THE OPTIMUM LINE WIDTH FOR A REFLECTION CAVITY MASER*

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Introduction

It has been pointed out previously (Troup 1959, hereinafter referred to as I) that there is an optimum line width for the transition used in a reflection cavity maser and that this optimum line width gives maximum bandwidth for a given gain. However, it was not explicitly deduced in I that this optimum line width actually gives the maximum value of $(\text{Gain})^{\frac{1}{2}} \times$Bandwidth product for a given cavity maser system. This note makes the appropriate deduction.

The Line Width for Maximum $(Gain)^{\frac{1}{2}}\times$*Bandwidth Product*

Commencing with equation (**3**) of I, we have that the gain G of a reflection cavity maser is given to a good approximation by

$$G \simeq (|Q_m|+Q_L)^2(|Q_m|-Q_L)^{-2}, \qquad \text{(1)}$$

where Q_L is the loaded Q of the cavity and $|Q_m|$ is the modulus of the negative molecular Q.

The bandwidth B of a reflection cavity maser, assuming a Lorentz line shape for the amplifying transition, is given by (equation (**4**) of I)

$$B \simeq \delta[(|Q_m|/Q_L)-1][1+\delta|Q_m|/f]^{-1}, \qquad \text{(2)}$$

where δ is the line width of the amplifying transition and f is the centre frequency of both cavity and molecular responses.

Using equations (**1**) and (**2**), we get

$$G^{\frac{1}{2}}B \simeq \delta[(|Q_m|/Q_L)+1][1+\delta|Q_m|/f]^{-1}. \qquad \text{(3)}$$

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Now for reasonably large gains ($G>10$ say) $|Q_m|\simeq Q_L$, so that

$$G^{\frac{1}{2}}B\simeq 2\delta(1+\delta\,|\,Q_m\,|/f)^{-1}. \qquad \qquad (4)$$

Now the molecular Q, Q_m, is of the form (equation (2) of I)

$$|\,Q_m\,|=K\delta/N^*, \qquad \qquad (5)$$

where K is a constant for the particular active material and cavity system, and N^* is the number density of excess molecules in the emissive state.

Using equation (5), we rewrite equation (4) as

$$G^{\frac{1}{2}}B\simeq 2\delta[1+(K\delta^2/N^*f)]^{-1}. \qquad \qquad (6)$$

Differentiation of this expression with respect to δ shows that $G^{\frac{1}{2}}B$ has a maximum value

$$(G^{\frac{1}{2}}B)_{\text{max.}}=(K/N^*f)^{-\frac{1}{2}}, \qquad \qquad (7)$$

when $\delta=(K/N^*f)^{-\frac{1}{2}}=\delta_{\text{opt.}}$. The quantity $\delta_{\text{opt.}}$ is identical with the value of δ deduced in I to give maximum bandwidth at a given gain. This means that suitable adjustment of the transition line width δ, keeping all other parameters constant, can maximize the $(\text{Gain})^{\frac{1}{2}}\times$Bandwidth product.

Conclusion

In the paramagnetic maser, the simplest way of adjusting δ is clearly to use a magnetic field deliberately made inhomogeneous. This of course requires that $\delta<\delta_{\text{opt.}}$ initially. The discovery of the narrowness of the lines of Fe^{3+} in Ti_2O_3 (Butcher and Gill 1959; Low 1960) makes the technique described above particularly applicable to this material. Moreover, Townes *et al.* at Columbia have obtained a $(\text{Gain})^{\frac{1}{2}}\times$Bandwidth product of 250 Mc/s at 4500 Mc/s using ruby (Cr^{3+} in Al_2O_3) as the active medium, by the use of magnetic field inhomogeneity (Butcher and Gill 1959). This latter result is taken as experimental confirmation of the existence of an optimum line width for a given maser system. It also confirms that the use of an inhomogeneous magnetic field is a practicable way of adjusting the line width.

Note added in Proof.—Maiman (1960) has come to the same conclusions regarding optimum line width, and has pointed out some of the consequences for paramagnetic masers.

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