DIRECTION-FINDING ON DIFFUSE SOURCES OF ELECTROMAGNETIC RADIATION

By D. G. Cartwright*

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Summary

It is shown that, for sources of large angular size, the response of an Adcock type direction-finder is independent of the extent of the source in altitude. On the other hand, the response of a rotating loop does depend on altitude. By combining the characteristics of both types of direction-finder, the position and size of an extended source can be found, provided that a brightness profile can be assumed.

I. Introduction

Observations of the time variations of very low frequency emissions from the Earth's upper atmosphere (Ellis 1959) have shown the need for a means of estimating the size of the apparent sources of the radiation. Investigations so far have been made at a frequency of about 5 kc/s, a long wavelength inconvenient for use with conventional high resolution antennae. However, information about the angular distribution of the recorded radiation can be obtained with relatively simple antennae such as loop and Adcock direction finders. For instance, using rotating loops, Ellis and Cartwright (1959) have found that the radiation sources have an apparent azimuthal size of between 60° and 90°.

In a recent paper (Wait 1959) it has been demonstrated that the characteristics of a downcoming radio wave may conveniently be measured by making use of rotating loop and Adcock direction-finders. Expressions were derived for finding the angle of arrival, the azimuth, and the polarization, in the case of a single plane wave-train incident on the observation point.

In the present paper the response of a loop and an Adcock system will be analysed for sources of large angular size. It will be shown that the direction-finding characteristics of an Adcock system are independent of the size of the source in elevation.

If the source is extensive in altitude as well as in azimuth, then the directional properties of a rotating loop are modified by a term containing the altitude.

II. Coordinates

The aerial systems will be considered relative to a spherical coordinate system, with the Earth a conducting surface in the x–y plane and the antenna in the x–z plane. An incident wave is specified in terms of its spherical coordinates : angle of elevation above the x–y plane, φ, and its azimuth measured from the x-axis, θ. This is shown in Figure 1.

* Upper Atmosphere Section, C.S.I.R.O., Camden, N.S.W.

III. Adcock Aerial

A pair of suitably connected antennae rotating about a vertical axis midway between them is equivalent to a conventional 4-wire Adcock and goniometer arrangement. The response of this system to a small source will be considered first. Ideally, it responds only to the vertical component of E of the incident wave. If the antennae are on the x-axis and close to a good conductor ($\sigma \gg \omega\varepsilon$, where σ is the conductivity and ε the permittivity of the surface beneath the

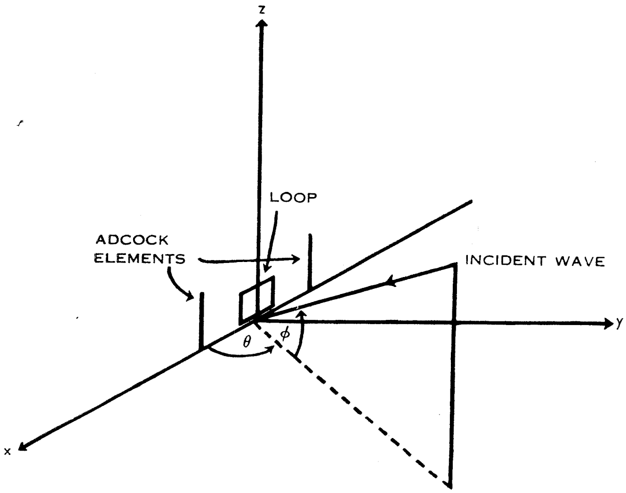

Fig. 1.—Coordinate system, showing position of loop and Adcock antennae.

loop in appropriate units, and ω is the angular frequency of the wave) the power transferred to the receiver is proportional to $\cos^4 \varphi \cos^2 \theta$, and the point source response can be written

$$p_A = \cos^4 \varphi \cos^2 \theta.$$

If the extended source is a random emitter with uniform brightness then the power at the receiver can be written

$$P_A = \int\int p_A \mathrm{d}\theta \mathrm{d}\varphi.$$

Assuming the source has a uniform brightness over an azimuthal width of 2Θ and extends in elevation from zero to Φ, then the maximum power recorded as the aerial rotates is

$$P_{\text{max.}} = 2\int_0^{\Theta}\int_0^{\Phi} p_A \mathrm{d}\theta \mathrm{d}\varphi.$$

This gives

$$P_{\text{max.}} = \alpha\left(\Theta + \frac{1}{2}\sin 2\Theta\right),$$

$$\alpha = \int_0^{\Phi} \cos^4 \varphi \mathrm{d}\varphi$$

$$= \frac{1}{4}\left(\frac{3}{2}\Phi + \sin 2\Phi + \frac{1}{8}\sin^4 \Phi\right),$$

and at the minimum

$$P_{\text{min.}}=2\int_{\frac{1}{2}\pi-\Theta}^{\frac{1}{2}\pi}\int_{0}^{\Phi} p_A \mathrm{d}\theta\mathrm{d}\varphi,$$

$$P_{\text{min.}}=\alpha(\Theta-\tfrac{1}{2}\sin 2\Theta).$$

Defining the modulation in power for the Adcock, M_A, resulting from the antenna rotation as

$$\frac{P_{\text{max.}}-P_{\text{min.}}}{P_{\text{max.}}+P_{\text{min.}}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

gives

$$M_A=\frac{\sin 2\Theta}{2\Theta}. \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

Since the spacing of the dipoles at very low frequency is very much smaller than a wavelength, the minimum power corresponds to the azimuthal centre of the source in the usual way.

Thus an Adcock gives information about the position and size in azimuth. The relationship is independent of the brightness profile in Φ.

IV. Loop Aerial

The case of a small rotating loop will now be considered. The magnetic components of the incident wave are H_p in the plane of incidence and H_n normal to this plane. For a loop in free space the Cartesian components due to H_p are

$$H_x=H_p \sin\varphi\cos\theta,$$
$$H_y=H_p \sin\varphi\sin\theta,$$
$$H_z=H_p \cos\varphi,$$

and those due to H_n are

$$H_x=H_n \sin\theta,$$
$$H_y=H_n \cos\theta,$$
$$H_z=0.$$

For random polarization, $H_n=H_p$ and the powers due to each component can be added. The loop absorbs power only from H_y, so the small source power response for a small loop in free space is proportional to

$$\cos^2\theta+\sin^2\varphi\sin^2\theta.$$

This is unaltered if the loop is close (in terms of the wavelength) to a good conductor in the x–y plane, so we can write

$$p_L=\cos^2\theta+\sin^2\varphi\sin^2\theta.$$

For an extended source of uniform brightness, the total power at the receiver is

$$P_L=\iint p_L \mathrm{d}\theta\mathrm{d}\varphi.$$

If, as before, the source is of width 2Θ in azimuth and extending from 0 to Φ in elevation, the maximum power recorded is

$$P_{\text{max.}}=(1+\beta)\Theta+\tfrac{1}{2}(1-\beta)\sin 2\Theta,$$

where

$$\beta=\int_0^\Phi \sin^2\varphi d\varphi$$
$$=\tfrac{1}{2}(\Phi-\tfrac{1}{2}\sin 2\Phi),$$

and as before

$$P_{\text{min.}}=(1+\beta)\Theta-\tfrac{1}{2}(1-\beta)\sin 2\Theta.$$

The modulation in power for the loop then becomes

$$M_L=\left(\frac{1-\beta}{1+\beta}\right)\frac{\sin 2\Theta}{2\Theta}. \quad \dots\dots\dots\dots\dots\dots \quad (3)$$

M_L is indistinguishable from M_A if $\beta \ll 1$, i.e. Φ is small.

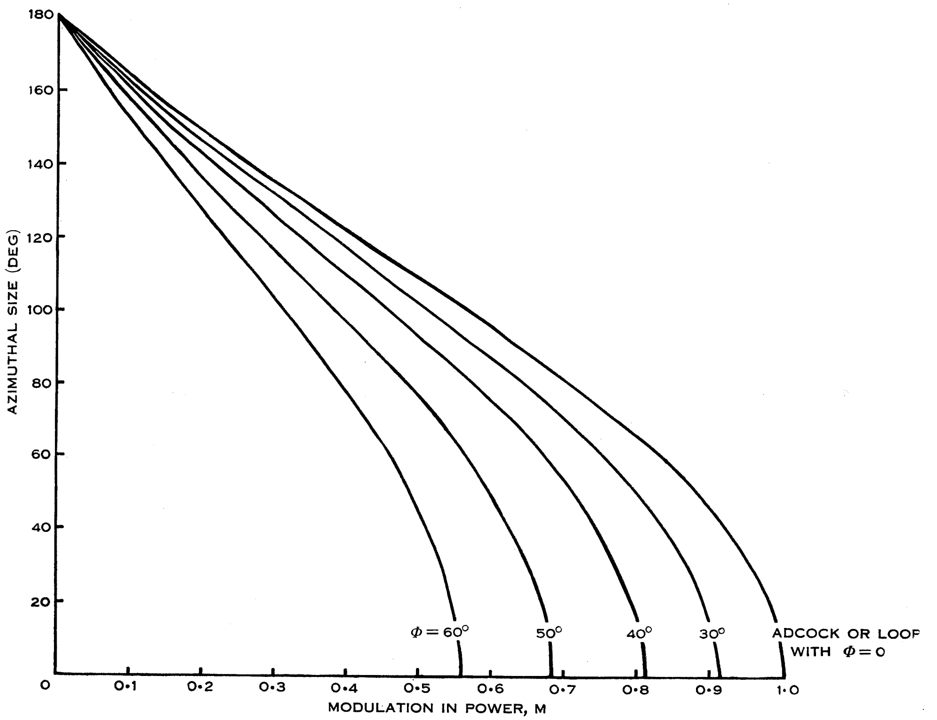

Fig. 2.—The azimuthal size 2Θ of a diffuse source, shown as a function of the modulation in power, M, produced by a rotating loop direction-finder, for different source heights Φ. The case $\Phi=0$ corresponds to an Adcock direction-finder.

V. Comparison of Loop and Adcock Results

The modulation in power of an Adcock direction-finder is independent of the brightness profile in elevation, but, if the brightness distribution in azimuth is symmetrical, the minimum recorded power corresponds to the direction of the centre of the source in the usual way.

The power modulation of a rotating loop, on the other hand, is affected by the distribution of brightness in elevation if the source extends more than about 5° above the horizon.

By comparing the records from an Adcock and a rotating loop the height of the source can easily be found.

For instance,

$$\frac{M_A}{M_L}=\frac{1+\beta}{1-\beta},$$

or

$$\frac{M_A-M_L}{M_A+M_L}=\beta=\tfrac{1}{2}(\Phi-\tfrac{1}{2}\sin 2\Phi).$$

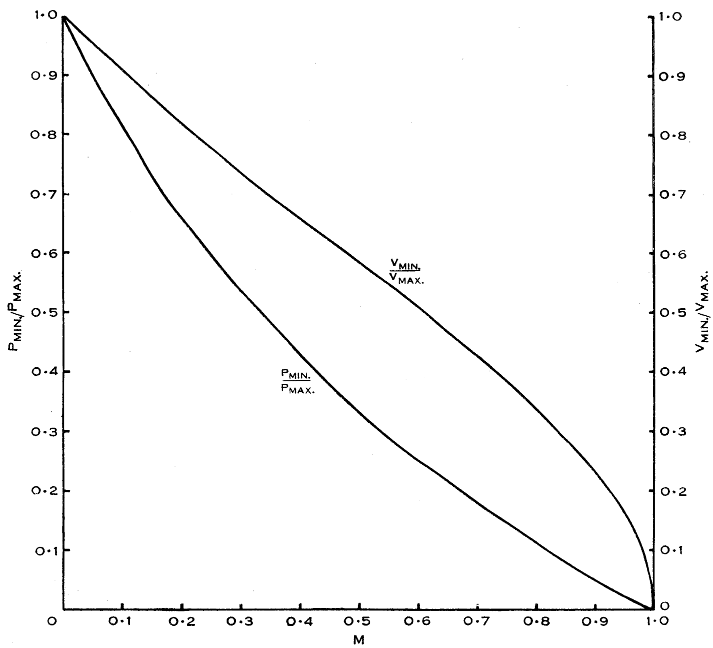

Fig. 3.—The relation between the modulation in power, $M=(P_{\max.}-P_{\min.})/(P_{\max.}+P_{\min.})$, and the ratio $P_{\min.}/P_{\max.}$, or $V_{\min.}/V_{\max.}=\sqrt{(P_{\min.}/P_{\max.})}$.

VI. Discussion

Figure 2 shows the expressions (**2**) and (**3**) with 2Θ plotted as a function of the modulation in power M. The most convenient quantity to record is the amplitude or voltage produced by the direction-finder. From (**1**) we have

$$M=(P_{\max.}-P_{\min.})/(P_{\max.}+P_{\min.}).$$

If we denote $V_{\min.}/V_{\max.}$ by v, then $v=\sqrt{(P_{\min.}/P_{\max.})}$, and so

$$v=\sqrt{\{(1-M)/(1+M)\}}. \quad \dots\dots\dots\dots\dots\dots \quad (4)$$

The relationship between v or $P_{\min.}/P_{\max.}$ and M is shown in Figure 3. Using either (**4**) or Figure 3 we can obtain the source size in azimuth as a function of v for any given Φ. This is shown in Figure 4.

This method is best suited to sources for which Φ is greater than about 30°. From Figure 2 it can be seen that the curves for Φ less than 30° are very closely spaced in comparison with curves for Φ greater than 30° and, in fact, the accuracy of measurement is not high enough to resolve curves for which Φ is less than 20°. An exception to this is when the source is of small size in azimuth, since the curves of Figure 4 are quite widely spaced below $2\Theta = 40°$, say.

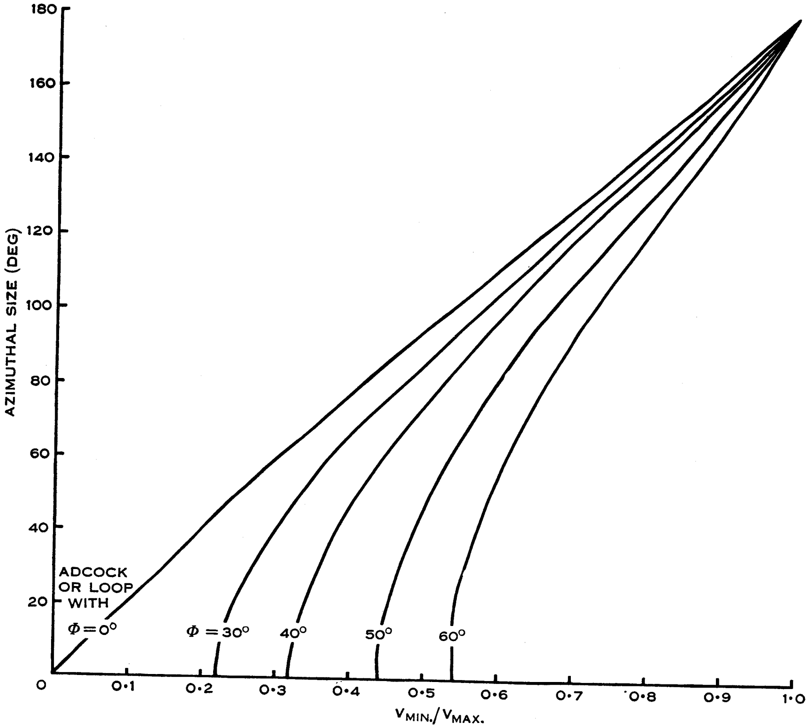

Fig. 4.—The azimuthal size 2Θ of a diffuse source shown as a function of the depth of the amplitude null, $V_{min.}/V_{max.}$, produced by a rotating loop direction-finder, for different source heights Φ. The case $\Phi = 0$ corresponds to an Adcock direction-finder.

VII. Acknowledgments

The author acknowledges many suggestions made during discussions with Dr. G. R. A. Ellis of this organization.

VIII. References

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