IONOSPHERIC REFRACTION IN RADIO ASTRONOMY

I. THEORY

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Summary

Expressions are derived for the apparent displacements of cosmic radio sources at transit resulting from ionospheric refraction. The derivation takes account of horizontal electron density gradients whose effects often outweigh the refraction expected from a spherically symmetrical ionosphere.

Using these results together with position measurements taken at 19·7 Mc/s, the " total thickness " of the ionosphere has been estimated. This quantity relates the total electron content of a vertical column to the maximum electron density. During the observing period it had a value of 355 km, in good agreement with moon-echo results. The observations also indicate that a considerable improvement in observed source positions may be achieved by applying the theory.

I. Introduction

Ionospheric refraction may seriously limit the accuracy of wide-aperture radio-astronomy aerials operating at low frequencies. A number of authors, notably Bailey (1948), Belyaev (1955), Link (1957), and Chvojkova (1958*a* and 1958*b*), have estimated this refraction assuming that the ionosphere is a spherical shell concentric with the Earth. Smith (1952), on the other hand, has shown that horizontal density gradients lead to appreciable displacements of sources at the zenith, indicating that an adequate theory must take account of departures from spherical symmetry.

The present calculation shows that, for a source at transit, the change of declination depends on the total electron content of a column of unit area through the ionosphere, together with the north-south gradient of this quantity. The change of Right Ascension depends to a first order only on the east-west gradient.

The investigation had a twofold aim. It was originally prompted by the need to eliminate refraction effects from galactic records taken with a Mills Cross operating as a transit instrument at 19·7 Mc/s. For this it was necessary to relate observed displacements of discrete sources to published ionospheric sounding data. This was accomplished, and also information was gained about the structure of the ionosphere above the F-region maximum.

This paper deals mainly with the theoretical aspect of the problem and includes an estimate of errors in the necessarily approximate treatment. A brief discussion of some experimental results shows that the derived value of ionospheric " total electron content " is in good agreement with estimates derived by

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other methods, and also indicates the improvement to the astronomical data which may be attained using the derived corrections.

A preliminary report on this work has been given previously (Komesaroff and Shain 1959).

II. Definitions of the Main Symbols Used

The more important symbols used are defined as follows:

N, number of electrons per cubic centimetre,
f, frequency of observation,
f_c, critical frequency of the F region,
X_m, f_c^2/f^2,
r, distance to the Earth's centre,
r_m, radius of the surface of maximum electron density,
r_b, radius of the lower ionospheric bounding surface,
φ, terrestrial latitude (radians),
Φ, latitude of observer,
L, longitude to the east of the observer (radians),
k, angle between ray tangent and radius vector,
Z, observed zenith angle,
δ, declination,
α, Right Ascension,
y_m, y_m', "semithicknesses" of regions above and below maximum density (parabolic model),
d, total effective thickness; $d=\frac{2}{3}(y_m+y_m')$.

III. Ray Paths in the Ionosphere

In discussing ionospheric parameters a spherical coordinate system, r, φ, L, is used with origin at the centre of the Earth, φ being latitude and L longitude measured to eastward from the observer. Records of critical frequency taken at different places show that the refractive index varies not only with r but also with φ and L. Thus the optical length of a ray path between points a and b is

$$P=\int_a^b \mu(r,\varphi,L)\mathrm{d}s$$
$$=\int_a^b [1+r^2\dot{\varphi}^2+r^2\dot{L}^2\cos^2\varphi]^{\frac{1}{2}}\mu(r,\varphi,L)\mathrm{d}r,$$

where $\dot{\varphi}=\mathrm{d}\varphi/\mathrm{d}r$ and $\dot{L}=\mathrm{d}L/\mathrm{d}r$.

From Fermat's principle, the differential of P with respect to changing ray paths is zero, and this leads to the following ray equations:

$$\frac{\mathrm{d}}{\mathrm{d}r}\left(\mu r^2\frac{\mathrm{d}\varphi}{\mathrm{d}s}\right)=\frac{\mathrm{d}s}{\mathrm{d}r}\left[\frac{\partial\mu}{\partial\varphi}-\mu r^2\frac{\mathrm{d}L}{\mathrm{d}s}\sin\varphi\cos\varphi\right], \quad \ldots\ldots\ldots \quad (1)$$

$$\frac{\mathrm{d}}{\mathrm{d}r}\left(\mu r^2\cos^2\varphi\frac{\mathrm{d}L}{\mathrm{d}s}\right)=\frac{\mathrm{d}s}{\mathrm{d}r}\frac{\partial\mu}{\partial L}. \quad \ldots\ldots\ldots \quad (2)$$

From these equations the refraction produced by any model ionosphere may be calculated.

Following Ratcliffe (1951) we will assume a density distribution of electrons below the maximum value N_m,

$$N = N_m(1 - y^2/y_m^2), \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

where y is the distance from the maximum density layer and the scale factor y_m is of the order of 100 km. A corresponding expression is adopted for the distribution above the maximum, y_m being replaced by y_m', where, according to the moon-echo work of Evans (1957), y_m' is several times larger than y_m, and

$$y_m + y_m' \gtrsim 0 \cdot 1 r_m,$$

r_m being the radius of the maximum density layer.

For a radio wave of frequency f, well above the electron gyro frequency (usually the case in radio-astronomical work), the refractive index is given by

$$\mu^2 = 1 - f_p^2/f^2 = 1 - X, \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

where f_p^2, the square of the "plasma frequency", is proportional to N. Thus from (3)

$$\mu^2 = 1 - X_m(1 - y^2/y_m^2). \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

In terms of the F-layer critical frequency f_c, X_m is defined by

$$X_m = f_c^2/f^2.$$

Now, assuming that the scale factors y_m, y_m' are substantially constant over large distances, then corresponding to horizontal density gradients we have refractive index gradients given by

$$\frac{\partial \mu}{\partial \varphi} = \frac{-1}{2\mu} \frac{\partial X_m}{\partial \varphi}\left(1 - \frac{y^2}{y_m^2}\right), \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (6)$$

$$\frac{\partial \mu}{\partial L} = \frac{-1}{2\mu} \frac{\partial X_m}{\partial L}\left(1 - \frac{y^2}{y_m^2}\right). \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (7)$$

In the subsequent argument it will be assumed that these partial derivatives of X_m are constant over small angular ranges.

IV. The Apparent Change of Declination

The discussion will be limited to the case of a source at transit. Let us first assume that there are no east-west refractive index gradients, so that the observed ray lies entirely in the meridian plane. Later it will be shown that the effect of these gradients on observed declinations is negligible.

(a) *The Two Refraction Components*

In Figure 1, ABC is part of a ray path. The ray tangent at B, in latitude φ, makes an angle k with the radius vector to the centre of the Earth O. It follows from the figure that the apparent declination of the source as seen from B is D, where

$$D = \varphi + k. \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (8)$$

It also follows that

$$\cos k = \mathrm{d}r/\mathrm{d}s, \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (9)$$

$$\sin k = \mathrm{r}\mathrm{d}\varphi/\mathrm{d}s, \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (10)$$

and

$$(\tan k)/r = \mathrm{d}\varphi/\mathrm{d}r. \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (11)$$

In Figure 2, *QRST* is a ray from a distant source to a terrestrial observer at *T*. The upper and lower ionospheric bounding radii are $(r_m + y_m')$ and $(r_m - y_m)$ respectively. The true declination δ of the source is indicated by the direction

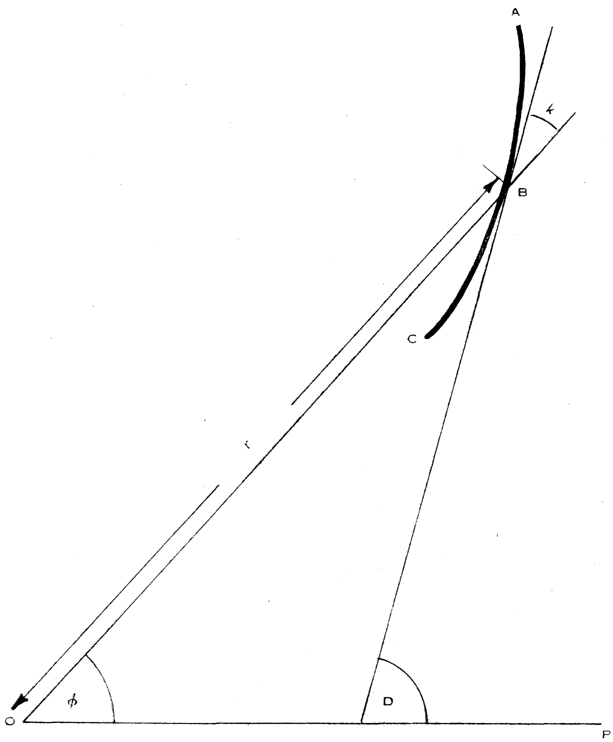

Fig. 1.—Section of a ray path in the meridian plane. *OP* is the equatorial plane and angle *D* is the apparent declination of the source seen from point *B* in latitude φ.

of the ray at the point *Q* at which it intersects the upper boundary. Denoting the values of angles at this level by the suffix *a*, it follows from (**8**) and (**11**) that

$$\delta = \Phi + \int_{r_e}^{r_m + y_m'} \frac{\tan k}{r} \mathrm{d}r + k_a, \quad \ldots\ldots\ldots\ldots\ldots \quad (12)$$

where Φ is the observer's latitude and r_e the radius of the Earth.

The straight line *TU* indicates the apparent declination δ_0 as seen from *T*. Denoting the angle between *TU* and the radius vector at any level by k_0, it may be seen that the value of δ_0 is given by a relation formally identical with (**12**), namely,

$$\delta_0=\Phi+\int_{r_e}^{r_m+y'_m}\frac{\tan k_0}{r}\mathrm{d}r+k_{0a}. \qquad \ldots\ldots\ldots\ldots \quad (\mathbf{13})$$

Thus the difference $\Delta(\delta)$ between the apparent and true declination is given by

$$\Delta(\delta)=\delta_0-\delta=k_{0a}-k_a+\int_{r_m-y_m}^{r_m+y'_m}\frac{(\tan k_0-\tan k)}{r}\mathrm{d}r, \quad \ldots\ldots \quad (\mathbf{14})$$

(since $k=k_0$ when r lies between r_e and r_m-y_m). The equation for k_0 is

$$r\sin k_0=r_e\sin Z=r_m\sin k_{0m}. \qquad \ldots\ldots\ldots\ldots \quad (\mathbf{15})$$

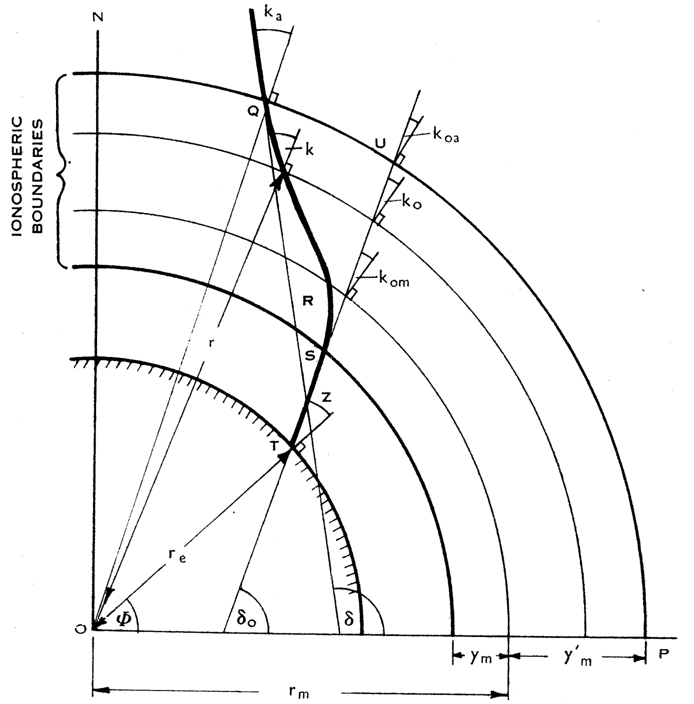

Fig. 2.—Path of a ray through the ionosphere to an observer at T. OP is the equatorial plane. (Not to scale.)

Here Z is the apparent zenith angle of the source and the suffix m denotes the value of a parameter at the level of maximum electron density. To calculate k we must refer to the ray equation (**1**). Since according to our assumption there are no east-west gradients, this reduces to

$$\frac{\mathrm{d}}{\mathrm{d}r}\left(\mu r^2\frac{\mathrm{d}\varphi}{\mathrm{d}s}\right)=\frac{\mathrm{d}s}{\mathrm{d}r}\,\frac{\partial\mu}{\partial\varphi}.$$

Integrating and combining this equation with (**9**), (**10**), and (**15**) yields

$$\mu r\sin k=r_m\sin k_{0m}+\int_{r_m-y_m}^{r}\sec k\frac{\partial\mu}{\partial\varphi}\mathrm{d}r. \qquad \ldots\ldots\ldots \quad (\mathbf{16})$$

Now taking into account that $\mu=1$ at the upper boundary, it follows from (**14**), (**15**), and (**16**) that

$$\Delta(\delta)=\Delta(\delta)_w+\Delta(\delta)_s,$$

where

$$\Delta(\delta)_w=k_{0a}-\sin^{-1}\left[\sin k_{0a}+\left(\frac{1}{r_m+y_m'}\right)\int_{r_m-y_m}^{r_m+y_m'}\sec k\frac{\partial\mu}{\partial\varphi}\mathrm{d}r\right], \quad \ldots (\mathbf{17})$$

$$\Delta(\delta)_s=\int_{r_m-y_m}^{r_m+y_m'}\frac{(\tan k_0-\tan k)}{r}\mathrm{d}r, \quad \ldots\ldots (\mathbf{18})$$

and

$$\sin k_{0a}=r_m\sin k_{0m}/(r_m+y_m'). \quad \ldots\ldots (\mathbf{19})$$

In substituting the value of k from (**16**) into (**17**) and (**18**), a simplifying approximation may be made. Provided k nowhere exceeds about 45°, and the values of $\partial\mu/\partial\varphi$ are such that the second term on the right-hand side of (**16**) is " small ", that is, provided

$$\frac{1}{\mu r}\int\sec k\frac{\partial\mu}{\partial\varphi}\lesssim\frac{\cos k_{0m}}{30}\mathrm{d}r, \quad \ldots\ldots (\mathbf{20})$$

we may neglect this term in (**16**), giving

$$\sin k=r_m\sin k_{0m}/\mu r. \quad \ldots\ldots (\mathbf{21})$$

The restriction imposed by inequality (**20**) will be discussed subsequently. It may be shown that the approximation which we have made results in an error equivalent to an underestimate of $\Delta(\delta)_w$ of the order

$$\{(y_m+y_m')/r_m\}\Delta(\delta)_w.$$

Thus the total north-south displacement of a source at transit is expressible as the sum of two terms given by (**17**) and (**18**). The term $\Delta(\delta)_w$, which will be designated the " wedge component ", depends on angular gradients of refractive index and vanishes in the limiting case of spherical symmetry. The term $\Delta(\delta)_s$ depends to a first order only on the radial distribution and represents the refraction calculated for a spherically symmetrical medium. Accordingly it is called the " spherical component ". Each of these terms is evaluated below. It is later shown that the wedge component is commonly predominant, indicating that previous treatments of the problem based on a spherically symmetrical model lead to erroneous results.

(*b*) *The Wedge Component*

Equation (**17**) may be simplified by a minor approximation. From (**19**),

$$k_{0a}\simeq k_{0m},$$

and, since the integral term has been assumed " small ", (**17**) may be restated thus

$$\Delta(\delta)_w=\frac{-\sec k_{0m}}{(r_m+y_m')}\int_{r_m-y_m}^{r_m+y_m'}\sec k\frac{\partial\mu}{\partial\varphi}\mathrm{d}r. \quad \ldots\ldots (\mathbf{22})$$

An expression for sec k may be derived from (**4**), (**5**), and (**21**). If M is defined by the relation

$$M=[1-X\sec^2 k_{0m}]^{-\frac{1}{2}}=[1-X_m\sec^2 k_{0m}(1-y^2/y_m^2)]^{-\frac{1}{2}} \quad .. \quad (\mathbf{23})$$

then it may be shown that

$$\sec k=\mu M\sec k_{0m}[1+(1-r_m^2/r^2)M^2\tan^2 k_{0m}]^{-\frac{1}{2}}. \quad .... \quad (\mathbf{24})$$

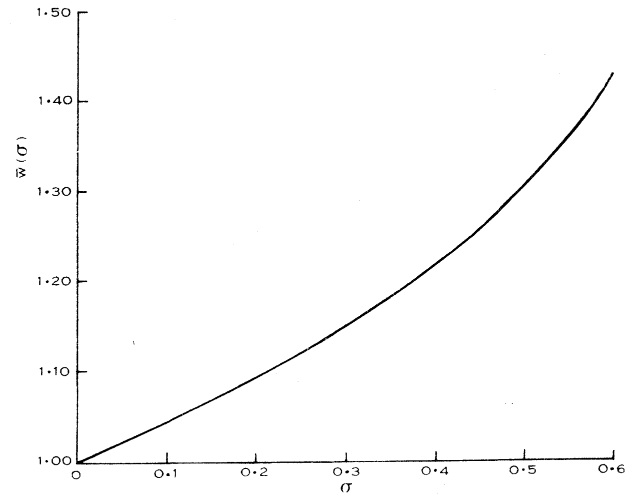

Fig. 3.—Graph of the function

$$\bar{w}(\sigma)=\frac{3}{4\sigma}\{[1+\sigma)/2\sigma^{\frac{1}{2}}]\ln[(1+\sigma^{\frac{1}{2}})/(1-\sigma^{\frac{1}{2}})]-1\},$$

where $\sigma=(f_c^2/f^2)\sec^2 k_{0m}$.

Now, provided $k_{0m}\lesssim 45°$ and $\sigma\lesssim\frac{1}{2}$, where σ is defined by

$$\sigma=X_m\sec^2 k_{0m},$$

the value of the surd in (**24**) is very nearly unity throughout the range of integration. Thus from (**22**) and (**24**) we derive the approximation

$$\Delta(\delta)_w=\frac{-\sec^2 k_{0m}}{r_m+y_m'}\int_{r_m-y_m}^{r_m+y_m'} M\mu\frac{\partial\mu}{\partial\varphi}\mathrm{d}r.$$

Substituting from (**5**), (**6**), and (**23**) and integrating,

$$\Delta(\delta)_w=\frac{(y_m+y_m')\cdot\bar{w}(\sigma)\sec^2 k_{0m}}{3(r_m+y_m')f^2}\cdot\frac{\partial(f_c^2)}{\partial\varphi}, \quad .... \quad (\mathbf{25})$$

where

$$\bar{w}(\sigma)=\frac{3}{4\sigma}\left\{\frac{(1+\sigma)}{2\sigma^{\frac{1}{2}}}\ln\left(\frac{1+\sigma^{\frac{1}{2}}}{1-\sigma^{\frac{1}{2}}}\right)-1\right\}. \quad .... \quad (\mathbf{26})$$

The parameter $\bar{w}(\sigma)$ is shown as a function of σ in Figure 3. For values of σ lying between 0 and $\frac{1}{2}$, $\bar{w}(\sigma)$ lies between 1 and 1·3.

The error of approximation in going from (**17**) to (**25**) is equivalent to an overestimate of about $\{y_m+y'_m)/r_m\}\Delta(\delta)_w$. Since the error mentioned in Section III (*a*) was of similar magnitude but opposite sign, the total approximation error is generally less than $\{(y_m+y'_m)/r_m\}\Delta(\delta)_w$, or one part in ten.

The foregoing results have been derived on the assumption that the vertical distribution of electron density has a parabolic form. However, it may be shown that (**25**) can be generalized to cover any form of vertical profile, provided that this is a smoothly varying function and has a reasonably sharp upper boundary. For the general form we introduce the " effective thickness " d of the ionosphere given by

$$dN_m=\int N\mathrm{d}r, \qquad \ldots\ldots \qquad (27)$$

so that for a parabolic layer

$$d=\tfrac{2}{3}(y_m+y'_m).$$

Equation (**25**) then becomes

$$\Delta(\delta)_w=\frac{d\bar{w}(\sigma)\sec^2 k_{0m}}{2(r_b+3d/2)f^2}\,\frac{\partial(f_c^2)}{\partial\varphi}, \qquad \ldots\ldots \qquad (28)$$

where r_b is the inner bounding radius of the ionosphere.

(*c*) *The Spherical Component*

To evaluate the integral in equation (**18**) we require an expression for $\tan k$. From (**21**) and (**24**),

$$\tan k=(r_m/r)M\tan k_{0m}[1+(1-r_m^2/r^2)M^2\tan^2 k_{0m}]^{-\frac{1}{2}}.$$

If now M is replaced by $\bar{M}$, its mean value for the range of integration, and the resulting value of $\tan k$ substituted in (**18**), it may be shown that this gives a close approximation to $\Delta(\delta)_s$,

$$\Delta(\delta)_s=\int_{r_m-y_m}^{r_m+y'_m}\frac{r_m\tan k_{0m}}{r^2}$$
$$\times\left\{\left[1+\left(1-\frac{r_m^2}{r^2}\right)\tan^2 k_{0m}\right]^{-\frac{1}{2}}-\bar{M}\left[1+\left(1-\frac{r_m^2}{r^2}\right)\bar{M}^2\tan^2 k_{0m}\right]^{-\frac{1}{2}}\right\}\mathrm{d}r$$
$$=\sin^{-1}\left[\frac{r_m\sin K}{r_m+y'_m}\right]-\sin^{-1}\left[\frac{r_m\sin K}{r_m-y_m}\right]-\sin^{-1}\left[\frac{r_m\sin k_{0m}}{r_m+y'_m}\right]+\sin^{-1}\left[\frac{r_m\sin k_{0m}}{r_m-y_m}\right]. \qquad \ldots\ldots \qquad (29)$$

Here K is given by

$$\tan K=\bar{M}\tan k_{0m},$$

and

$$\bar{M}=\frac{1}{y_m}\int_{-y_m}^{0}M\mathrm{d}y=\frac{1}{y'_m}\int_{0}^{y'_m}M\mathrm{d}y=\frac{1}{y_m+y'_m}\int_{-y_m}^{y'_m}M\mathrm{d}y$$
$$=\frac{1}{2\sigma^{\frac{1}{2}}}\ln\left(\frac{1+\sigma^{\frac{1}{2}}}{1-\sigma^{\frac{1}{2}}}\right). \qquad \ldots\ldots \qquad (30)$$

The parameter $\bar{M}$ is shown as a function of σ in Figure 4.

The error in approximation (**29**) is

$$\int_{r_m-y_m}^{r_m+y'_m} \frac{r_m \tan k_{0m}}{r^2}$$
$$\times\left\{\bar{M}\left[1+\left(1-\frac{r_m^2}{r^2}\right)\bar{M}^2\tan^2 k_{0m}\right]^{-\frac{1}{2}}-M\left[1+\left(1-\frac{r_m^2}{r^2}\right)M^2\tan k_{0m}^2\right]^{-\frac{1}{2}}\right\}\mathrm{d}r.$$

By expressing the integrand as a first-order Taylor expansion in $(\bar{M}-M)$, it may be shown that the ratio of this error term to $\Delta(\delta)_s$, given by (**29**), is considerably less than $(y_m+y'_m)/r_m$.

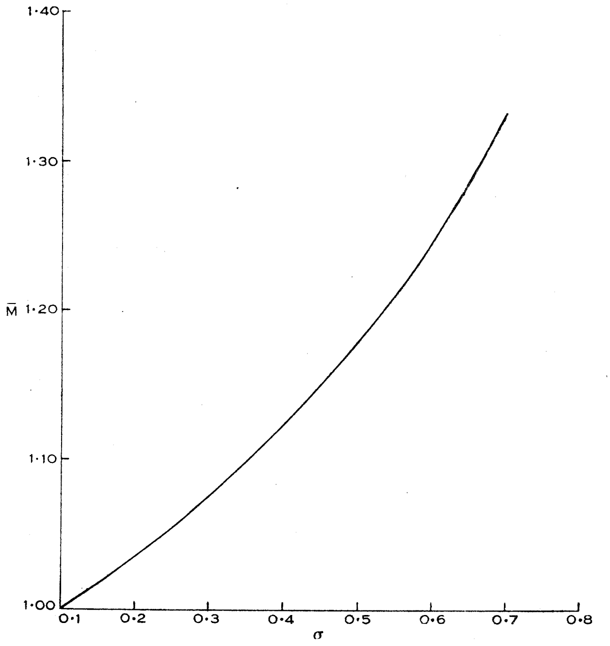

Fig. 4.—Graph of the function $\bar{M}=(1/2\sigma^{\frac{1}{2}})\ln[(1+\sigma^{\frac{1}{2}})/(1-\sigma^{\frac{1}{2}})]$.

In Figure 5, $\Delta(\delta)_s$ is shown as a function of k_{0m} according to equation (**29**). This figure also includes a graph relating k_{0m} to the apparent zenith angle Z (from (**15**)). Figure 5 is based on a model ionosphere for which

$$y_m=0\cdot018r_m=120 \text{ km},$$
$$y'_m=0\cdot062r_m=410 \text{ km}.$$

For values of y_m and y'_m differing slightly from the above, $\Delta(\delta)_s$ may be calculated by assuming a linear relation with the total thickness.

In some applications a less accurate expression than (**29**) is adequate. Thus at high frequencies and for moderate values of k_{0m}, the following relation may be derived from (**29**)

$$\Delta(\delta)_s=-\frac{d}{2r_m}\frac{f_c^2}{f^2}\sec^2 k_{0m}\tan k_{0m}. \qquad (\mathbf{31})$$

This equation, due originally to Chvojkova (1958*a*, 1958*b*), is very convenient for rapid numerical calculation.

Numerical values of $\Delta(\delta)_s$ may be computed by graphical integration of the general refraction integral

$$\int \frac{\tan i}{\mu} \mathrm{d}\mu$$

taken in conjunction with equation (**21**). In Table 1 a number of values calculated in this way are presented, together with corresponding values from equation (**29**). For comparison, results given by Chvojkova's equation (**31**) and an equation due to Bailey (1948), are also quoted (the last example quoted is one

TABLE 1

THE SPHERICAL COMPONENT

k_{0m}	$\frac{f_c^2}{f^2}$	y'_m (km)	y_m (km)	$r_m - r_e$ (km)	$-\Delta(\delta)_s$			
					Bailey	Chvojkova	Eqn. (**29**)	Graph. Integ.
25° 35·9′	0·25	330	100	300	11·1′	11·0′	13′	13·1′
30°	0·1875	561	198	300	19·1′	19′	21′	21·4′
45°	0·25	495	99	300	51·9′	51·6′	1° 5·7′	1° 7·6′
45°	0·25	660	165	300	1° 12′	1° 11·6′	1° 29′	1° 30′
70° 42·2′	0·0576	375	75	375	1° 8′	1° 4′	1° 25·1′	1° 26·6′

which Bailey considers). It can be seen that equation (**29**) yields an accuracy five to ten times better than the other equations. However, world maps of critical frequency show that in moderate latitudes $\partial(f_c^2)/\partial\varphi$ often far exceeds f_c^2, so that from (**28**) and (**31**)

$$\Delta(\delta_w) \gg \Delta(\delta)_s.$$

Under these conditions, $\Delta(\delta)$ may be calculated using the less accurate form of $\Delta(\delta)_s$ without significant loss of overall accuracy.

(d) The Total Change of Declination

Combining equations (**28**) and (**31**), the angle $\Delta(\delta)$ by which the observed declination of a source at transit exceeds its true declination is given by

$$\Delta(\delta) = \frac{d \sec^2 k_{0m}}{2(r_b + 3d/2)f^2}\left[\bar{w}(\sigma)\frac{\partial(f_c^2)}{\partial\varphi} - \frac{(r_b+3d/2)}{r_m}f_c^2 \tan k_{0m}\right], \quad .. \quad (\mathbf{32})$$

which may be written

$$\Delta(\delta) = \frac{d \sec^2 k_{0m}}{2(r_b + 3d/2)f^2} R(\delta) \quad \ldots\ldots\ldots\ldots \quad (\mathbf{33})$$

(for k_{0m}, d, and $\bar{w}(\sigma)$ see equations (**15**) and (**27**) and Figure 3).

Equation (**32**) has an accuracy of about one part in ten for sources up to about 45° from the zenith assuming that f is not less than $2\frac{1}{2}f_c \sec k_{0m}$. This is not a serious restriction since radiation of frequencies less than $f_c \sec k_{0m}$ cannot be observed.

The derivation of (**32**) also assumes that the first term on the right-hand side considerably outweighs the second. When this condition is not satisfied, a more accurate approximation to the second term, given by (**29**), should be used. When the condition is satisfied, however, it is noteworthy that the source displace-

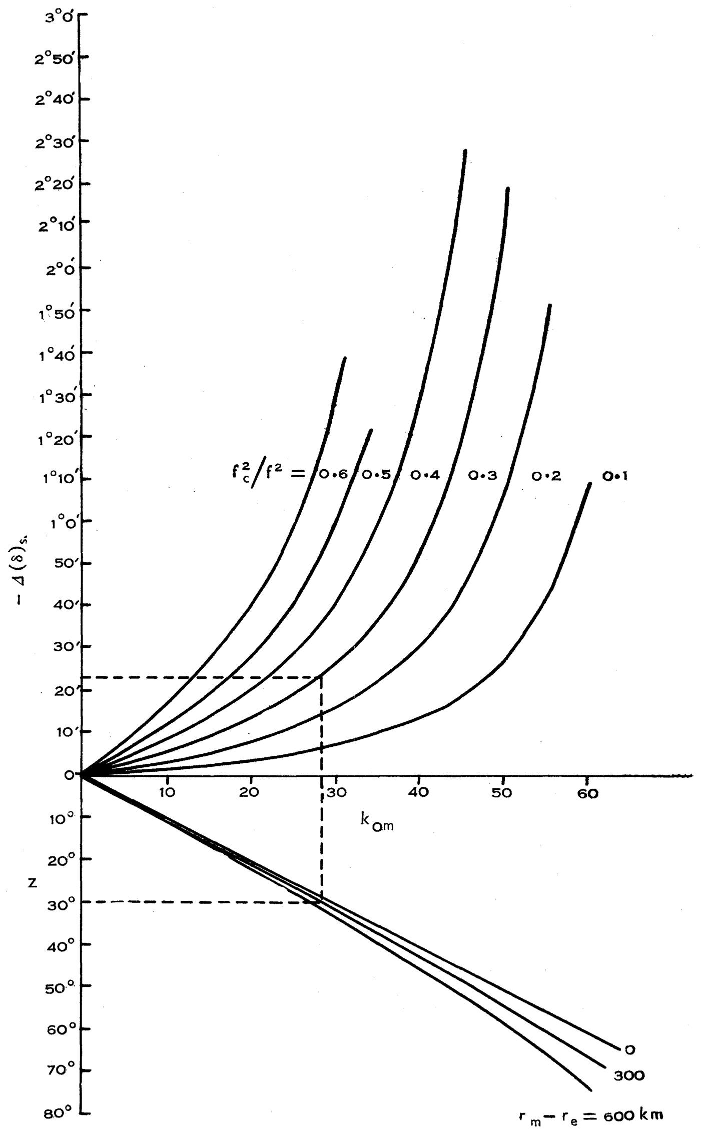

Fig. 5.—Curves showing the relationship between $\Delta(\delta)_s$ and observed zenith angle Z for various values of the parameters f_c^2/f^2 and r_m-r_e, according to equations (**29**) and (**15**). The dotted line illustrates the way in which the figure is used to determine $\Delta(\delta)_s$. In the example shown $Z=30°$, $r_m-r_e=300$ km, $f_c^2/f^2=0\cdot3$, and $\Delta(\delta)_s=-23'$.

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ment is in the direction in which f_c increases. By contrast, in a spherically symmetrical medium the displacement is always towards the zenith.

Finally, certain restrictions have been imposed on the magnitude of the horizontal gradients. From inequality (**20**) and equation (**22**), it follows that the first term in (**32**) (the wedge component) should not exceed about 1/30 radian or 2°. In the next section the magnitude of the change in Right Ascension is expressed in terms of east-west gradients. Provided this change also does not exceed a few degrees, the foregoing derivation, which has neglected east-west gradients, is still valid.

V. The Apparent Change of Right Ascension

In general the refractive index gradient has an east-west component. We will designate the resulting change in Right Ascension by $\Delta(\alpha)$. From equation (**2**), by an argument similar to that outlined in Section IV (*b*),

$$\Delta(\alpha)=\frac{d\cdot\bar{w}(\sigma)\sec\delta\cdot\sec\varphi_a\sec k_{0m}}{2(r_b+3d/2)f^2}\cdot\frac{\partial(f_c^2)}{\partial L}. \quad \ldots\ldots \quad (\mathbf{34})$$

The angle φ_a is the latitude of the point at which the ray intersects the upper ionospheric boundary. To sufficient accuracy $\sec\varphi_a$ is given by

$$\sec\varphi_a=\sec\Phi\left[1-\tan k_{0m}\tan\varphi_T\left(\frac{r_b-r_e+3d/2}{r_m}\right)\right]^{-1}.$$

Provided

$$\frac{\cos\delta}{\cos\Phi}\cos k_{0m}\gtrsim 1,$$

(**34**) has an accuracy of about 5 per cent.

VI. Applications of the Theory

The results just derived have been applied to a number of discrete source observations taken with the Sydney 19·7 Mc/s Mills Cross with a beamwidth of 1·4° (Shain 1958).

The observing site (lat. 34° S., long. 151° E.) is well placed for calculating f_c^2 and its north-south gradient, since there are a number of ionospheric sounding stations 800–900 km apart and close to the 150th meridian. The situation is less favourable, however, for calculating the east-west gradient. The nearest stations in this direction are at Christchurch (New Zealand) and Watheroo (Western Australia), each more than 2000 km away. Furthermore, it was found on comparing sounding records taken in longitude 150° E. with Watheroo records for the same local times, that the time derivative of f_c^2 (as used by Smith (1952)), was not a sufficiently accurate measure of the east-west gradient. The method of computation finally arrived at required data from at least three stations. Since there were very few days on which these were all available at the appropriate times, much of the information had to be based on monthly medians. Consequently, the east-west component estimates are considerably less accurate than those of the north-south component.

Figure 6 shows declination measurements of the source 03S3A, taken between September and November 1957. The local times of transit were between 2330 and 0330. It is clear that there is good correlation between observed declination δ_0 and the quantity $R(\delta)$, given by equations (**32**) and (**33**). (Since the source is close to the zenith the second term on the right-hand side of (**32**) may be neglected.) From the " least squares " regression line also shown on this figure, the " true declination " is $-37° 23'$. The declination measured at 85 Mc/s is $-37° 23' \pm 3'$ (Sheridan 1958), so that the two results agree within the limits of observational accuracy.

From the slope of the regression line, taking r_b as 6500 km, we calculate that d=355 km. This is only about 10 per cent. lower than a value derived by combining Evans's (1957) moon-echo results taken in the pre-dawn period at

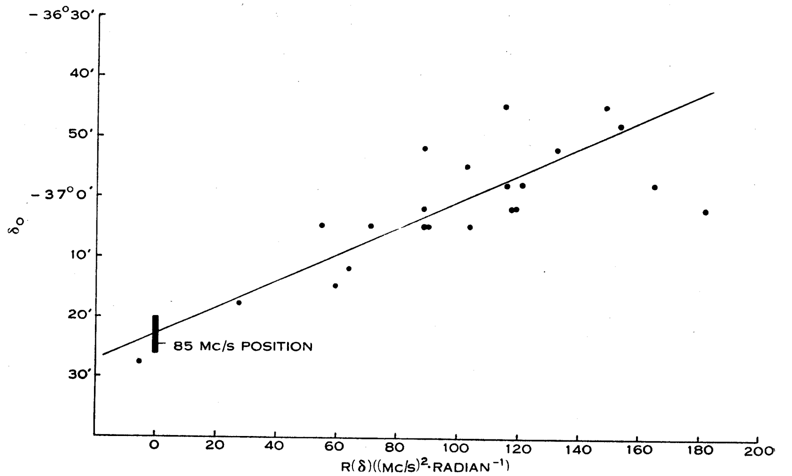

Fig. 6.—Observed declination (epoch 1950) of the source 03S3A related to ionospheric parameters according to equation (**32**). $R(\delta)=\bar{w}(\sigma)\partial f_c^2/\partial\varphi$.

Manchester with simultaneous critical frequency measurements made at Slough. In an earlier report on this work Komesaroff and Shain (1959) quoted a value of 550 km. The lower value is based on a more careful analysis of the experimental results.

The derived value of d was used to compute Right Ascension corrections for the same observations, and Figure 7 (a) shows the observed and corrected positions of 03S3A. The improvement in declination is very marked; that in Right Ascension is by no means as good, although the mean error in Right Ascension has been reduced from $0^m{\cdot}4$ to less than $0^m{\cdot}1$.

Figure 7 (b) is a similar diagram for the source 09S1A based on observations taken between the beginning of October and mid December 1957. During this period the source was observed within two and a half hours of sunrise when ionospheric conditions were changing rapidly, making the period difficult for refraction corrections. Nevertheless, although the corrected declinations show considerable " scatter ", their mean is within 5′ of the 85 Mc/s position (Mills, Slee, and Hill 1958). On the other hand, there is still a systematic displacement in Right Ascension from the high frequency position. In the absence of more detailed ionospheric data, it is not possible to assess the significance of this shift.

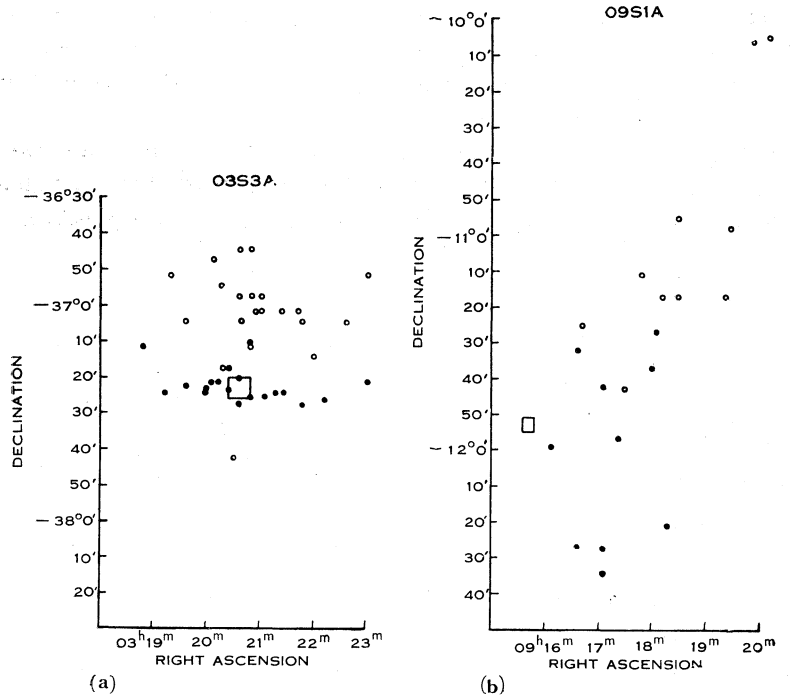

Fig. 7.—19·7 Mc/s positions (epoch 1950) of two sources : (*a*) 03S3A, (*b*) 09S1A. Observed positions are shown by open circles and positions obtained after applying corrections for ionospheric refraction are shown by full circles. The rectangles indicate 85 Mc/s positions.

VII. Conclusions

The main results of the present paper are expressed by equations (**32**) and (**34**), relating the displacement components of a radio source at transit to ionospheric parameters. In order to compute these displacements under commonly prevailing ionospheric conditions, account must be taken, not only of the total electron content of a vertical column through the ionosphere but also of the horizontal gradients of this quantity.

Application of the results to preliminary observations, taken mainly in the early morning, indicates that over several months the effective ionospheric thickness d was apparently constant at a value of about 355 km, in substantial agreement with the results of Evans (1957) for another time and place.

Combining this value of d with sounding data on the basis of equations (**32**) and (**34**), a considerable improvement in measured source positions has been effected. Even under the somewhat unfavourable conditions around sunrise time, it has been possible to achieve a ten to one reduction in mean declination error. To make the declination correction it was necessary to derive detailed information relating to north-south gradients from the published data of several sounding stations along the eastern Australian coast. No such detailed information about east-west gradients is available, however, and the improvement in Right Ascension which could be achieved was correspondingly less marked.

Since these observations were made, many more position measurements have been taken, and these, together with a more detailed account of the experimental procedure, will be given in a subsequent paper.

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