APPROXIMATIONS FOR THE ELECTRON DENSITY IN METEOR TRAILS*

By A. A. Weiss†

Herlofsen (1947) has shown that an exact solution of the equations governing the evaporation of a meteor particle during its flight in the upper atmosphere can be obtained in the special case of a spherical meteor in an isothermal atmosphere. In terms of the number n of meteor atoms evaporated in unit length of trail, the electron density α is

$$\alpha=\beta n. \qquad (1)$$

β is the ionizing probability, defined as the probability that an evaporated meteor atom will produce a free electron. If a power-law dependence of β upon velocity v is assumed, so that

$$\beta=\beta_0 v^{\eta}, \qquad (2)$$

the electron density may be written

$$\alpha=K(\eta)\left(\frac{v}{v_\infty}\right)^{\eta}\left(\frac{\rho H \sec\chi}{r_\infty \rho_m}\right)\left(\frac{v^2}{12l}\right)\left(\frac{r}{r_\infty}\right)^2, \qquad (3)$$

$$K(\eta)=4\cdot 5\beta_0 m_\infty v_\infty^{\eta}\cos\chi/\mu H.$$

In these expressions m, r, and ρ_m are the mass, radius, and density of the meteor; μ the mass of an individual meteor atom; l the latent heat of evaporation of a meteor atom, corrected for the efficiency of heat transfer; ρ the density of the atmosphere and H the atmospheric scale height; and χ is the zenith angle of the meteor radiant. The subscript ∞ refers to the initial state of the meteor.

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† Division of Radiophysics, C.S.I.R.O., at Department of Physics, University of Adelaide.

If it is assumed that the deceleration of the meteor is small over the whole flight, and that v is sufficiently large so that $v^2 \gg 12l$ and also $v^2 \gg 3l(2+\eta)$, the following expressions give a good approximation to the electron density profile:

$$\alpha = (\beta/\mu H) m_\infty \cos\chi (\rho/\rho_{\max.})[1-\tfrac{1}{3}(\rho/\rho_{\max.})]^2, \quad \ldots\ldots \quad \textbf{(4a)}$$

$$\rho_{\max.} = (8l/3v_\infty^2)(\rho_m r_\infty/H) \cos\chi. \quad \ldots\ldots \quad \textbf{(4b)}$$

These approximations have passed into general use, and a good deal of the theory underlying the interpretation of radio-echo observations of meteors has been based on them.

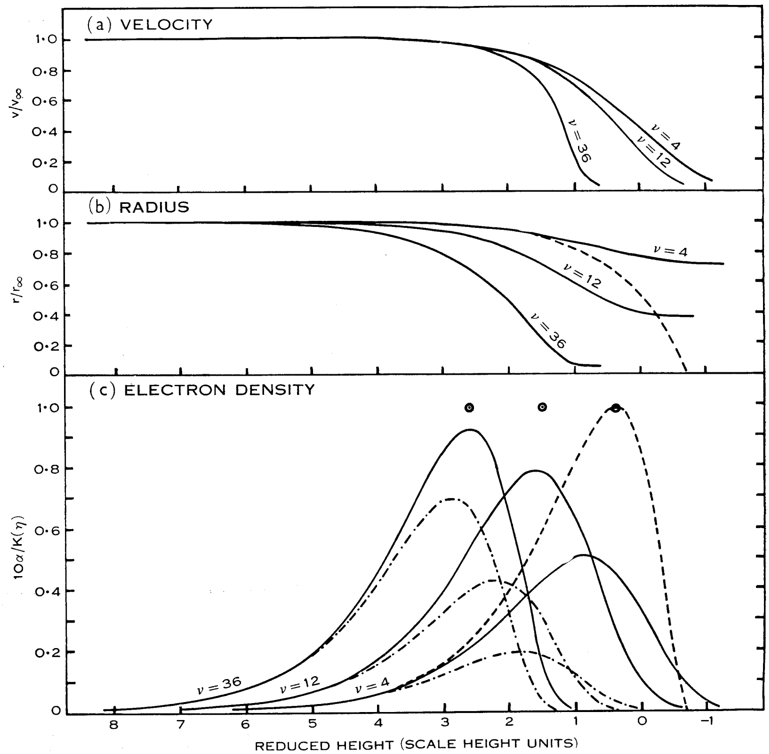

Fig. 1.—Comparison of exact and approximate solutions of the evaporation and ionization equations for a spherical meteor particle in an isothermal atmosphere. η=ionizing probability exponent of an evaporated meteor atom; $\nu = v_\infty^2/l$. See text for constant $K(\eta)$. Exact solution: $\eta=0$, ———; $\eta=5$, —·—·—·. Usual approximation, all η, - - - - - -. Maximum electron density for all η with old approximation, ⊙. Velocity and radius are not functions of η.

Calculations of deceleration with Herlofsen's exact solution and insertion of numerical values into the other two assumptions involving v, l, and η, show that all three assumptions are justified for fast meteors. The validity of their application to slow meteors is open to serious doubt unless $l < 10^{11}$ ergs/g and $\eta \sim 0$. Jacchia (1949) has given a mean value of $l \sim 3 \times 10^{11}$ ergs/g, and several independent evaluations of η (Whipple 1955; Hawkins 1956; Weiss 1957) suggest a value in the vicinity of 5. For slow meteors the divergence of the approximate expressions (4) from the exact solution (3) therefore deserves careful consideration.

The electron density profiles and the velocity and radius of the meteor during its flight, as given by the exact solution (3) and the approximations (4), are compared in Figure 1 for three values of the parameter $\nu = v_\infty^2/l$, namely, $\nu = 4$, 12, 36. To avoid confusion, only one set of approximate curves, for $\nu = 4$, appears in this diagram. The shape of the approximate curves is independent of ν and η. For other values of ν the electron density curves may be located by reference to the circled dots which show the positions of the maxima. In Figure 1, and also Table 1, the reduced height is equivalent to ln $(\rho H \sec \chi/\rho_m r_\infty)$.

TABLE 1

COMPARISON OF APPROXIMATIONS WITH EXACT SOLUTION FOR POINT OF MAXIMUM ELECTRON DENSITY

v_∞^2/l	4	4	12	12	36	36
η	0	5	0	5	0	5
(*a*) $10\alpha_{\mathrm{max.}}/K(\eta)$*						
Exact solution ..	0·504	0·192	0·782	0·423	0·917	0·692
New approximation ..	0·772	0·220	0·939	0·467	0·980	0·726
Old approximation ..	0·988	0·988	0·988	0·988	0·988	0·988
(*b*) Reduced Height*						
Exact solution ..	0·90	1·89	1·57	2·21	2·63	2·88
New approximation ..	1·10	1·91	1·79	2·27	2·71	2·93
Old approximation ..	0·39	0·39	1·50	1·50	2·60	2·60

* See text for $K(\eta)$ and for definition of reduced height.

The approximations (4) evidently give a poor representation of the exact solution for large values of η and small values of v_∞^2/l. A more realistic description of conditions near the point of maximum electron density is afforded by the following improved set of approximations:

$$\alpha_{\mathrm{max.}} = 12(\beta_0 v_{\mathrm{max.}}^{\eta}/\mu H) m_\infty F(\eta)^2 [1+2F(\eta)]^{-3} \cos\chi, \quad \ldots \quad \mathbf{(5a)}$$

$$\rho_{\mathrm{max.}} = (8l/v_\infty^2)(\rho_m r_\infty/H)[1+2F(\eta)]^{-1} \cos\chi, \quad \ldots\ldots\ldots \quad \mathbf{(5b)}$$

$$v_{\mathrm{max.}}^2 = v_\infty^2 + 12l \ln (r_{\mathrm{max.}}/r_\infty), \quad \ldots\ldots\ldots \quad \mathbf{(5c)}$$

$$r_{\mathrm{max.}}/r_\infty = 2F(\eta)[1+2F(\eta)]^{-1}, \quad \ldots\ldots\ldots \quad \mathbf{(5d)}$$

where

$$F(\eta) = 1 + 3(2+\eta)l/v_\infty^2. \quad \ldots\ldots\ldots \quad \mathbf{(6)}$$

In arriving at these approximations, the assumption $v^2 \gg 3l(2+\eta)$ has been abandoned. Deceleration has been neglected in equations (**5b**), (**5c**), and (**5d**) relating to the evaporation process, but is introduced into (**5a**) for the electron density. The extent of the improvement over the old approximations may readily be seen from Table 1.

The shape of the electron density profile depends on the deceleration along the whole meteor path. Empirical improvements to the profile (**4a**), which fails only over the last third of the meteor trail, could be suggested. Departures of the actual atmosphere from the assumed isothermal model and the effects of possible fragmentation of the meteor body (Jacchia 1955) will limit the value of further refinements in this direction. For the same reason, there is little point in pushing the approximations for the maximum point beyond the degree of accuracy attained in the set of approximations (**5**).

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