# FOCUSING OF RADIO WAVES REFLECTED FROM A ROUGH CURVED IONOSPHERE

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*Summary*

The reflection of radio waves from a partially rough, curved ionosphere is considered. The relationship between the amplitude of the echo, $A$, and phase path $P$ when the ionosphere moves overhead with a horizontal velocity $V$ at a height $h$ is the same as that for a smooth curved ionosphere, i.e.

$$A^2 \propto 1 - \frac{h}{2V^2}\frac{\mathrm{d}^2P}{\mathrm{d}t^2},$$

although because of the different physical conditions the large increases in echo amplitude observed when reflection takes place from a smooth ionosphere are not expected for reflection from a rough ionosphere.

A method of testing the relationship experimentally is suggested.

## I. Introduction

It occasionally happens that the ionosphere behaves as a specular reflector of radio waves incident normally on it, so that for many purposes it may be treated as a smooth mirror. However, Munro (1950) and Bramley and Ross (1951) have shown that this smooth " mirror " is not plain but has ripples moving through it ; the ripples having a wavelength of about 100 km and an amplitude of a few kilometres. As they move overhead, the ripples have the effect of producing large increases in the amplitude of the reflected radio waves : this effect is known as focusing and the author (Whitehead 1956) has shown that, for vertical incidence, the amplitude $A$ and phase path $P$ are related by the equation

$$A^2 \propto 1 - \frac{h}{2V^2}\frac{\mathrm{d}^2P}{\mathrm{d}t^2}, \qquad (1)$$

when $h$ is the average height of the reflector and $V$ is its horizontal velocity.

However, it is more usual for the ionosphere to be a rough reflector, so that the echo fades and the amplitude has a Rayleigh distribution (Mitra 1949). It is of interest to enquire whether " focusing " is to be expected under these circumstances if the rough reflector is curved. It is indeed found that the mean amplitude fluctuates slowly (Meadows and Moorat 1957), and it is of interest to see if there is a similar relationship to (1) between the mean amplitude and the phase path. It is the purpose of the present paper to derive this relationship. However, it is shown that the large increases in mean amplitude are not to be expected because of the difference in the physical situations.

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## II. Derivation of the Equation

Briggs and Phillips (1950) have shown that, for radio wavelengths shorter than about 200 m, the ionosphere can usually be treated as a partially rough reflector so that most of the power returned to a transmitter on the ground is contained within a cone of semi-angle $\theta_0$ if the ionosphere is flat. $\theta_0$ was found to be about 5°, so that the effective area of the ionosphere for reflection is that contained within a circle of radius $\theta_0 h$, about 10–30 km, small compared with the wavelength of the ripples.

Now suppose the rough reflector is slightly curved concave downwards with a radius of curvature $R > h$, the curvature being in one dimension only so that it forms part of a cylinder. Power is now returned from a greater area of the ionosphere, an ellipse with semimajor axes $\theta_0 h$ and $\varphi h$, where $\varphi$ is the angle an incident ray makes with a line drawn perpendicular to the reflecting surface from the transmitter when the specularly reflected ray makes an angle $2\theta_0$ to the incident ray in the plane of curvature of the ionosphere (Fig. 1). Thus the power returned is increased in proportion to $\varphi/\theta_0$. It is seen that

$$\frac{\sin\varphi}{R}=\frac{\sin\theta_0}{R-h},$$

therefore

$$\frac{\sin\varphi}{\sin\theta_0}=\frac{R}{R-h}=1+\frac{h}{R-h},$$

so that the square of the amplitude is given approximately by

$$A^2 \propto \varphi/\theta_0 \approx 1+\frac{h}{R-h}.$$

This is identical with the relation derived for the amplitude of a wave reflected from a curved mirror: thus it follows that when the rough curved reflector moves with velocity $V$ at right angles to the axis of the cylinder,

$$A^2 \propto 1-\frac{h}{2V^2}\frac{\mathrm{d}^2P}{\mathrm{d}t^2}.$$

In this equation $A^2$ and $\mathrm{d}^2P/\mathrm{d}t^2$ are to be interpreted as mean values taken over several rapidly fading cycles.

## III. Discussion

Although the same equation has been derived for both rough and smooth ionospheres, there is an important physical difference. For a mirror reflection, the principal contribution to the amplitude comes from the first few Fresnel zones: the amplitude is proportional to the area of the zones, or for cylindrical curvatures, to the length of the (elongated) zones. However, for a rough reflector it is the power or the *square* of the amplitude which is proportional to the area, or length, of the effective reflecting part of the ionosphere. With a smooth flat reflector at a height of 200 km, the first Fresnel zone has a radius of about 5 km at a radio wavelength of 100 m: if the zone is elongated to a half length of 15 km, small compared to the wavelength of a ripple, the amplitude is increased by a factor of three. For a rough ionosphere at the same height, for which $\theta_0=5°$, the radius of the effective area for reflection is 20 km : if the amplitude is to be increased by the same factor, the half length of the effective area has to be increased to 180 km, which is no longer small. Thus it is to be expected that equation (**1**) will apply to a rough ionosphere when the amplitude increases above normal are quite small : for large values of the right-hand side of equation (**1**), the amplitude will be less than that given by the equation.

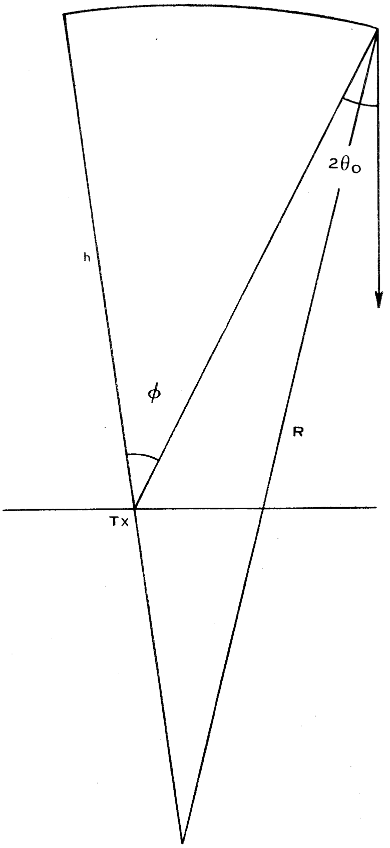


Fig. 1

## IV. Proposed Experimental Test

The difficulty of testing the relationship is that, under rapid fading conditions, the changes in phase path are also rapid, and the usual method of measuring phase path (Findlay 1951) is unsatisfactory, partly because of the tedious work involved in counting wavelength changes from a confused film record and partly because the large and rapid changes in echo amplitude make it difficult to obtain a continuous record.

The Findlay method consists of beating an echo from a pulse transmitter with a c.w. oscillator output phase locked to the transmitter at a frequency of about 50 kc/s greater than the transmitter frequency. The rectified output is differentiated and recorded on film.

It is proposed to overcome the difficulties associated with this method by feeding the rectified output into a phase-sensitive rectifier whose comparison phase is derived from a 50 kc/s oscillator phased with respect to the original transmitted pulse by the rotation of a condenser. The output from the phase-sensitive rectifier will be used to drive the condenser in such a direction as to reduce this output. The rotation of the condenser will be recorded: this gives the changes in phase path. To ensure that a sufficiently pronounced beat is produced at the receiver output, the output amplitude of the c.w. radio-frequency oscillator will be made proportional to the echo amplitude.

This system of measurement should also enable us to distinguish between the rapid fading produced by small irregularities and that produced by a deep phase screen of the type discussed by Hewish (1951).

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