feature is an extremity of the well-known flare emanating from the galactic plane near $l=0°$.

It is clear that between Dec. $-15°$ and $+22°$ emission from the supergalaxy is not apparent at $3 \cdot 5$ m in regions where earlier observations might have led one to expect it. Thus, if radio emission from the supergalaxy does exist in the region discussed, it is overshadowed by the feature described above.

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THE CLOCK PARADOX IN SPECIAL RELATIVITY*

By H. Jeffreys†

Dr. G. Builder (1957) has produced a new analysis of this problem, which is criticized by Professor Dingle (1957). I think that both introduce concealed hypotheses, and that the methods of the special theory cannot produce a unique answer.

Standard works on relativity still start from the postulated invariance of the velocity of light, which can be stated in the form

$$\mathrm{d}s=0 \text{ is equivalent to } \mathrm{d}s'=0 \quad \ldots\ldots\ldots\ldots\ldots \quad (1)$$

for the observers, and infer that there is a linear relation between the coordinate systems, leading to the Lorentz-Einstein transformation. It was, I think, first shown by E. Cunningham that the conclusion does not follow; there are infinitely many relations that satisfy the equivalence, which do not even need to be linear. I have given additional conditions that are sufficient to lead to the transformation (Jeffreys 1957). The first is that for two observers of the same body

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2}=\frac{\mathrm{d}^2y}{\mathrm{d}t^2}=\frac{\mathrm{d}^2z}{\mathrm{d}t^2}=0 \text{ is equivalent to } \frac{\mathrm{d}^2x'}{\mathrm{d}t'^2}=\frac{\mathrm{d}^2y'}{\mathrm{d}t'^2}=\frac{\mathrm{d}^2z'}{\mathrm{d}t'^2}=0. \ldots \quad (2)$$

This amounts to saying that two observers will agree on what particles move with uniform velocity in straight lines. It does not say that there are no other

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particles. Nor does it say that it holds for all pairs of observers; it, like (**1**), picks out a class of observers whose measures satisfy the rules. I shall call them unaccelerated observers. (**1**) and (**2**) together can be shown to imply

$$ds' = k\,ds, \qquad (3)$$

where k is a constant for a given pair of unaccelerated observers.

The second hypothesis is that

$$k = 1. \qquad (4)$$

This requires some actual comparison of scales. One postulate, experimentally verifiable in principle, that suffices to justify it is that observers will attach the same measures to displacements normal to their relative velocity. The essential point is that the Lorentz-Einstein relation between systems of reference needs hypotheses equivalent to Newton's first law and the establishment of some comparison of scales; it has no relevance to accelerated particles without further hypothesis.

In the conditions of the problem R is an unaccelerated particle. M is initially in contact with R and they have been together long enough to synchronize their clocks, so that $k=1$. R continues in its original path. M moves away from it in a straight line with uniform velocity. At some stage M rebounds from an obstacle, and ultimately meets R again, and is brought to relative rest; then the clocks are again compared. Builder argues that there is an asymmetry between R and M, Dingle that there is not. Now the conditions considered in the special theory contemplate a whole background of unaccelerated observers, who would agree that R is unaccelerated, and that M has three impulsive changes of velocity. (There is no objection to supposing R so massive in comparison with M that its two impulsive changes of velocity can be neglected.) It is not true, as Dingle supposes, that R would be the only authority for the change of direction of motion of M; consequently his *a priori* argument for symmetry falls to the ground. But, since M is an accelerated particle, we cannot apply the special theory to it without further hypothesis, even though the accelerations are confined to the duration of the impulses.

Even if R and M were the only observers Dingle's argument still fails. He says "The principle of relativity allows us with equal justification to suppose that R is moving and M stationary . . . The two clocks will agree on reunion. That this must be so follows immediately from the symmetry of the situation and the principle of relativity of motion." It would be possible to infer a change of relative motion from a change in the Doppler effect; the two observers would then infer that *one* of them was accelerated, but could not say which. The symmetry is not one of fact but of knowledge. To make it into one of fact they would have to agree to attribute precisely half the change to each observer, but this is not what Dingle says. If a difference is to be expected at all, the observers would both expect that the clocks would differ in return, and that the sign of the difference would reveal which was on the accelerated body. Not to know which way the difference will be is not the same thing as to know that it is 0; this would be like saying that if $x^2=1$, then $x=\pm 1$, therefore $x=0$. This statement is still too favourable to Dingle's point of view, since it supposes that the observers themselves have no means of detecting which has changed its velocity, before the return. Actually the reversal of M's velocity implies an impulse; M would presumably have seen the body it collided with or at least felt the bump.

Suppose that in R's system M proceeds out with velocity u_1 to a distance X and returns with velocity $-u_2$. There is no need for u_1 and u_2 to be equal, since the obstacle may be moving or restitution may be imperfect. Then in R's system the total time is

$$T_R = X\left(\frac{1}{u_1}+\frac{1}{u_2}\right). \qquad (5)$$

In each part of the path M is in uniform motion. If $(xyzt)$ refer to R's system, $(x'y'z't')$ to M's, we have for the outward journey

$$\mathrm{d}s' = k_1 \mathrm{d}s, \quad \mathrm{d}x/\mathrm{d}t = u_1, \quad \mathrm{d}x'/\mathrm{d}t' = 0, \qquad (6)$$

whence

$$\mathrm{d}s' = c\,\mathrm{d}t' = k_1(c^2 - u_1^2)^{\frac{1}{2}}\mathrm{d}t. \qquad (7)$$

Similar relations hold for the return journey; then in M's system the total time is

$$T_M = X\left\{\frac{k_1}{u_1}\left(1-\frac{u_1^2}{c^2}\right)^{\frac{1}{2}} + \frac{k_2}{u_2}\left(1-\frac{u_2^2}{c^2}\right)^{\frac{1}{2}}\right\}. \qquad (8)$$

If $k = (1-u^2/c^2)^{-\frac{1}{2}}$, $T_M = T_R$ and Dingle's result follows. If $k=1$, Builder's result follows. The question is which, if either, is right. The time τ taken by the impulses does not matter since it is small and anyhow presumably does not increase indefinitely with X.

$k=1$ in the special theory supposes some method of comparison of scales. Here M's clock is compared with R's before departure and $k=1$. But as M's velocity changes the only obvious method of comparison after departure is by measures of transverse displacements by both. M *could* always take scales to keep $k=1$, but the question is whether, if he did, he would still assign the same length to the same scale. If there is a change, R could detect it. Suppose the linear dimensions of M are of order a, and its density ρ. Then, if it acquires velocity u in travelling through its linear dimensions, momentum per unit area $\rho a u$ is acquired in time a/u, and the force needed per unit area is of order ρu^2. Before the expulsion this is counterbalanced by other reactions, during the expulsion it is not. Then the stress implies elastic strains of order $\rho u^2/\mu$, where μ is some elastic constant of the material. If α is the velocity of elastic waves the strains are of order u^2/α^2. Now the general theory makes the greatest possible α of order c; actual values are much less than c. Hence the expulsion gives changes of length differing from 1 by far more than $(1-u^2/c^2)^{\frac{1}{2}}$ does. This concerns lengths in the direction of motion, but transverse lengths will be altered in a comparable ratio by the Poisson's ratio effect.

The conclusion seems to be that Builder's result cannot be right in any case. Dingle's might be right if M is made of the most rigid material that is possible, subject to the condition that the velocities of elastic waves do not exceed that of light, though his way of getting it is fallacious. With actual materials it also will be wrong. The change of k is of course one of the phenomena that could be covered by the general theory applied to elasticity.

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THE CLOCK PARADOX IN RELATIVITY*

By E. F. FAHY†

I have been following with great interest the discussions on the above topic which have been published in several journals. On reading a recent letter to *Nature* by Professor Herbert Dingle (1957) in reply to a previous letter by Sir Charles Darwin (1957), I noticed that it is feasible to perform astronomical observations which could provide an experimental basis for choosing between the two points of view. In fact, these observations may have already been made.

Dingle (1957), in the course of his analysis of the particular aspect of this problem which was introduced by Darwin (1957), concludes that " S_1 will not observe S_0's flashes to change until after he has fired his rocket ". It is evident from the context that the length of the delay is

$$t_1-\tfrac{1}{2}T_1=\tfrac{1}{2}T_1(1-\beta)/(1+\beta).$$

In the light of this result, consider an observer on this Earth who is interested in one of the distant nebulae. He sees a red-shift in the spectral lines and can think of himself as being a traveller who left that nebula many years ago, thereby interpreting the red-shift in terms of the velocity which he believes he gave himself at the beginning of his journey. He now decides that he will return to the nebula and builds a rocket which will take him from the Earth and produce a violet-shift in the nebula's spectrum equal in magnitude to the previous red-shift. Dingle's result indicates that this traveller will have to wait for a long time before he will observe the violet-shift; if D light years is the distance to the nebula, it indicates that the delay is about $D/2$ years.

On advancing the argument further, one arrives at the following aspect of the above situation, which is simpler from the experimental point of view. Consider any star which lies approximately in the plane of the Earth's orbit. Because of the Earth's orbital motion, an earth-bound observer would expect to

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