THE DESIGN OF PHOTOGRAPHIC OBJECTIVES OF THE TRIPLET FAMILY

II. THE INITIAL DESIGN OF COMPOUND TRIPLET SYSTEMS

By F. D. CRUICKSHANK*

[*Manuscript received September* 28, 1959]

Summary

Part II. The general method developed for the type 111 triplet is extended to the initial design of triplet systems having compound components. The possibility of replacing a single lens by an equivalent doublet or triplet allows the single lens to be regarded as not being restricted to values of N and V associated with known glass types. It follows that the glass parameters α, β, γ, and ξ may be treated as continuously variable over limited ranges. The general effects of the variation of these parameters on the powers and separations of the basic initial solutions are shown in a set of diagrams. As an example, the initial design of a triplet with compound members is given in detail.

I. INTRODUCTION

In Part I of this series (Cruickshank 1958) the basic equations and procedures have been given for the initial design of the parent triplet objective, type 111, for a given selection of glasses and prescribed values of the power, the paraxial chromatic aberrations, and the third-order monochromatic aberrations. The purpose of the present paper is to extend the general method to the initial design of other members of the triplet family (Fig. 1 of Part I) having compound components. The first step necessary is to consider the equivalent glass properties of such components.

II. EQUIVALENT DOUBLETS AND TRIPLETS

Consider the thin lens (φ,N,V) of power φ made from glass of refractive index N and V-number V. Suppose this lens is to be replaced by n thin lenses $(k_1\varphi,N_1,V_1)$, $(k_2\varphi,N_2,V_2)$, . . . , $(k_n\varphi,N_n,V_n)$ in contact, the replacement being made in such a way that the compound group has the same power, the same Petzval sum, and the same paraxial chromatic aberrations. This requires that

$$\sum_{j=1}^{n} k_j = 1, \qquad (1)$$

$$\sum_{j=1}^{n} k_j\omega_j = \omega, \qquad (2)$$

$$\sum_{j=1}^{n} k_j\psi_j = \psi, \qquad (3)$$

where $\omega=1/N$, $\omega_j=1/N_j$, $\psi=1/V$, $\psi_j=1/V_j$. Two particular cases arise for detailed consideration.

* Department of Physics, University of Tasmania, Hobart.

The first of these is the case in which the replacing group is a doublet, i.e. the thin lens (φ, N, V) is replaced by the thin lenses $(k_1\varphi, N_1, V_1)$, $(k_2\varphi, N_2, V_2)$ in contact. The parameters k_1, k_2 must be chosen so that

$$k_1 + k_2 \quad = 1, \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

and either

$$\omega_1 k_1 + \omega_2 k_2 = \omega, \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

or

$$\psi_1 k_1 + \psi_2 k_2 = \psi. \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (6)$$

Many possibilities now arise. Suppose, for example, that the single *positive* thin lens (φ, N, V) is replaced by a doublet consisting of a thin *positive* lens $(k_1\varphi, N_1, V_1)$ and a thin *negative* lens $(k_2\varphi, N_2, V_2)$ in contact. Then k_2 will be negative, $|\, k_2/k_1 \,| < 1$, and both N_1 and N_2 must be greater or less than N according as $N_2 \gtrless N_1$. Similarly, both V_1 and V_2 must be greater or less than V according as $V_2 \gtrless V_1$.

Looking at the matter from the opposite point of view, suppose a thin doublet of positive power is constructed from the positive thin lens $(k_1\varphi, N_1, V_1)$ and the negative thin lens $(k_2\varphi, N_2, V_2)$ in contact. This doublet is equivalent as regards power, Petzval sum, and paraxial chromatic aberrations to a single thin lens (φ, N, V) where N and V are given through equations (**5**), (**6**). These values N, V will not correspond in general to those of any known glass and may be varied widely by intelligent choice of the two glasses and the value of k_2/k_1. In particular, by selecting a pair of glasses of the same index for some mean wavelength and varying k_2/k_1, the effective value of V may be varied over a wide range without change of index. Such an iso-N doublet can be very useful. For example, if we combine a positive thin lens of the DBC glass 658508 with a negative thin lens of EDF 654335 then the equivalent thin lens has an effective N-value of $1\cdot 66$ approximately and an effective V-value anywhere between 50 and infinity depending on the value of k_2/k_1. Similarly, by choosing an iso-V pair of glasses the effective value of N may be varied substantially without change of V. More generally, the choice of glass pairs in which both N and V are dissimilar allows the effective values of N and V to be changed simultaneously over a considerable range.

Consider next the special case in which the replacing group is a triplet. Thus suppose the thin lens (φ, N, V) is replaced by a compound triplet consisting of the three thin lenses $(k_1\varphi, N_1, V_1)$, $(k_2\varphi, N_2, V_2)$, $(k_3\varphi, N_3, V_3)$ in contact. The replacement may now be made in such a way that the triplet has the same power, the same Petzval sum, and the same chromatic aberrations as the singlet it replaces. Hence the equations (**1**)–(**3**) become

$$k_1 + k_2 + k_3 \qquad = 1, \qquad \ldots\ldots\ldots\ldots\ldots \quad (7)$$

$$\omega_1 k_1 + \omega_2 k_2 + \omega_3 k_3 = \omega, \qquad \ldots\ldots\ldots\ldots\ldots \quad (8)$$

$$\psi_1 k_1 + \psi_2 k_2 + \psi_3 k_3 = \psi, \qquad \ldots\ldots\ldots\ldots\ldots \quad (9)$$

whence

$$k_1 = \Delta_1/\Delta\,; \qquad k_2 = \Delta_2/\Delta\,; \qquad k_3 = \Delta_3/\Delta, \qquad \ldots\ldots\ldots \quad (10)$$

where

$$\Delta_1 = \begin{vmatrix} 1 & 1 & 1 \\ \omega & \omega_2 & \omega_3 \\ \psi & \psi_2 & \psi_3 \end{vmatrix}, \qquad \Delta_2 = \begin{vmatrix} 1 & 1 & 1 \\ \omega_1 & \omega & \omega_3 \\ \psi_1 & \psi & \psi_3 \end{vmatrix},$$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 1 \\ \omega_1 & \omega_2 & \omega \\ \psi_1 & \psi_2 & \psi \end{vmatrix}, \qquad \Delta = \begin{vmatrix} 1 & 1 & 1 \\ \omega_1 & \omega_2 & \omega_3 \\ \psi_1 & \psi_2 & \psi_3 \end{vmatrix}.$$

It is thus formally possible to design a triplet component which is equivalent as regards power, Petzval sum, and chromatic aberrations to a thin lens having, within a certain range, any value of N, and, independently thereof, any value of V. Therefore in the development of a photographic objective we may ignore the restrictions imposed by the existence of only a limited number of glass types if we are prepared to use cemented doublets and triplets to replace components which are single thin lenses in the initial stages of the design. For example, if for some reason one requires for the negative component of a triplet a glass for which $N=2\cdot 0$ and $V=15$, it can be assumed that this fictitious glass exists and in the next stage of design these effective values can be achieved by replacing the fictitious component by an actual cemented triplet.

In the design of triplet objectives of type 111 we are limited to available glass types for the three components and hence the values of the glass parameters,

$$\alpha = V_a/V_b\,, \qquad \xi = V_a/V_c\,,$$
$$\beta = N_a/N_b\,, \qquad \gamma = N_a/N_c\,,$$

are not continuously variable quantities. In the light of the considerations of this section, however, those members of the triplet family of objectives which have compound components may be considered at first as simple type 111 triplets in which one or more components have fictitious glass constants. This simplifies the problem of designing such systems as the theory of the initial design of type 111 triplets given in Part I is immediately applicable since each equivalent simple triplet has the same power, the same corrector power χ, and the same Petzval sum and chromatic aberrations as the compound system to which it is equivalent. The appearance in this way of glass constants which do not correspond necessarily to known types removes substantially the restrictions on the glass parameters and it is neither artificial nor without physical meaning to regard α, β, ξ, and γ as continuously variable over limited ranges at least.

III. The Effects of the Variation of the Glass Parameters

The first stage in the development of the design of a triplet objective is the determination of the powers φ_a, φ_b, φ_c and the separations t_1, t_2 of the three thin lenses a, b, c constituting the initial arrangement of the system. From the point of view of this first stage the variety of constructions in the triplet family is largely the consequence of the variation of the glass parameters α, β, ξ, and γ. If an understanding of the range of properties of the whole family of objectives is to be reached, or, indeed, an understanding of the detailed properties of any one member, it is necessary to know the effect on the initial solution of the variation of α, β, ξ, and γ. It is not assumed that this is the whole story as regards glass selection, because each of these parameters is a ratio and therefore any given values of the parameters might be achieved in a variety of ways. However, it represents a systematic approach to what is a complicated problem, and, once this general approach is outlined, it may be supplemented by a closer study of the effects of achieving any of these parameters in different ways in connexion with different members of the triplet family.

When the changes in the powers and separations due to given changes in the glass parameters are known there remains still the problem of determining whether the new basic solution is more favourable or less favourable than its predecessor. This is effectively the second stage of the design. Broadly speaking, an initial solution must be scrutinized in three different ways. The inherent characteristics of the system in respect of spherical aberration of all orders must be examined, then the asymmetry errors, i.e. the comas of all types and orders thereof, and finally those characteristics which control the field correction, i.e. the astigmatism of different orders and what is generally of over-riding importance in photographic objectives the oblique spherical aberrations.

In this laboratory extensive use is made of the aberration coefficients introduced by Buchdahl (1954). The general method developed for the use of these coefficients is described in another paper by Cruickshank and Hills (1960). Anyone who has used these coefficients will readily concede that they provide the most powerful means available at present in geometrical optics for predicting the image quality of a system and for analysing the way in which the particular construction achieves its performance. The laboratory's computing room furnishes a standard service of the computation of all the aberration coefficients of third, fifth, and seventh orders for any system. Formerly these coefficients were computed on electric desk calculating machines, but recently Ford (1959) has arranged the computation on a digital computer. In these computations the total value of each aberration coefficient of the complete system is obtained as the sum of a set of partial coefficients, one for each surface of the system. If, for example, a variation of the basic parameters leads to a general reduction of the partial coefficients of the spherical aberration at the several surfaces of the system, it is permissible to infer that this represents a step towards the development of reduced zonal spherical aberration and hence the possibility of increased aperture for the system if needed. One has only to compare the partial aberration coefficients of a few first class objectives designed for apertures of, say $f/3 \cdot 5$, $f/2 \cdot 5$, $f/2$, and $f/1 \cdot 5$ respectively, to see that this is so. No successful high aperture systems are developed from initial arrangements with characteristically large partial coefficients of spherical aberration. Now, broadly speaking, reduced partial coefficients of spherical aberration go hand in hand with reduced powers of the components, which in turn lead to shallower curves throughout the system. Of course, reduced powers cannot be sought without consideration of the separations of the components which may become so small that the system cannot be built or so large that there is an unacceptable degree of vignetting. The criteria by which the inherent properties of a system in respect of asymmetry errors and field errors are to be judged are similarly based on an examination of the appropriate aberration coefficients. The calculation of the aberration coefficients provides an analysis of how these aberrations arise within any particular system, but just how these are linked with simpler quantities in the initial solution requires deeper investigation. Further work is in progress in this connexion, in the course of which it is planned to make a broad study of the characteristic distribution of these aberration coefficients within the various types of construction which constitute the triplet family.

(*a*) *Objectives for which* $\xi=\gamma=1$

Consider first those objectives in which the effective glass constants of the front and back components are the same. It is common practice to construct type 111 objectives from two glasses, using the same crown glass for both positive lenses. This makes $\xi=1=\gamma$ and leaves only α and β as the effective glass parameters. Extending this idea to triplets with compound components it is possible to make systems like types 212, 222, 313, 323, etc. on this pattern provided the effective N and V values for the front and back components are the same.

Choosing some typical values for the aberration residuals which enter into the fundamental equations

$$\chi=-0\cdot25, \qquad L=+0\cdot20,$$
$$P=+0\cdot50, \qquad T=+0\cdot05,$$

the variation of the powers and separations of the thin lenses in the initial arrangement have been calculated for a range of values of α and β that are quite practical. Figures 1 (*a*)–1 (*f*) present a summary of the results.

It is immediately clear that increasing the value of β leads to substantial reductions in the powers of the first two lenses and to a lesser degree reduces the power of the back lens also. Intelligent choice of β can lead, therefore, to shallower curves throughout the system, with consequent reductions in the values of the partial aberration coefficients at the individual surfaces of the system. As β increases from lower values the front airspace at first widens but subsequently diminishes, while the rear airspace decreases steadily throughout the range. For any value of β the $|\varphi_b|$ versus α curve exhibits a shallow minimum somewhere towards the higher end of the α range. In addition, φ_a is decreasing steadily as α increases, while the accompanying increase in φ_c is not nearly so large. This may suggest, in general, the selection of an α-value near to that at which $|\varphi_b|$ attains its minimum. However, the very considerable increases in t_1 and t_2 as the value of α increases must be weighed against this.

There is another important matter to be considered in connexion with the choice of α, and that is the correction of the spherical aberration of the system. It was shown in Part I of this study that, once the glass types and values of desired residuals have been chosen for a triplet, the shapes of the three components can be adjusted to secure control of the primary astigmatism, distortion, and coma, while the primary spherical aberration, σ_1, is controlled by the parameter χ, the power of the corrector of the system. The whole placing of the σ_1 versus χ parabola is often controlled by the choice of α as is seen in Figure 2.

It cannot be assumed, however, that everything is necessarily satisfactory provided the σ_1 versus χ parabola comes near to or intersects the χ-axis. As α decreases and the corresponding parabola drops towards the χ-axis, the general magnitudes of the partial aberration coefficients of the surfaces increase. Varia-

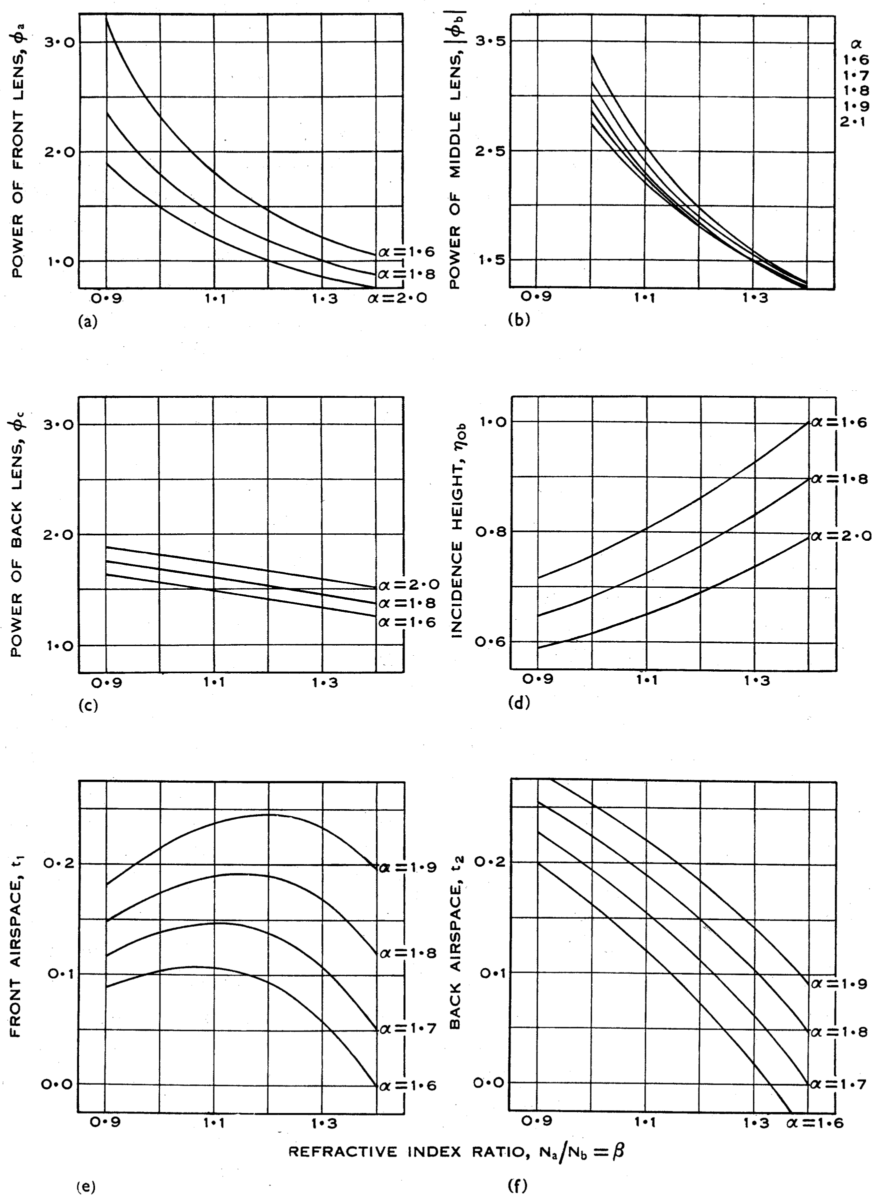

Fig. 1.—The effects of the variation of β on the powers and separations of the thin components of unit power triplets having different values of α are shown. In all triplets $\xi=1$, $\gamma=1$, and $\chi=-0\cdot25$. The residuals of Petzval curvature and the chromatic aberrations are in all cases $P=0\cdot50$, $L=0\cdot20$, $T=0\cdot05$.

tion of χ for a fixed α-value leaves the partial coefficients of spherical aberration of the negative lens almost unchanged, while the partial coefficients of the positive lenses change considerably in accordance with the parabolic law. On the other hand, the partial coefficients for the negative lens, and hence also those for the positive lenses, increase very substantially as α is decreased. This is due principally to the effect of α upon the front airspace. A decrease in α lowers the front airspace, raises η_{0b}, and hence the spherical aberration coefficients at the

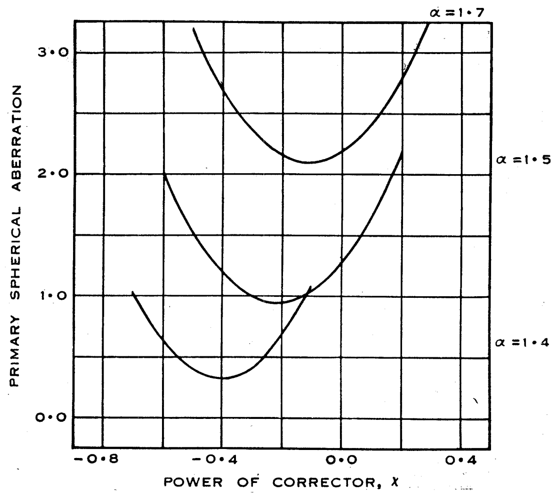

Fig. 2.—This figure shows the effect of the variation of α on the placing of the σ_1 *v.* χ parabola in a type 212 triplet.

surface of the negative lens. If, then, it is desired to secure as high an aperture as is consistent with a certain type of triplet construction, it is important to consider the location of the σ_1 versus χ parabola during selection of glass for the system, for the lower order partial aberration coefficients largely determine the magnitude of the higher order aberration coefficients of the system. A thorough study of Figures 1 and 2 will enable the reader to grasp some of the essential significance of glass selection in the design of this class of triplets.

(*b*) *Objectives for which* $\xi \neq \gamma$. *The General Case*

Apart from the construction of a series of three-dimensional models, it is difficult to present the effects of the simultaneous variation of the four glass parameters on the powers and separations in the general case of three-glass objectives. What has been done, therefore, is firstly to set $\beta=1=\gamma$ and investigate the effect of the variation of ξ. The results are presented for selected values of α in Figures 3 (*a*)–3 (*f*). It is seen that an increase in the value of ξ increases the powers of the lenses in the corrector system and slightly decreases the power of the back component, the changes being almost linear over this range in each case. Over against this are fairly rapid decreases in both airspaces. Secondly, setting $\beta=1=\xi$, the variation of γ has been investigated over the range 0·8–1·2. The results are presented in Figures 4 (*a*)–4 (*f*), from which it is seen that an increase of γ increases all powers, decreases the front airspace, and increases the back airspace. At smaller values of α the effect on the front airspace is very small. It is noticeable in all these variations that the effects

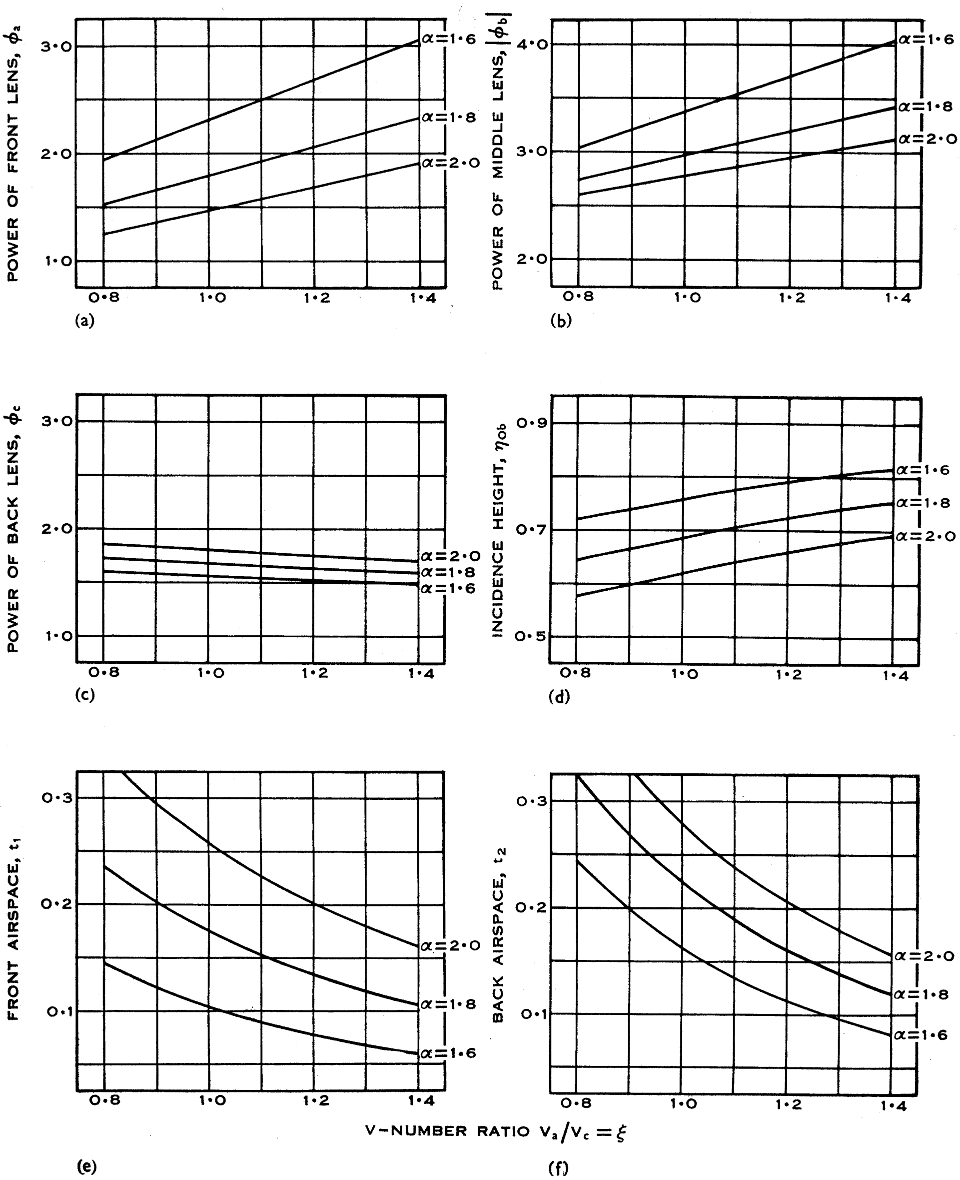

Fig. 3.—These figures show the effects of the variation of ξ on the powers and separations of the thin components of unit power triplets having different values of α. In each triplet $\beta=1$, $\gamma=1$, and $\chi=-0\cdot25$. The residuals of Petzval curvature and chromatic aberration are $P=0\cdot50$, $L=0\cdot20$, $T=0\cdot05$.

on the powers of the first two lenses are very much greater than on the power of the back component. This component behaves as a fairly stable feature of the construction, while the front portion of the system may undergo wide changes. This strengthens the view adopted by the writer that the first two lenses fulfil the function of a corrector system.

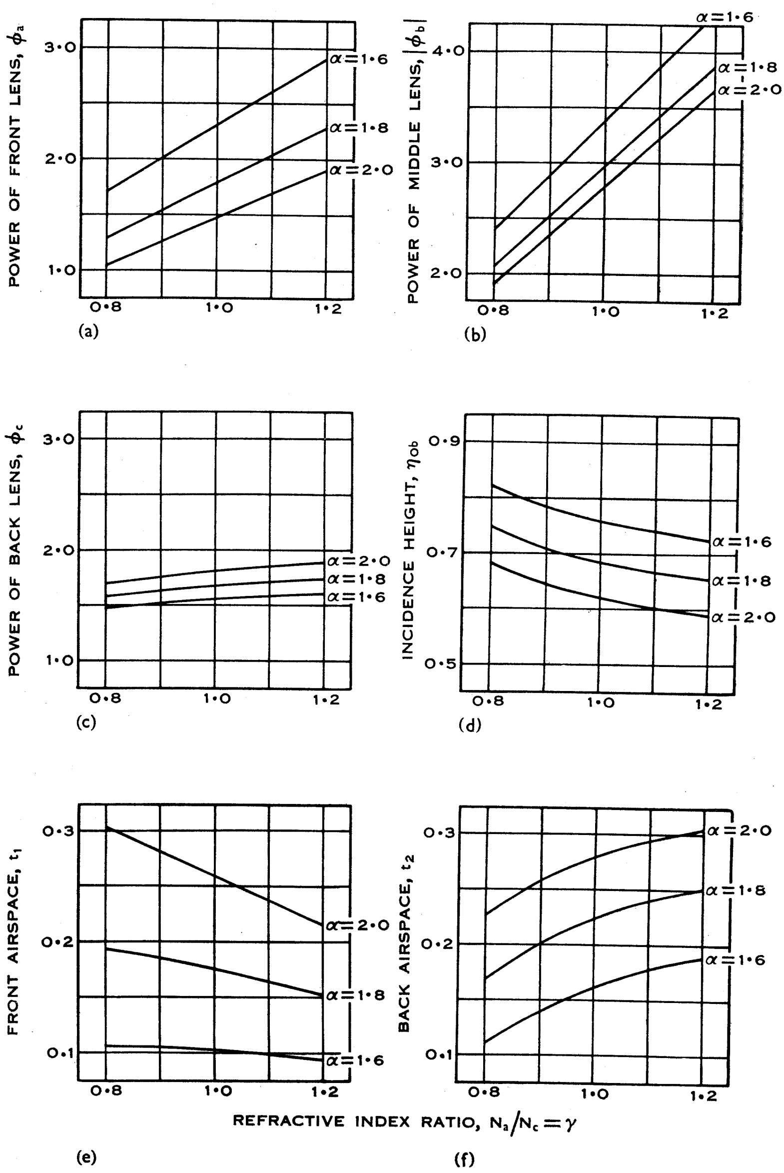

Fig. 4.—These figures show the effect of the variation of γ on the powers and separations of the thin components of unit power triplets having different values of α. In each triplet $\beta=1$, $\xi=1$, and $\chi=-0\cdot25$. The residuals selected are $P=0\cdot50$, $L=0\cdot20$, and $T=0\cdot05$.

IV. An Example of Initial Triplet Design

It will be useful now to discuss in detail an actual example of an initial design to illustrate the general method. Suppose we consider what could be achieved in a type 212 lens in which the front and rear components are iso-index doublets. We choose from the glass catalogue the approximately iso-index pair of glasses $1\cdot65695$, $50\cdot80$ and $1\cdot6538$, $33\cdot55$ for this purpose. The equivalent refractive index of a doublet from this pair will be approximately $1\cdot66$ but the equivalent V-number will depend on the relative values of k_1 and k_2. Equations (**4**), (**6**) show, for instance, that for the range $0 \leqslant |k_2/k_1| \leqslant 0\cdot6$ the corresponding equivalent V-number range is $50\cdot8 \leqslant V \leqslant 222$, so that we may give V almost any value we please.

Figures 1 (*a*)–1 (*c*) show that the powers of all components are reduced by making β as high as possible. In addition, the powers of the first two components are reduced by using a high value of α. Having fixed the value of N_a at approximately $1\cdot66$ by the choice of the iso-index pair, there remains only one way of increasing β and that is the choice of a low index glass for the central negative lens. Accordingly we choose the BSC glass $1\cdot5095$, $64\cdot4$ for this. It may seem rather unusual to choose such a crown glass for the main dispersive member of the triplet, but at least it does provide the increased value of β which seems desirable. We do not know yet what value of α will be required to make the correction of spherical aberration possible. Initially, a high value of α is desirable to give further reduction in the powers of the first two components. Quite arbitrarily we choose $\alpha=2$ and see what happens. Now $\alpha=V_a/V_b$ whence $V_a=2\times64\cdot4=128\cdot8$. Equations (**4**), (**6**) therefore give $k_2/k_1=-0\cdot54083$ and $k_1=2\cdot17785$, $k_2=-1\cdot17785$. With these k values equation (**5**) gives $N_a=1\cdot66068$ and hence $\beta=N_a/N_b=1\cdot1002$.

Since we are using the same iso-index pair for the back doublet, the equivalent refractive index of this doublet will be also approximately $1\cdot66$ and hence $\gamma=N_a/N_c$ will be very close to unity. It only remains, then, to settle the value of ξ. Figures 1 (*e*) and 1 (*f*), which are drawn for $\xi=1$, $\gamma=1$, show that the choice of $\alpha=2$ and $\beta=1\cdot100$ results in high values for the airspaces if $\xi=1$. Figures 3 (*e*) and 3 (*f*) show, however, that if ξ is increased the airspaces will be reduced substantially. Figures 3 (*a*)–3 (*c*) show also that this reduction in the airspaces will be obtained at the expense of some increase in the powers of the first and second components. A compromise is therefore called for and so, tentatively, we put $\xi=1\cdot5$. Since $\xi=V_a/V_c$, this gives $V_c=128\cdot8/1\cdot5=85\cdot867$ and thus $k_1=1\cdot79439$, $k_2=-0\cdot79439$, and $k_2/k_1=-0\cdot44270$. It then follows that the equivalent index of the back doublet is $N_c=1\cdot65946$ and thus $\gamma=N_a/N_c=1\cdot0007$.

The next matter is to select the values of P, L, and T. Putting $P=0\cdot6$ makes the Petzval sum $P/N_a=0\cdot6/1\cdot66068=0\cdot3613$. This is fairly high but can be tolerated if the field angle required is not too great. If we make L slightly positive the longitudinal chromatic aberration will be corrected at an outer zone of the aperture rather than paraxially. We therefore put $L=0\cdot50$, and similarly $T=0\cdot045$. Now, using equations (**21**)–(**32**) of Part I, initial solutions are made for three values of the corrector power, say $\chi=-0\cdot25$, $-0\cdot375$, and $-0\cdot50$.

The resulting equivalent thin solutions are summarized in the first five lines of Table 1. Using the values $k_2/k_1=-0\cdot54083$ and $-0\cdot44270$ for the front and back doublets respectively, the powers of the members of these doublets are calculated in each case and are shown in the next four lines of Table 1. This completes the calculation of the powers and separations of the thin components for these three systems.

TABLE 1

DETAILS OF DESIGN EXAMPLE DURING THIN LENS STAGE

χ	$-0\cdot250$	$-0\cdot375$	$-0\cdot500$
φ_a	$1\cdot2723$	$1\cdot2016$	$1\cdot1322$
φ_b	$-2\cdot1306$	$-2\cdot1710$	$-2\cdot2156$
φ_c	$1\cdot6705$	$1\cdot7857$	$1\cdot9040$
t_1	$0\cdot2244$	$0\cdot2279$	$0\cdot2326$
t_2	$0\cdot1351$	$0\cdot1169$	$0\cdot1023$
φ_{a1}	$2\cdot7708$	$2\cdot6168$	$2\cdot4657$
φ_{a2}	$-1\cdot4985$	$-1\cdot4152$	$-1\cdot3335$
φ_{c1}	$-1\cdot3270$	$-1\cdot4184$	$-1\cdot5124$
φ_{c2}	$2\cdot9975$	$3\cdot2041$	$3\cdot4164$
S_{a1}	$0\cdot1853$	$0\cdot2276$	$0\cdot2718$
S_{a2}	$0\cdot4992$	$0\cdot4213$	$0\cdot3401$
S_b	$-0\cdot2744$	$-0\cdot2027$	$-0\cdot1232$
S_{c1}	$-1\cdot9954$	$-1\cdot9220$	$-1\cdot8501$
S_{c2}	$0\cdot3324$	$0\cdot2998$	$0\cdot2678$
σ_1	$0\cdot9350$	$0\cdot8910$	$0\cdot9249$
σ_2	$+0\cdot0007$	$-0\cdot0002$	$-0\cdot0001$
σ_3	$-0\cdot0005$	$-0\cdot0005$	$-0\cdot0004$
$2\sigma_4$	$0\cdot3613$	$0\cdot3613$	$0\cdot3613$
σ_5	$0\cdot0000$	$0\cdot0000$	$0\cdot0000$

We come now to the determination of the shapes of the thin lenses. Assuming that the two doublets will be cemented, the shapes of the lenses 1, 2 of each doublet must satisfy the relation

$$\varphi_1(S_1-1)/(N_1-1)=\varphi_2(S_2+1)/(N_2-1). \quad \ldots\ldots\ldots \quad \mathbf{(11)}$$

There are thus only three independent shapes, say, S_{a1}, S_b, and S_{c2}. The condition for zero third-order astigmatism is now set up using equation (**44**) of Part I, the diaphragm being assumed to be coincident with the central negative lens b. In the case of $\chi=-0\cdot375$, for example, we obtain

$$2\cdot8746S_{a1}^2-6\cdot1068S_{a1}+S_{c2}^2+3\cdot4401S_{c2}+0\cdot11992=0.$$

Similarly, the condition for zero third-order distortion is derived from equation (**45**) of Part I, and for $\chi=-0\cdot375$ we find

$$-4\cdot3148S_{a1}^2+14\cdot1855S_{a1}+S_{c2}^2+5\cdot8447S_{c2}-4\cdot8468=0,$$

the solution of these two quadratics giving

$$S_{a1}=0\cdot22759, \qquad S_{c2}=0\cdot29975.$$

Equation (**11**) then gives $S_{a2}=0\cdot42134$, and the corresponding relation for the back doublet gives $S_{c1}=-1\cdot92195$. The condition for zero third-order coma is then written down from equation (**43**) of Part I and this gives S_b directly. As a check on the calculations up to this point the third-order aberrations are calculated for the thin systems thus obtained. The computations up to this stage are summarized in the third and fourth blocks of Table 1. Assuming a quadratic relation between the third-order spherical aberration σ_1 and the corrector power χ, the three values of σ_1 in Table 1 give

$$2\cdot4896\chi^2+1\cdot9074\chi+1\cdot2562=\sigma_1,$$

which should be roughly plotted as a guide to the possible correction of spherical aberration.

The next stage involves the replacement of the thin lenses by lenses of finite thickness. To do this we introduce axial thicknesses sufficient to permit a reasonable aperture, say, $f/2\cdot3$, and hence put $t_{a1}=0\cdot130$, $t_{c2}=0\cdot140$, and for the negative lenses $t_{a2}=t_b=t_{c1}=0\cdot03$. The replacements are made in the standard way which leaves the refraction of the axial paraxial ray the same as in the thin system. The third-order aberrations are now recalculated and attain the values shown in the first block of Table 2. It is apparent that substantial changes have occurred, necessitating readjustment of the shapes.

The shapes are readjusted as shown in the second block of Table 2 and the resulting aberration coefficients are given in the third block of the table. It will be seen that now the shapes have been chosen to ensure a small negative residual of primary coma and a sufficient value of σ_3 to flatten the tangential field. A small positive residual of primary distortion has been left to offset the negative secondary and tertiary distortion coefficients. The adjustment of the coefficients σ_2 to σ_5 is practically the same in each case, and the only third-order quantity that varies with χ is the spherical aberration coefficient. Assuming, as usual, the quadratic relation between χ and σ_1, we find

$$1\cdot3293\chi^2+1\cdot7971\chi+0\cdot7608=\sigma_1.$$

This expression is plotted on a ten-times scale in Figure 5.

Up to this point all the computation has been done on a desk machine. Now the complete coefficients of third, fifth, and seventh orders are computed on the digital computer, the process requiring about 5 minutes computing time per system. This checks all the earlier third-order work and gives the higher order coefficients as well. In Figure 5 the variation of the secondary coefficients with χ is plotted. Each curve is based on three computed points and quadratic variation with χ is assumed. In addition the dotted curves of $10\sigma_1$ and $10\sigma_2$ are added. Because of the way the shapes are chosen the remaining primary coefficients will be practically constant, as instanced by the plot of $10\sigma_2$. They are therefore not plotted.

This diagram should be well studied. It summarizes the correction possibilities of this type of construction over a wide range of χ values. As χ approaches zero from the negative side the following properties should be noticed.

(1) *Spherical aberration.*—The third-order coefficient σ_1 is positive and increasing, while the fifth-order coefficient μ_1 is negative and decreases in absolute value: values of χ can therefore be found for which the spherical aberration can be corrected at different apertures.

(2) *Coma.*—The fifth-order circular coma specified by μ_2, μ_3 is positive and is decreasing rapidly as $\chi \to 0$; it is advantageous therefore to proceed as far in this direction as possible. The fifth-order elliptical coma (μ_7, μ_8, μ_9) is positive

TABLE 2

DETAILS OF DESIGN EXAMPLE AFTER THE INTRODUCTION OF FINITE AXIAL THICKNESSES

χ	—0·250	—0·375	—0·500
σ_1	0·7701	0·7217	0·7423
σ_2	—0·3078	—0·2932	—0·2886
σ_3	—0·1104	—0·1204	—0·1322
$2\sigma_4$	0·3759	0·3789	0·3825
σ_5	0·0235	0·0451	0·0657
S_{a1}	0·0742	0·1384	0·2091
S_{a2}	0·7037	0·5855	0·4554
S_b	—0·3490	—0·3244	—0·3073
S_{c1}	—2·0990	—2·1363	—2·1760
S_{c2}	0·3785	0·3951	0·4128
σ_1	0·3946	0·2738	0·1945
σ_2	—0·0231	—0·0247	—0·0207
σ_3	—0·0620	—0·0604	—0·0600
$2\sigma_4$	0·3969	0·3971	0·3980
σ_5	0·0152	0·0123	0·0112
μ_1	—5·008	—7·168	—9·293
μ_2	1·548	2·850	4·489
μ_3	1·075	1·930	3·013
μ_4	—4·675	—5·595	—6·485
μ_5	—2·163	—2·576	—2·967
μ_6	—2·935	—3·316	—3·731
μ_7	2·034	1·959	1·939
μ_8	1·273	1·218	1·196
μ_9	0·7955	0·777	0·775
μ_{10}	—1·076	—1·054	—1·054
μ_{11}	—0·6291	—0·5948	—0·5678
μ_{12}	—0·0593	—0·0481	—0·0342

and, as it changes little with χ, nothing much can be done about it. Some degree of balance can be obtained by offsetting the effects of these positive coefficients by adjusting σ_2 to a negative value. This explains why σ_2 was adjusted to a small negative value earlier.

(3) *Curvature of field.*—The quantities which must be considered are the third-order astigmatism σ_3 and the Petzval curvature σ_4, the fifth-order

astigmatism coefficients μ_{10}, μ_{11}, and then the very important oblique spherical aberration terms involving μ_4, μ_5, μ_6. Usually it is the values of μ_4, μ_5, μ_6 which set a limit to the extent of the field of a photographic objective. In the development of this example we have already made some adjustment of the field by balancing σ_3 and σ_4 to give an approximately flat tangential field as

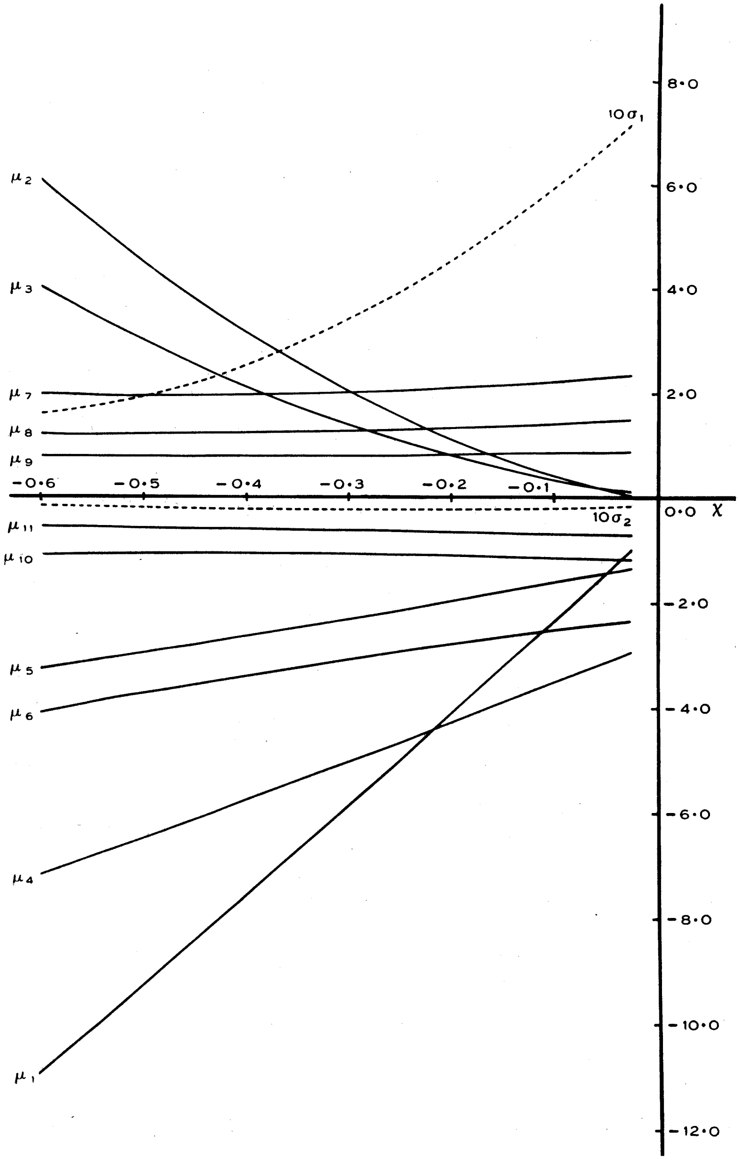

Fig. 5.—The variation of the twelve fifth-order aberration coefficients μ_1 to μ_{12}, with χ are shown for the group of type 212 triplets considered in the example. The upper dotted line shows the corresponding variation of $10\sigma_1$, the third-order spherical aberration plotted on a $10\times$ scale. The lower dotted line shows the variation of $10\sigma_2$, σ_2 being the third-order coma.

far as third order is concerned. Figure 5 shows that there is a significant decrease in μ_4, μ_5, μ_6 as χ tends towards zero. It will be advantageous therefore to move as far as possible in this direction for field correction as well as for the diminution of fifth-order circular coma.

Examining the spherical aberration coefficients up to the seventh order, it becomes clear that, if we select $\chi=-0\cdot27$ approximately, we can achieve spherical correction at an aperture of about $f/2\cdot3$. We can now read off the values of all the aberration coefficients of this system and thus are able to assess accurately the correction state of this system without any further detailed computation. If this is not quite what is wanted, a slight shift in χ may give

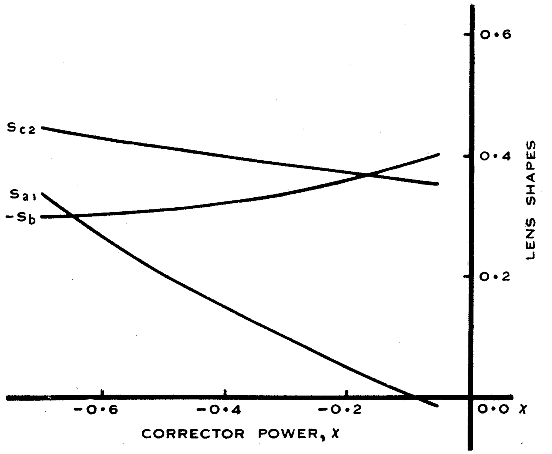

Fig. 6.—This figure shows the relations between the corrector power χ and the shapes of the thick lenses a_1, b, and c_2 which give simultaneous correction of third-order coma, astigmatism, and distortion in the type 212 triplet systems considered in the example. The shapes of the lenses for any selected χ value may be read directly from these graphs.

a more useful system. The point to emphasize is that Figure 5 and a similar diagram showing the corresponding trends among the seventh-order coefficients summarize completely the possibilities in this type of system with this selection of glasses and residuals. If, for example, the work were repeated with some other set of glasses, a corresponding diagram would result, and a direct comparison of the two diagrams would show what advantages or disadvantages had resulted from the changes introduced. In this way areas of interest can be explored with precise knowledge of what is happening to all the aberration coefficients and why the changes are taking place.

Suppose now that it is decided that the system with $\chi=-0\cdot270$ is the solution appropriate to a particular purpose or requirement. The detailed specification of this system can be obtained with very little further computation. The initial solution is computed with this value of χ, and this gives

$$\varphi_a=1\cdot2608, \quad \varphi_b=-2\cdot1367, \quad \varphi_c=1\cdot6887$$
$$t_1=0\cdot2249, \quad t_2=0\cdot1319.$$

Using the values $k_2/k_1 = -0 \cdot 54083$ for the front doublet and $k_2/k_1 = -0 \cdot 44269$ for the back doublet, the powers of the five thin lenses of the system are obtained. Figure 6 shows the relations between the corrector power χ and the shapes of the lenses a_1, b, and c_2 for which simultaneous correction of third-order coma, astigmatism, and distortion is achieved. These curves are based on the data contained in the second block of Table 2, i.e. the final shape values used in the three computed examples to adjust the third-order coma, astigmatism, and distortion. Quadratic relations between the shapes and χ have been assumed in the curve fitting. Hence the shapes required for the lenses a_1, b, c_2 for the $\chi = -0 \cdot 270$ system may be read off directly. The values of the shapes of lenses a_2 and c_1 follow from the cementing conditions, e.g. equation (**11**). Assuming the same axial thicknesses as used already in the example and the shapes determined in the manner just described, the radii of the refracting surfaces of each lens may be computed and thus the specification completed. Running down the intersections of the curves of Figure 5 with the ordinate $\chi = -0 \cdot 270$, the aberration coefficient values expected for the system are then read off. These may then be confirmed by a 5-minute computation on the digital computer.

V. Acknowledgments

The author wishes to acknowledge gratefully the help of Mrs. B. J. Brown, Senior Computer at the laboratory, in much of the computation associated with this work and in its preparation for publication. Grateful thanks are also due to Miss Jennifer James, who carried out the computations for the example and drew all the diagrams. The financial support of this work by the Department of Defence Production is also gratefully acknowledged.

VI. References

Buchdahl, H. A. (1954).—" Optical Aberration Coefficients." (Oxford Univ. Press.)
Cruickshank, F. D. (1958).—*Aust. J. Phys.* **11** : 41.
Cruickshank, F. D., and Hills, G. A. (1960).—*J. Opt. Soc. Amer.* **50** : (In press.)
Ford, P. W. (1959).—*J. Opt. Soc. Amer.* **49** : 875.