at both limbs precludes the possibility that the changes in the relative penumbral widths at either limb are due to a persistent elongation of the umbra on one side of the spot. For, since the two fixed lines of measurement in the spot differ by only 7°, the effect of any such asymmetry would be reversed at the two limbs and the results would be correspondingly discordant. The substantial agreement between the results obtained at the two limbs confirms the reality of the effect.

It is also possible that the observed effect could be due to short-lived elongations of the umbra in the direction of measurement, which develop and decay during a period of a few days or less. However, in order to explain the appearance of the limb photographs any such distortion would have to extend over at least a quarter of the umbral boundary. No such distortion can be seen on the photographs for May 21 or 22, although on the 23rd the umbra-penumbra boundary shows some irregularities. The east limb results can therefore be accepted with confidence. Such a distortion does in fact develop in the umbra on May 24. It so happens that the lines fixed in the spot corresponding to the directions of measurement at the two limbs pass through this region. However, although the distortion had increased even further by May 25, it appears to be declining on May 26. The change in this region is reflected in the discordance of the measurements for May 24 and 25 (cf. Tables 1(a) and 1(b)). But it seems

* The two small bright regions bordering the left-hand side of the umbra on this photograph are phenomena of the type recently described elsewhere (Bray and Loughhead 1957).

unlikely that the projection, even if it had continued until the spot reached the west limb on May 30, could account for the magnitude of the observed effect. In fact the measurements show that the reduction in the penumbral widths on the side of the spot remote from the limb due to this projection is probably insufficient to account for the observed effect.

V. Remarks on Interpretation

Although there has been controversy about the reality of the effect, previous observations have generally been explained on the hypothesis that sunspots are shallow, saucer-shaped depressions in the photosphere (Young 1895 ; Menzel 1949 ; Cowling 1953, cf. pp. 569–72, 576, 578 ; Kiepenheuer 1953 ; Abetti 1957, cf. pp. 65–7, 245). However, such a geometrical interpretation is inadequate, since the whole question of the changing appearance of a sunspot during its passage across the disk is essentially a problem in the theory of radiative transfer. The principal factor in causing the Wilson effect is the greater transparency of the spot compared with the surrounding photosphere (Michard 1953 ; Sweet 1955), so that a line-of-sight from the observer penetrates to a greater depth in the spot than in the surrounding photosphere. Accordingly, the spot appears as a depression, whose depth depends on the heliocentric angle and on the relative transparencies of the spot and the photosphere.

Theoretically, a knowledge of the emission and absorption coefficients in the photosphere, umbra, and penumbra would yield the intensity profile of the spot along the line AA' (Fig. 1), and hence the Wilson effect. Conversely, measurements of the intensity profile at different heliocentric angles might provide some information about the emission and absorption coefficients in the spot and their variation with depth. However, any attempt at a detailed comparison of theory and observation would face serious difficulties, both observational and theoretical. The observations would require correction for scattered light and for distortion by the combined instrumental profile of the telescope and atmosphere. These corrections become very large near the limb, where the width of the foreshortened penumbra may be less than 2 sec of arc. On the theoretical side there is the difficulty of matching the scales of optical depth in the spot and photosphere respectively (Michard 1953, cf. p. 280 ; Sweet 1955).

Some insight into the cause of the Wilson effect can be obtained by considering a simple model in which the sunspot is represented as a cylindrical structure extending through the photosphere, and in which the absorption coefficients $\varkappa_1$, $\varkappa_2$, $\varkappa_3$ of the photosphere, penumbra, and umbra respectively are constant (Fig. 2). The photosphere and the spot are assumed to have a sharp upper boundary. Making the reasonable assumption that $\varkappa_1 > \varkappa_2 > \varkappa_3$ and taking the directions of the apparent photosphere-penumbra and umbra-penumbra boundaries to be such that the corresponding lines-of-sight pass through unit optical thickness, it is easy to show that the apparent width of the penumbra on the side of the spot remote from the limb decreases more rapidly than a geometrical foreshortening law would imply, while on the other side of the spot the reverse applies. This is the Wilson effect. Moreover, in agreement with observation (Waldmeier 1955, cf. p. 163), the area of the whole spot decreases faster than implied by geometrical foreshortening. This simple model therefore explains qualitatively not only the Wilson effect but also the anomalous foreshortening of sunspot areas.* However, values of $\varkappa_2$ and $\varkappa_3$ obtained from the observations on the basis of this model probably have little meaning. The observations indicate that in this particular sunspot the Wilson effect is due mainly to the

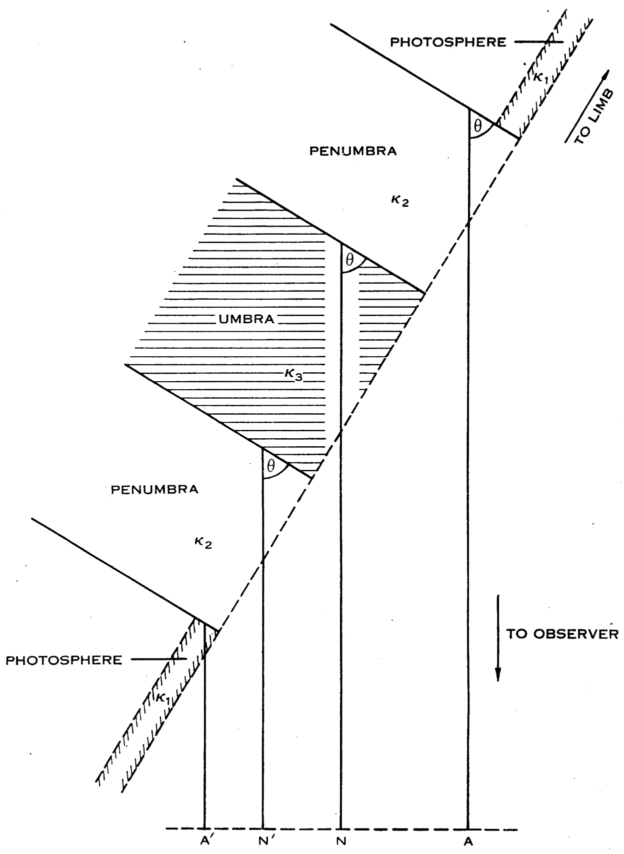

Fig. 2.—A simple model of a sunspot. The figure shows a section of the spot by the plane of the great circle containing the solar radius and the line-of-sight to the spot.

reduced foreshortening on the limb side of the spot and not to the increased foreshortening on the other side. If this is true for spots in general then the model implies that the dominant factor is the great transparency of the umbra, rather than that of the penumbra, compared with that of the photosphere.

VI. Acknowledgments

The authors wish to thank Mr. V. R. Burgess for assistance in securing some of the observations and Mr. H. Gillett for processing the films and making the enlargements.

* Hotinli (1957) attributes both the Wilson effect and the failure of the cosine law to an elevated ring of faculae surrounding the spot.

VII. References

Abbot, C. G. (1912).—" The Sun." p. 199. (Appleton : London and New York.)

Abetti, G. (1957).—" The Sun." (Faber and Faber : London.)

Bray, R. J., and Loughhead, R. E. (1957).—*Observatory* **77** : 201.

Cowling, T. G. (1953).—" The Sun." (Ed. G. Kuiper.) Ch. 8. (Univ. Chicago Press.)

Hotinli, M. (1957).—*Ann. Astrophys.* **20** : 45.

Kiepenheuer, K. O. (1953).—" The Sun." (Ed. G. Kuiper.) pp. 324, 330. (Univ. Chicago Press.)

Loughhead, R. E., and Burgess, V. R. (1958).—*Aust. J. Phys.* **11** : 35.

Menzel, D. H. (1949).—" Our Sun." pp. 124–5. (Churchill : London.)

Michard, R. (1953).—*Ann. Astrophys.* **16** : 217.

Sweet, P. A. (1955).—" Vistas in Astronomy." Vol. I. (Ed. A. Beer.) p. 677. (Pergamon : London and New York.)

Unsöld, A. (1955).—" Physik der Sternatmosphären." 2nd Ed. p. 559. (Springer : Berlin.)

Waldmeier, M. (1955).—" Ergebnisse und Probleme der Sonnenforschung." 2nd Ed. (Geest u. Portig : Leipzig.)

Young, C. A. (1895).—" The Sun." pp. 128–31. (Kegan Paul : London.)

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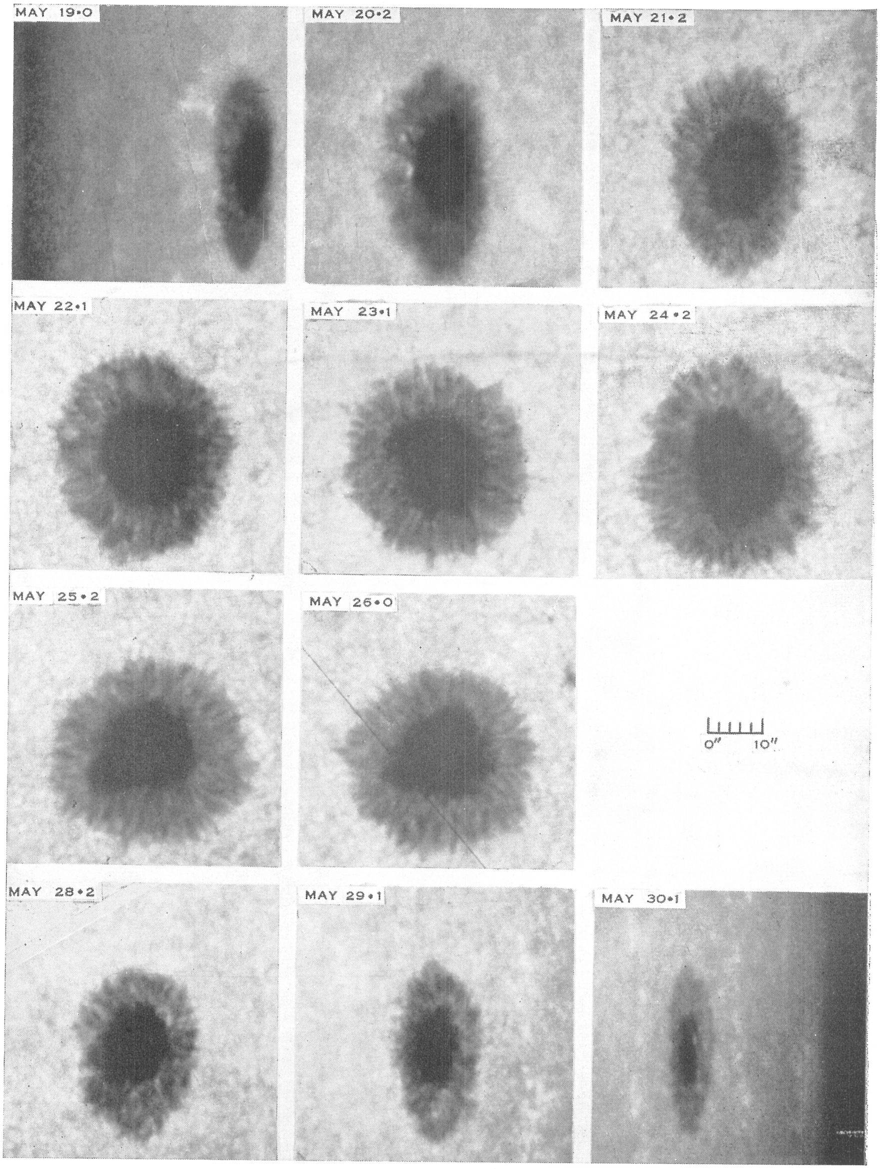

The changing appearance of the sunspot during its passage from the east to the west limb. Each photograph is orientated so that the direction of greatest foreshortening is horizontal. It will be noticed that, when the spot is near the limb, the apparent width of the penumbra on the side remote from the limb is less than that on the limb side. The heliographic coordinates on May 24 were 10 °S., 204 °W.