WAVELENGTHS OF KRYPTON 86, MERCURY 198, AND CADMIUM 114

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Summary

The vacuum wavelengths and the spectral line profiles of four lines of krypton 86, four lines of mercury 198, and four lines of cadmium 114 have been measured. One line, the radiation $2p_{10}$–$5d_5$ of krypton 86 (6057 Å), has been used as the reference standard in the wavelength measurements.

A variable gap Fabry-Perot interferometer with electromagnetically controlled plate holders was used throughout under vacuum conditions. The use of photoelectric recording methods and mechanical scanning made it possible to compare wavelengths with an accuracy of better than 1 part in 10^8, and half-intensity widths of lines were measured with an accuracy of $0\cdot5$ mK ($0\cdot0005$ cm^{-1}).

The 6057 Å line of krypton 86 was examined under different operating conditions of the Engelhard-type lamp and small wavelength shifts due to variation of temperature, pressure, and current density were measured. The Doppler shift and interatomic Stark shift annul each other if the lamp is viewed in the direction cathode to anode to observer, and is operated in a liquid air bath with the temperature near the capillary surface of the lamp at 63 ± 1 °K and with a current density of $0\cdot28\pm0\cdot05$ A/cm^2. The Doppler shift under these conditions was found to be $+0\cdot014\pm0\cdot003$ m^{-1} (-50 ± 10 μÅ) and the interatomic Stark shift $-0\cdot014\pm0\cdot003$ m^{-1} ($+50\pm10$ μÅ). Under these conditions also the half-intensity width is $13\cdot5\pm0\cdot5$ mK, and the wavelength emitted is that for the unperturbed state of the atoms to 2–3 parts in 10^9. This line is superior to the other lines examined for sharpness and reproducibility and other wavelengths may be established in terms of it to at least 1 part in 10^8. This is the line that has been recommended as the new primary standard of length.

I. Introduction

The recommendation of the Advisory Committee of the International Committee of Weights and Measures on the Definition of the Metre (1957) that the metre be redefined in terms of a krypton 86 wavelength emphasizes the importance of obtaining exact knowledge of the characteristics of this line and other lines suitable as secondary standards in metrology and spectroscopy.

The chosen line is the radiation $2p_{10}$–$5d_5$ of krypton 86 for the unperturbed state of the atoms and by definition it has been recommended that the metre be 1 650 763·73 times the vacuum wavelength of this radiation. The vacuum wavelength is then 6057·802106 Å (1 Å $=1\times10^{-10}$ m) and it is this value that has been used as a reference standard in the wavelength measurements described here.

A photoelectric Fabry-Perot interferometer was developed using mechanical scanning for the precise comparison of vacuum wavelengths and for the accurate recording of the spectral line profiles. An electrodeless cadmium 114 lamp has been prepared and shows promise as a satisfactory source of secondary standards.

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II. Light Sources

The krypton 86 lamp, supplied by E. Engelhard of the Physikalisch-Technische-Bundesanstalt (PTB) (Engelhard 1952), was of the hot cathode type and was normally operated at 20 mA d.c., the current density being 0·28 A/cm². The lamp was cooled by immersion in a bath of liquid air which was continuously evacuated. The temperature of the lamp could be satisfactorily regulated by controlling the rate of pumping. The temperatures in the liquid air bath were measured at various points, using an oxygen gas thermometer and specially calibrated thermocouples. Two thermocouple junctions were attached to different points on the capillary of the lamp and were thus very close to the gas discharge used as the light source. Another thermocouple was placed in the liquid air close to the bulb of the oxygen thermometer. Under stable conditions, with no fluctuations in the pumping rate, all three thermocouples and the gas thermometer indicated the same temperature to 1 degK. However, any variation in the pumping rate led to differences of several degrees Kelvin between the gas thermometer and the thermocouples. The cold junctions of the thermocouples were placed in an ice bath and the e.m.f.'s generated were measured with a precision potentiometer. The temperatures indicated by the thermocouples attached to the capillary of the lamp are those referred to in this report, and these are reliable to within $\pm 0 \cdot 1$ degK. The lamp was viewed, in general, from the anode side. That is, the direction of propagation of light may be said to be from cathode to anode to observer (CAO). In Doppler shift measurements, the lamp was also viewed in the opposite sense (ACO). This method of designating light direction will be used throughout this article.

Table 1

Electrodeless Mercury Lamps

Lamp	Mercury 198	Carrier Gas	Tube Material
NBS 32	2 mg	Argon at 3 mmHg	"Vycor"
NBS 99	1 mg	Argon at 3 mmHg	"Vycor"
NRC	2 mg	Argon at ½–1 mmHg	"Vycor"
NSL 7	1 mg	Argon at 2 mmHg	Quartz

The mercury 198 lamps consisted of a series of the electrodeless type, two of which were Meggers lamps from the National Bureau of Standards (NBS) (Meggers and Westfall 1950), one from the National Research Council (NRC), and one made in this Laboratory (NSL). All were excited at a frequency of 100 MHz and cooled with water to 10–12 °C during operation. One lamp (NSL 7) was examined at temperatures in the range 4–15 °C. The power output from the high frequency oscillator, in general, was about 60 W and the lamps were observed broadside on. The details of these lamps, as given by the makers, are in Table 1.

NBS 32 was an old lamp that had been used extensively for several years. NBS 99 was a new lamp. NRC had an isotopic content as follows : 198/98·0% ; 199/1·51% ; 200/0·16% ; 201/0·08% ; 202/0·12% ; and 204/0·06%.

The cadmium 114 lamp was of the electrodeless type made in this Laboratory and consisted of a small amount of cadmium 114 in a quartz tube containing argon at a pressure of 2 mmHg. This lamp was then surrounded by a further quartz jacket which was evacuated to a pressure below 10^{-6} mmHg. Considerable care was taken in the baking and degassing of the lamp tube and its jacket to ensure that the pressure in the surrounding jacket was as low as possible. The form of the lamp is shown in Figure 1. The techniques required to make this lamp were established by first making several lamps with natural cadmium. One special form of test lamp was made with a thermocouple sealed through the outer jacket and attached to the outer surface of the quartz tube of the lamp. The temperature at this point was then measured for different power outputs from the oscillator. With about 2–3 mg of natural cadmium and with argon at 1–2 mmHg pressure, the lamp emitted the cadmium spectrum very intensely for power outputs from 40 W upwards. The intensity of the red line 6440 Å

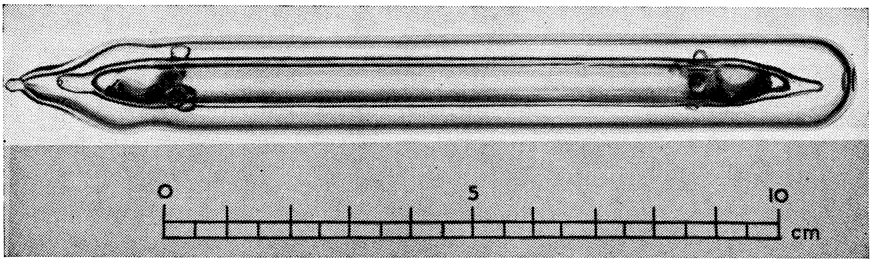

Fig. 1.—Cadmium 114 electrodeless lamp. Outer jacket evacuated to less than 10^{-6} mmHg pressure.

was about twenty times that emitted by a cadmium Osira-type lamp running at 1 A, and the fringes obtained with the lamp were of comparable visibility. The temperature at the centre of the lamp was in the range 180–200 °C. It was found that, with an oscillator power of about 50 W, the lamp operated very satisfactorily. Using a 2400 MHz oscillator, a lamp was life tested over 100 hr and was still continuing to emit a clean cadmium spectrum when excited at either 100 MHz or 2400 MHz. The vacuum jacket around the lamp therefore maintains the temperature quite efficiently for the satisfactory emission of the cadmium spectrum.

A lamp was then prepared using cadmium 114 isotope. The supply of this isotope had to be distilled to obtain pure metal and consequently the lamp prepared was estimated to contain only a small fraction of a milligram of the pure cadmium isotope. The lamp was filled with argon at 2 mmHg pressure. A power output from the oscillator of about 60–80 W was used to obtain an intense cadmium spectrum. The intensity of the red line was about three times as great as that of the Osira-type lamp containing natural cadmium and operating at 1 A.

The experiments with lamps containing natural cadmium indicate that a few milligrams of cadmium are desirable for a lamp of high intensity and stability, and more lamps will be made with "Vycor" glass when further supplies of cadmium 114 become available.

III. The Interferometer System

The interferometer was a variable gap Fabry-Perot type in which the optical flats were mounted in specially designed electromagnetic plate holders. The carriages for the plate holders were made of invar and supported in a V-shaped trough of invar. The plate holders could be adjusted for parallelism initially by coarse and fine mechanical controls.

One holder provided means for moving one optical flat through a small distance by remote electrical control without appreciable change in parallelism. This plate could also be oscillated at any desired frequency by applying an a.c. input to the exciting coil. A schematic diagram is shown in Figure 2. The unit

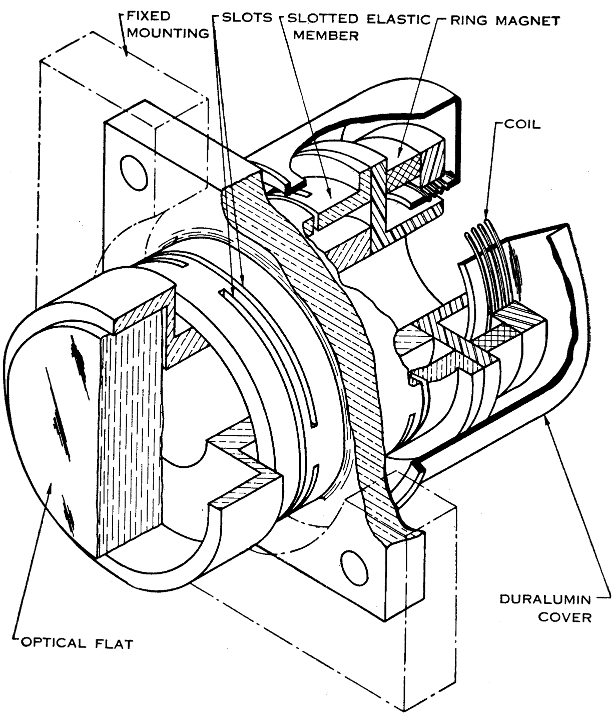

Fig. 2.—Schematic diagram of electromagnetic plate holder for translating and oscillating one interferometer plate.

consists essentially of a fixed coil in a magnetic field produced by a permanent ring magnet and soft iron ring assembly. The assembly is attached to a brass elastic member which holds the optical flat. The movement of the assembly is controlled by the current passing in the coil. The slotted elastic member was designed to give adequate displacement of the optical plate without passing too great a current through the coil and without being excessively sensitive to external vibration. A direct current of 180 mA moved the plate through about four orders of interference.

The other holder provided a means of tilting the second optical flat to a very fine degree in two mutually perpendicular directions. This unit consists essentially of four electromagnets, with four polarizing magnets acting on a soft iron ring assembly. The elastic member is attached to the soft iron assembly so that it is tilted when current passes through the coils. A schematic outline is shown in Figure 3. Details of these holders will be reported elsewhere.

The whole of the interferometer assembly was mounted in a vacuum chamber from which the electrical leads passed to the electrical controller situated close to the observer. A temperature-controlled water jacket was designed to enclose the chamber, but this was not available in time for the present measurements. External vibrations were entirely eliminated by mounting the interferometer on inflated motor tubes (pressure of only a few pounds per square inch) which were sandwiched between two flat plates. The whole of this assembly was mounted on a heavy concerete block.

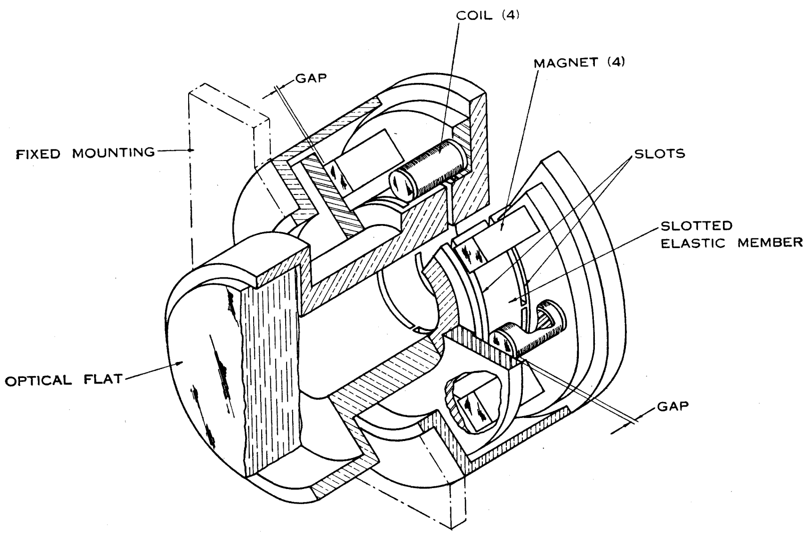

Fig. 3.—Schematic diagram of electromagnetic plate holder for tilting other interferometer plate.

The interferometer could be illuminated with either collimated or convergent light and the Haidinger fringes were projected on to the slit of a prism spectrograph. With a camera lens of focal length 640 mm the dispersion at 5000 Å was 17 Å/mm, using glass prisms.

A special photoelectric head was made to permit simultaneous observation of two spectral lines with two photomultiplier tubes. This head was substituted for the photographic plate holder. With this unit two fairly close lines could be observed, photoelectrically, at the same time. With a camera lens of focal length 1600 mm, even closer lines could be observed.

The plate separation of the interferometer was established conveniently to a few orders of interference by the use of air gauging. A double jet unit was placed between the two optical plates and held in position by means of a spring clip attached to the plate holder so that the gap between one jet and one optical flat was of a convenient value (about 5μ). The other plate could then be brought up towards the second jet by means of a micrometer head controlling the position of the plate in the chamber until a given reading was obtained on a single head Solex air controller. Figure 4 shows the arrangement. Calibration scales had been prepared for jets of different lengths corresponding to different plate separations. The air input was disconnected at the interferometer when the chamber was evacuated but the jet attachment remained in position between the optical plates during measurements. The air gauge reading was checked at

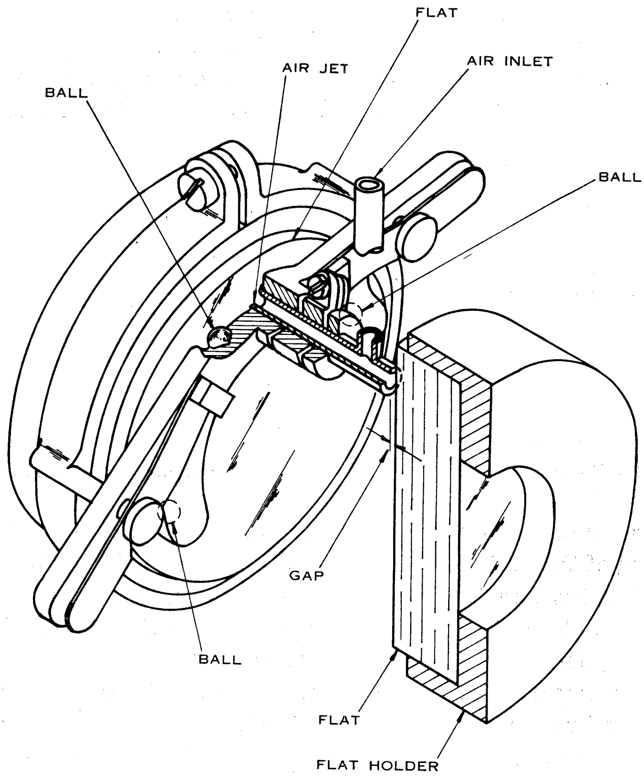

Fig. 4.—Schematic diagram of air-gauging jet unit for setting separation of interferometer plates.

the end of a series of measurements. The air jet unit is situated near the edge of the optical plates, but this is of little consequence since the final air gauge readings are taken when the plates are accurately parallel, as observed interferometrically.

The optical plates were of high quality fused quartz whose flatness over the 25 mm area used was within the limits λ/50–λ/100. Multilayer dielectric and silver films were used on the plates. The seven-layer dielectric films had a reflectance ranging from 92% at 5000 and 6500 Å to a peak of 96% over the region 5560–5880 Å. The absorption was less than 0·8%. These films were specially useful in line profile studies. The silver plates had a reflectance of about 80–85%.

The linearity of the electromagnetic plate holders used for the translation of the optical flat was carefully tested by measuring the displacement of sharp multiple-beam Fizeau fringes for given currents passing through the exciting coil. The currents were determined by measuring the potential difference across a standard resistor in the coil circuit with a high precision potentiometer. A further and final check was obtained from the recorded line profiles of the very sharp spectral lines where the abscissa was found to be strictly proportional to changes in plate separation. The plate holder behaved in a strictly linear fashion and would return to a particular position when the exciting current was switched off and then on. No deterioration in the sharpness of the Haidinger fringes was observed when the plate was moved over several orders of interference.

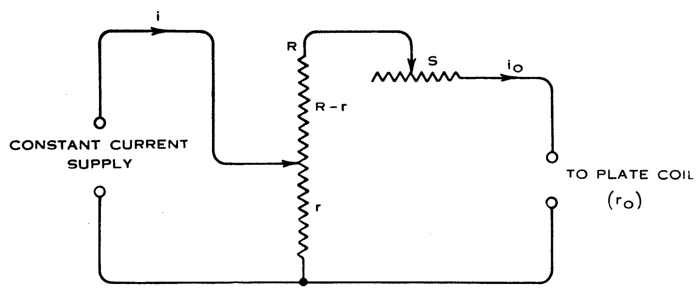

Fig. 5.—Basic electrical circuit of controller for changing plate separation.

The heating effect of the small currents in the exciting coils for the translation and tilting of the optical plates was investigated. With maximum currents of the order of 200 mA in the translation plate coils, there was an initial drift in plate separation of about 0·3 of a fringe in 30 min but a steady state was then reached. In wavelength intercomparisons, currents, on the average, did not exceed 20 mA (power dissipation 0·04 W). Further, the methods used enabled setting points for two wavelengths to be obtained almost simultaneously. The heating effects of the small currents were thus not significant in practice. This was consistently verified throughout measurements.

The current in the coils for the translation and tilt of the optical plates was controlled by variable resistances. The only requirement for the tilt mechanism is that the current control should be fine enough to obtain precise parallelism of the plates, these having already been set mechanically to quite a high degree of accuracy. The translation control is required for accurate and rapid measurement of fringe fractions in wavelength measurement and for recording spectral line profiles in which the abscissa is directly proportional to wave number or plate separation. The important requirement is linearity between the setting of the variable resistance and the current passing through the excitation coil to better than 1 in 1000. The basic circuit is shown in Figure 5, where R is the variable resistance controlling the fringe displacement, r_0 the resistance of the exciting coil, and S is the sensitivity control. The current i_0 through the exciting coil is $ir/(R+S+r_0)$ and thus, if i is constant, $i_0=Kr$, where K is a constant. A constant current power supply provided a current of 200 mA which was constant to better than 1 in 10^4.

For the recording of spectral line profiles, the resistance R was a precision helical potentiometer whose spindle was connected through gears to the drive of a pen recorder. For wavelength measurement, the resistance R became a set of three decade series of special resistors enabling fringe fractions to be measured. Two sets were built into a controller with a switch for selecting either set and with dials for reading to any desired fraction of a fringe. The sensitivity controls were designed specifically to give direct readings of fringe fractions to 0·001 fringe but appropriate changes in the design can give any desired accuracy. Subsequent work showed that a controller designed to read directly to 0·0001 fringe is justified.

IV. Spectral Line Profiles

The spectral line profiles were recorded for the main radiations of krypton 86, mercury 198, and cadmium 114 using a Sefram spot-follower recorder in conjunction with a galvanometer having a sensitivity of 10^{-9} A/mm. The Fabry-Perot etalon was *in vacuo* (less than 0·01 mmHg) and the path difference was 100 mm, giving a spectral range of 0·1 cm^{-1}. Dielectric reflecting films were used for wavelengths down to 4800 Å. Silver films were used for shorter wavelengths. The paper speed of the recorder was 120 mm/min, which through the helical potentiometer gave a uniform scanning speed for the Fabry-Perot etalon of 0·008 fringe/s at 5462 Å. The centre of the ring system of fringes was focused in the centre of a small aperture (0·4 mm diameter) by the camera lens of the spectrograph. Appropriate corrections for the error in centering of the ring system in the aperture were considered and applied to the results. It can be shown that asymmetric profiles, occurring if there is a decentering between aperture and ring pattern, can be used to find half-widths, since, for small decentering errors, only one half of the profile shows asymmetry. Figure 6 shows typical recorded spectral line profiles for the 6057 Å line of krypton 86, the 5462 Å line of mercury 198, and the 6440 Å line of cadmium 114 (nominal vacuum wavelength values).

The instrumental half-width was determined by recording the line profiles at plate separations of less than 1 mm using the various wavelengths from the isotope lamps. At this separation, the true line profile of the spectral lines makes a negligible contribution to the profile, which is purely instrumental. The true half-widths of the spectral lines were subsequently computed using the method of Minkowski and Bruck (1935).

The intensity fluctuations at the centre of the ring pattern were observed with photomultiplier tubes using low noise stabilized power supplies. A tube with a tri-alkali cathode was specially useful for the relatively weak lines in the red region. The linearity of response of the photomultipliers was measured using a calibrated wedge filter, and relative intensities for given lines under different operating conditions were also measured.

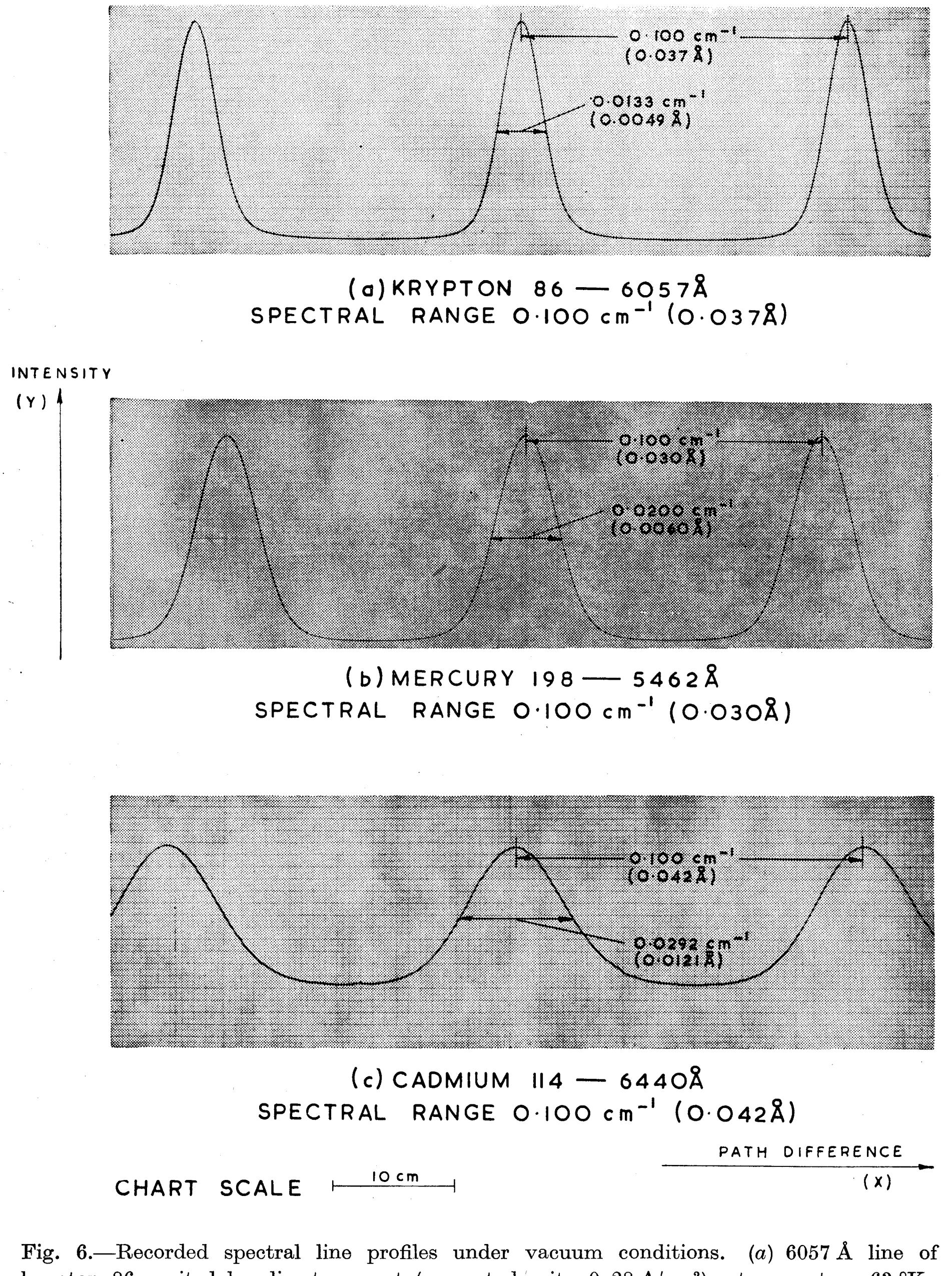

Fig. 6.—Recorded spectral line profiles under vacuum conditions. (*a*) 6057 Å line of krypton 86 excited by direct current (current density 0·28 A/cm^2); temperature 63 °K; (*b*) 5462 Å line of mercury 198 excited at 100 MHz; oscillator output 60 W; temperature 285 °K; (*c*) 6440 Å line of cadmium 114 excited at 100 MHz; oscillator output 60–80 W; temperature about 470 °K.

The half-widths could be obtained from the profiles to $0 \cdot 0005$ cm^{-1} and it is considered that the values for the half-intensity widths given in Table 2 are accurate to $0 \cdot 5$ mK ($0 \cdot 0005$ cm^{-1}).

The krypton 86 line 6057 Å was studied in further detail, and the effects of current density and temperature of the lamp on half-intensity width and intensity were measured from the recorded profiles. The results are shown in Figures 7, 8, 9, and 10. Results obtained by the Bureau International des Poids et Mésures (BIPM) (Terrien 1957*a*) and PTB (Engelhard 1957*a*) are indicated on the figures for comparison.

Table 2

HALF-INTENSITY WIDTHS

Lamp	Details	t (°C)	Wave-length (Å)	Half-intensity Width (mK)			
Krypton 86 Hot cathode (PTB)	Excited with d.c.; current density 0·28 A/cm²; temperature at capillary surface of lamp 63 °K; viewed end-on; light direction CAO		6458	15·8			
			6057	13·3			
			5651	14·3			
			4503	18·8			
Mercury 198 Electrodeless	Excited at 100 MHz; oscillator output 50 W; viewed broadside on; cooled with running water to t °C			NBS 32	NBS 99	NRC	NSL 7
		12	5792	20·1	20·6	23·1	19·5
		12	5771	19·8	20·4	23·0	20·1
		5	5462	—	—	—	19·7
		15		20·4	20·2	22·2	21·3
		12	4359	18·2	20·0	19·3	19·0
Cadmium 114 Electrodeless (NSL)	Excited at 100 MHz; oscillator output 60–80 W; viewed broadside on; lamp enclosed in outer vacuum jacket; temperature at outer surface of lamp about 200 °C		6440	29·2			
			5087	33·6			
			4801	33·3			
			4679	33·4			
Cadmium natural Osira	Excited at 140 V a.c., 1 A		6440	36·9			
			5087	49·8			
			4801	43·2			
			4679	42·5			

The effect of temperature (as measured by the thermocouples on the capillary of the lamp) on the half-intensity width is shown in Figure 7 for a current density of $0 \cdot 28$ A/cm². The half-width of the line at 63 °K is about 30% broader than the theoretically computed Doppler width. Further, the half-width is not proportional to the square root of the measured temperature over the range of temperatures studied (55–70 °K) but within 1 mK was proportional to the square of the temperature. If pure Doppler broadening is assumed the " emission temperature " corresponding to the energy of the excited atoms is about 108 °K when the temperature of the lamp surface is 63 °K. This was computed using the Doppler relation

$$T = M\omega^2 c^2/5 \cdot 578 R\nu^2,$$

where T=absolute temperature,
M=molecular weight of the gas,
ω=half-width in cm^{-1},
c=velocity of light,
R=universal gas constant,
ν=wave number in cm^{-1}.

The relationship for the Doppler line broadening considers only the physical properties of the gas and does not consider Stark broadening that may occur from the current density existing during measurement. Figure 8 shows the relationship between the half-width and current density, and from this graph the half-width at zero current density is about 10·8 mK, which corresponds quite closely to a temperature of 63 °K.

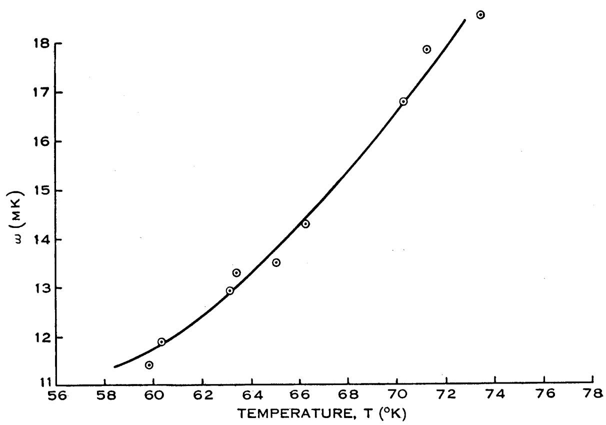

Fig. 7.—Effect of temperature T on line half-width ω for krypton 86–6057 Å; current density 0·28 A/cm².

Over the temperature range 60–65 °K the relation between half-width and temperature is fairly linear and the relation $\omega = 0 \cdot 38T + 11$ (ω in millikaysers and T in degrees Kelvin) fits the curve to well within 0·5 mK. It follows that the half-width value is constant to within $\pm 0 \cdot 5$ mK for a tolerance of ± 1 degK on the temperature T.

The effect of current on half-width (Fig. 8) is fairly linear over the range 0–40 mA and the relation $\omega = 0 \cdot 12i + 11$ fits the curve to within 0·5 mK where i is in milliamperes and ω in millikaysers. As the capillary diameter of the lamp was 3 mm, a tolerance of $\pm 0 \cdot 05$ A/cm² on current density would establish the half-width to within 0·5 mK.

Appropriate limits on current density and temperature as measured at the surface of the lamp would therefore be $(0 \cdot 25 \pm 0 \cdot 05)$ A/cm², and (63 ± 1) °K respectively. This establishes the half-width to within 1 mK.

The effect of temperature on the intensity of the 6057 Å line (Fig. 9) was found to be in good agreement with the results of Engelhard (1957*b*). However, the effect of current on intensity was different from that found by Engelhard (1957*c*). For this reason a second set of independent readings was taken and this is shown in Figure 10. The relation between intensity I and current i which fitted the observed points best was $I = ki^m$, where m was about 0·33.

The half-width obtained for the cadmium 114 6440 Å line as emitted by the NSL type electrodeless lamp was compared with results reported by Terrien (1957*b*), who used a lamp supplied by the Institute of Metrology (I.M.) of the U.S.S.R. (Batarchoukova, Kartachev, and Romanova 1954). The temperature of the I.M. lamp was maintained by an electric furnace and the oscillator was excited at 300 MHz. The NSL lamp, as already mentioned, maintained itself

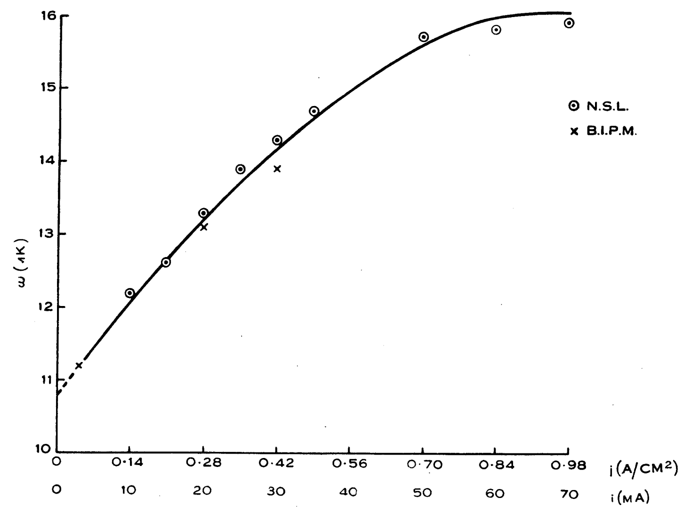

Fig. 8.—Effect of current density j on line half-width ω for krypton 86–6057 Å ; temperature 63 °K.

at a high enough temperature by the use of a surrounding vacuum jacket and was excited at 100 MHz. The observed half-widths and furnace temperatures reported for the I.M. lamp are plotted in Figure 11 and there is clearly a linear relationship between half-width and temperature over the range 230–270 °C. Using this graph and the observed half-width value for the NSL lamp, the temperature at the surface of this lamp would be about 195 °C. This value agrees very well with the measured value at the surface of the natural cadmium lamp in its vacuum jacket.

The half-width of the 6440 Å line was about 30% sharper than the same line emitted by an Osira-type lamp containing natural cadmium and operated at 140 V and 1 A alternating current. The intensities of the lines from the cadmium 114 lamp were several times greater than those from the Osira lamp while the power required to excite the lamp was only a fraction of that used for the Osira lamp. Therefore, this lamp seems to be a very satisfactory source of secondary wavelength standards in metrology. The fact that cadmium 114 emits four strong lines well distributed and separated in the visible spectrum makes it particularly attractive as a source of working optical standards of length.

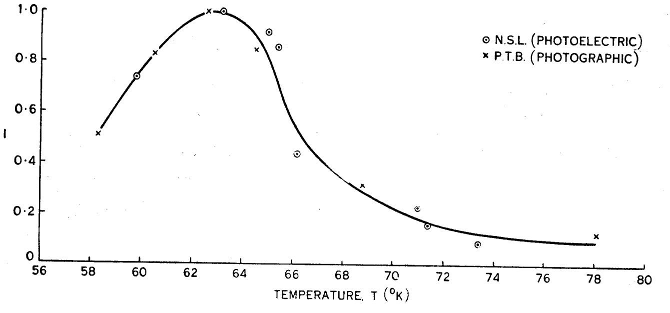

Fig. 9.—Effect of temperature T on intensity I for krypton 86–6057 Å ; current density 0·28 A/cm².

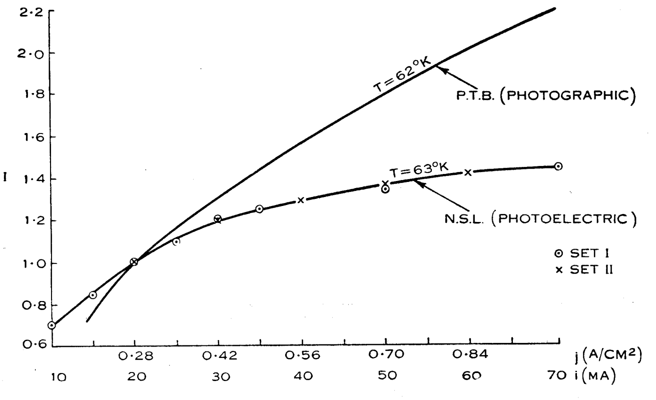

Fig. 10.—Effect of current density j on intensity I for krypton 86–6057 Å ; temperature 63 °K.

V. Wavelength Measurements

(a) *Photoelectric Method*

The vacuum wavelengths of four main radiations from each of the isotope lamps were measured using the photoelectric recording method described in Section III.

Throughout, the reference wavelength was obtained from the radiation corresponding to the transition between the levels $2p_{10}$ and $5d_5$ of the atom of krypton 86. This radiation has been accepted internationally as the primary standard, and it has been recommended that the metre be defined as 1 650 763·73 wavelengths in vacuo of this radiation. The wavelength in vacuo is then 6057·802106 Å where 1 Å$=10^{-10}$ m. This is the standard value used throughout.

One interferometer plate was oscillated at a frequency of about 20 c/s, and the intensity fluctuations at the centre of the ring system were observed for two different wavelengths with two photomultiplier tubes mounted on the spectrograph. A small aperture situated at the principal focus of the telescope lens of the spectrograph restricted, to a desired amount, the area of the central spot of the fringe system observed photoelectrically. The obliquity effect due to the integration of light from the finite area was negligible. The fringes were brought to a sharp focus in the plane of the viewing aperture. The light emerging from the apertures was collimated by a small lens on to the photocathodes of the photomultiplier tubes and the output from a selected photomultiplier tube was passed through a tuned amplifier and then to the Y-plates of an oscillograph. The output from the oscillator producing the scanning frequency of the interferometer plates was connected to the X-plates of the oscillograph. The oscillograph is thus a synchronous detector that can point on a bright fringe maximum with high sensitivity.

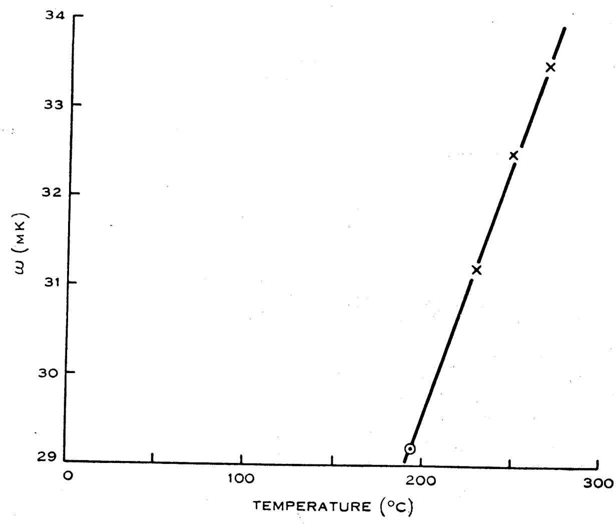

Fig. 11.—Effect of temperature I on line half-width ω of cadmium 114–6440 Å. × BIPM; ⊙ NSL.

With one interferometer plate scanning over a double amplitude of about 0·1 fringe, the plate was gradually moved by the electrical controls until the Lissajous figure on the oscillograph indicated zero signal on the Y-plates. This indicated precise setting on the peak of a fringe. The change in interferometer plate separation, or, change in order of interference, required to set on a bright maximum could be read directly from the dials of the electrical controller.

The fine control and measurement of the change in the plate separation necessary to set on the peak of a bright fringe was obtained by replacing the resistance R, Figure 5, by three sets of 10 resistance coils making up three decades. The basic circuit is shown in Figure 12. If r is the resistance of each coil in R_1, then $R_1=10r=10R_2=100R_3$. The resistances in each decade were made equal and $R_1/R_2=10$ and $R_1/R_3=100$ to the required accuracy. The resistance R_3'

is a compensating resistance linked with R_3 in order to keep the total resistance in the circuit constant. This arrangement was duplicated and the two groups incorporated in a controller so that, by the throw of the selector switch, observa-

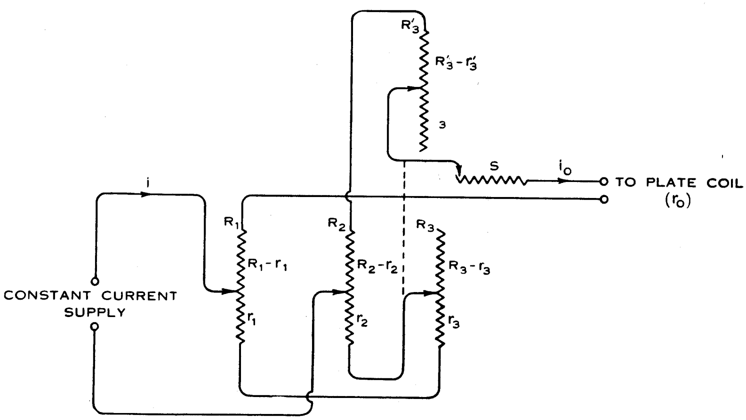

Fig. 12.—Basic electrical circuit of a single channel of the controller for precise measurement of fractional orders of interference.

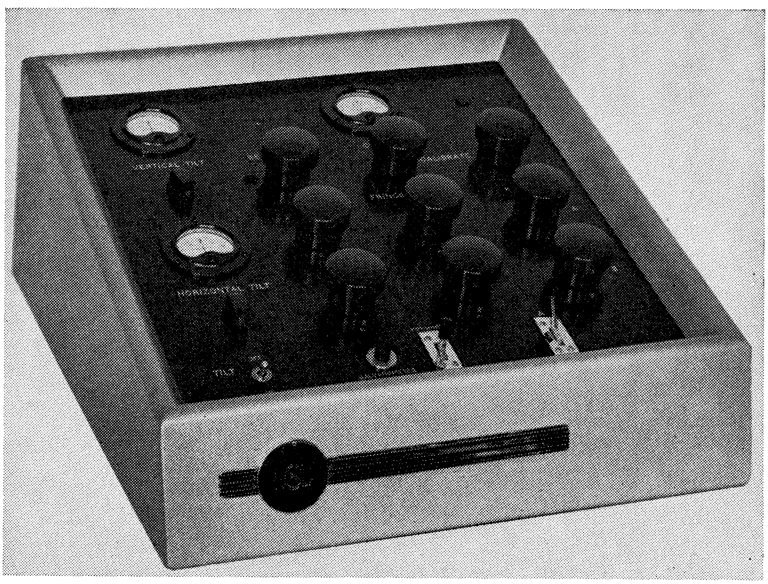

Fig. 13.—Control panel of electrical controller.

tions could be taken at one wavelength and then the other. Thus two wavelengths could be compared almost simultaneously.

Figure 13 is a photograph of the instrument panel of the controller and Figure 14 is a block diagram of the optical, scanning, and photoelectric recording systems.

The procedure adopted was to obtain the fringe spacing for the standard wavelength before and after a set of independent settings as a calibration check for both the standard and unknown wavelengths.

If the setting on the standard λ_A is A and that on the unknown λ_B is B, then the series of settings is $A_1\ B_1\ A_2\ B_2\ A_3\ B_3\ A_4\ B_4\ A_5$. If the fringe spacing is C, then the fringe fraction for λ_A is taken to be $(A_1+A_2)/2C$, $(A_2+A_3)/2C$, etc., and the corresponding fraction for λ_B is B_1/CR, B_2/CR, etc., where $R=\lambda_B/\lambda_A$ and need only be known approximately. Obviously, by adjustment of the sensitivity of the controller, C may be made unity and then the dial readings are direct fractional orders for λ_A. In practice, it was found convenient to make C slightly less than unity. A single wavelength determination involving settings ABA took about 15 s.

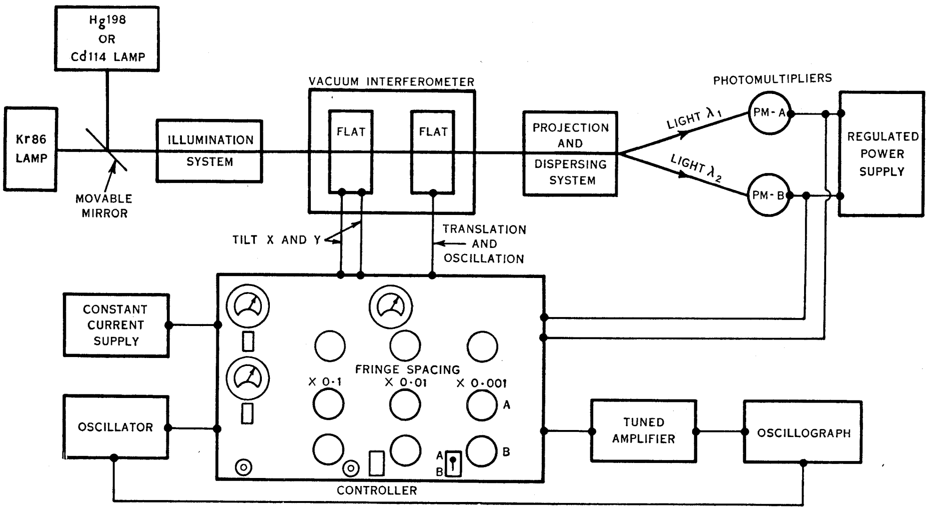

Fig. 14.—Block diagram of the optical, mechanical scanning, and photoelectric recording system used in wavelength measurement.

The measurements were done at a path difference of 100 mm, and at 16 mm to eliminate phase dispersion effects. The integral order of interference was established from the fairly exact knowledge of the plate separation by air gauging, as described earlier, and the observed fractional orders at different wavelengths.

The mean measured values of the vacuum wavelengths are given in Table 3 (a).

The mercury 198 wavelength values are those obtained at a temperature of 283 °K and are uncorrected for pressure shift.

The cadmium 114 wavelength values are those obtained for a temperature of about 470 °K and are uncorrected for pressure shift.

(b) *Photographic Method*

Some photographic measurements were done by the conventional methods in order to compare the accuracy of the photoelectric methods with them. The fringes were photographed on high speed panchromatic plates, the complete visible spectrum for each lamp being recorded in the sequence Kr 86–Hg 198–Kr 86–Hg 198–Kr 86–Hg 198–Kr 86 on one plate. This sort of sequence can be used to reduce temperature effects in comparing the Hg 198 spectrum results with the standard Kr 86 orange-red line. However, different exposure times were necessary for different lines in the one spectrum. In such cases, temperature drift during the time intervals involved (up to 60 s) could influence the position of the centre of density of the recorded image of the fringe on the photographic plate. This effect was in fact detectable on analysing the records. The photoelectric method on the other hand ensured that the observations on standard and unknown were almost simultaneous.

TABLE 3 (*a*)

MEASURED VACUUM WAVELENGTHS

Reference wavelength: krypton 86, $6057 \cdot 802106 \times 10^{-10}$ m

Radiation	Lamp	Measured Wavelength (Unit: 1×10^{-10} m)		Standard Deviation ∇ (10^{-10} m)	N*
		Whole Part	Fractional Part		
Krypton 86 . .	PTB	6458	07240	0·000 09	15
		5651	12851	0·000 07	15
		4503	61553	0·000 04	14
Mercury 198 . .	NSL 7	5792	26816	0·000 15	14
	NBS 99		26797	0·000 13	8
	NRC		26799	0·000 07	8
	NBS 32		—	—	—
Mercury 198 . .	NSL 7	5771	19772	0·000 08	11
	NBS 99		19826	0·000 09	14
	NRC		19856	0·000 08	15
	NBS 32		—	—	
Mercury 198 . .	NSL 7	5462	27029	0·000 03	16
	NBS 99		27064	0·000 04	8
	NRC		27053	0·000 02	8
	NBS 32		27063	0·000 02	8
Mercury 198 . .	NSL 7	4359	56227	0·000 04	16
	NBS 99		56239	0·000 03	16
	NRC		56157	0·000 06	14
	NBS 32		56145	0·000 15	8
Cadmium 114 . .	NSL	6440	24659	0·000 10	16
		5087	23849	0·000 11	8
		4801	25358	0·000 12	6
		4679	45526	0·000 40	4

* N=number of determinations.

The photographic plates were measured on two comparators by two observers using 3–5 rings in the pattern. The fractional orders of interference were obtained in the usual way by the method of least squares.

The photographic results are given in Table 3 (*b*) together with photoelectric results for the krypton 86 lines and the mercury 198 green line from three different lamps.

The krypton 86 lamp was operated as described earlier, the capillary surface temperature being 63 °K and the current density 0·28 A/cm^2. The light direction for viewing was CAO.

TABLE 3 (*b*)

COMPARISON OF MEASURED VACUUM WAVELENGTHS

Reference wavelength: krypton 86, 6057·802106 $\times 10^{-10}$ m

A, photoelectric; B, photographic

Radiation	Lamp	Measured Wavelength (Unit: 1×10^{-10} m)		Standard Deviation ∇	N
		Whole Part	Fractional Part		
Krypton 86 ..	PTB	6458 A	07240	0·000 09	15
		B	07278	0·000 37	9
		5651 A	12851	0·000 07	15
		B	12865	0·000 28	8
		4503 A	61553	0·000 04	14
		B	61567	0·000 13	10
Mercury 198 ..	NSL 7	5462 A	27029	0·000 03	16
		B	27034	0·000 39	9
	NBS 99	5462 A	27064	0·000 04	8
		B	27083	0·000 21	9
	NRC	5462 A	27053	0·000 02	8
		B	27094	0·000 19	10

VI. THE KRYPTON 86 RADIATION $2p_{10}$–$5d_5$ ($\lambda_{vac.}=6057$ Å)

The recommended value by convention of 1 650 763·73 vacuum wavelengths per metre of the krypton orange-red radiation refers to the wavelength as radiated from the atoms of krypton 86 in the unperturbed state. In practice, the wavelength actually emitted is influenced by (*a*) the motion of the radiating atoms in the line of sight (Doppler effect) and (*b*) the pressure and current density in the lamp (interatomic Stark effect).

It is of importance, therefore, to obtain exact knowledge of the change in wavelength due to the Doppler effect (Doppler shift $(\Delta\sigma)_D$) and the change due to the effect of pressure and current density (Stark shift $(\Delta\sigma)_S$). The effects are small and they may be expressed as a wavelength shift $\Delta\lambda$. However, having in mind the recommended definition of the metre, it is more appropriate to express the effects as a change in the number of waves per metre $(\Delta\sigma)_D$ and $(\Delta\sigma)_S$.

If $\Delta\lambda$ is expressed in 10^{-6} Å and $\Delta\sigma$ in m^{-1}, then $\Delta\lambda = -3669\,\Delta\sigma$ for the radiation under consideration here.

The Doppler and Stark shifts were measured using the photoelectric recording interferometer described in Section III, at different temperatures and current densities, using an Engelhard-type hot cathode krypton 86 lamp. It was operated in the same manner as already described by Engelhard and Terrien (1960), except that there was no stirrer within the liquid air.

The shift in wavelength due to a change in the direction of viewing or to a change in current density was immediately visible on the cathode-ray oscillograph. The change in fringes was read directly from the dials of the electrical controller when the adjustment was made to give zero signal. A path difference of 100 mm was used throughout and shifts of a few parts in 10^9 could be measured.

(*a*) *Doppler Shift* $(\Delta\sigma)_D$

The lamp was viewed first from the anode side of the lamp (CAO) and then from the cathode side of the lamp (ACO) and the dial readings of the controller taken for a setting on a fringe maximum for both methods of viewing. The viewing direction could be changed rapidly by the movement of a single mirror. Due precautions were taken to see that correct imaging and illumination existed for both methods of viewing.

The wavelength λ for the undisturbed state of the atom shifts to $\lambda+\Delta\lambda$ when the lamp is viewed from the cathode side (ACO). The corresponding shift in wavelength when the direction of propagation is in the opposite sense (CAO) is $\lambda-\Delta\lambda$.

Table 4

DOPPLER SHIFT $(\Delta\sigma)_D$ FOR KRYPTON 86–6057 Å LINE

Temperature 63 °K

No.	Laboratory	$(\Delta\sigma)_D$ (Light Direction CAO) (m^{-1})	Reference
1	NSL	$+0\cdot014$	
2	PTB	$+0\cdot024$	Engelhard (1957*d*)
3	NPL	$+0\cdot009$	Barrel (1957)
4	NRC	$+0\cdot013$	Baird and Smith (1959)
5	PTB–BIPM	$+0\cdot019$	Engelhard and Terrien (1960)

With the surface of the capillary of the lamp at $63\cdot0$ °K, and a current density of $0\cdot28$ A/cm^2, the value of $2\Delta\lambda$ was measured and found to be 100×10^{-6} Å. The value of $\Delta\lambda$ is therefore -50×10^{-6} Å if the light direction is CAO, and $+50\times10^{-6}$ Å if the light direction is ACO. The standard deviation for 50 determinations was 10×10^{-6} Å. The value of $(\Delta\sigma)_D$ is therefore $+0\cdot014$ m^{-1} if light direction is CAO, and $-0\cdot014$ m^{-1} if light direction is ACO, with a standard deviation of $0\cdot003$ m^{-1}. The result is compared with results from other laboratories in Table 4.

(i) *Effect of Temperature and Current Density.*—There was no significant variation in $(\Delta\sigma)_D$ with current density over the range $0 \cdot 1$–$1 \cdot 0$ A/cm^2.

The effect of temperature was also very small. Table 5 reports observed changes in $(\Delta\sigma)_D$ for a change in temperature from 63 to 61 °K for light direction CAO. Results of PTB and BIPM are also given for comparison.

TABLE 5

CHANGE IN DOPPLER SHIFT WITH TEMPERATURE

No.	Laboratory	Change in Doppler Shift for Change in Temperature from 63 to 61 °K (m^{-1})	Reference
1	NSL	$+0 \cdot 006$	
2	PTB	$+0 \cdot 001$	Engelhard and Terrien (1960)
3	BIPM	$+0 \cdot 006$	Engelhard and Terrien (1960)

(*b*) *Interatomic Stark Shift* $(\Delta\sigma)_S$ (*Pressure and current density effect*)

The procedure adopted here was again to use one lamp and observe the wavelength shift on changing the current density by a fixed amount over a range of temperatures and the shift on changing the current density by different amounts at a constant temperature. The direction of light propagation throughout was CAO.

The wavelength shift is towards the red and follows the approximate law

$$(\Delta\lambda)_S = K(pj)^{2/3}, \tag{1}$$

when p is the pressure of the krypton in the lamp and j is the current density. The pressure in the lamp is computed from the temperature T (°K) at the capillary surface of the lamp, using the Meihuizen (1940) formula

$$\log P_{\mathrm{cm}} = -607 \cdot 69/T + 7 \cdot 2955 - 0 \cdot 0026675T.$$

(i) *Absolute Value of* $(\Delta\sigma)_S$.—The absolute value of $(\Delta\sigma)_S$ was established in three ways.

(1) The wavelength shift was observed when the current density was changed from $0 \cdot 28$ A/cm^2 to $1 \cdot 12$ A/cm^2 (20–80 mA current) with the temperature at 63 °K, and the value of K calculated assuming equation (1) is valid. It was found to be 255 ($\nabla=35$, $N=15$) if $(\Delta\lambda)_S$ is in 10^{-6} Å, p in mmHg, and j in mA/mm^2. With $K=255$, the absolute value of $(\Delta\sigma)_S$ was calculated for $T=63$ °K and $j=0 \cdot 28$ A/cm^2.

(2) A $(\Delta\sigma)_S$–$j^{2/3}$ straight line graph was obtained from the plotted values of $(\Delta\sigma)_S$ (relative) and of $j^{2/3}$ (absolute), and the graph extrapolated to $j=0$. This established the abscissa and therefore absolute values for $(\Delta\sigma)_S$.

(3) A $(\Delta\sigma)_S$–$p^{2/3}$ straight line graph was plotted as in (2) and the graph extrapolated to $p=0$ and the absolute value of $(\Delta\sigma)_S$ again was found.

The results are given in Table 6, together with results from other laboratories.

TABLE 6

STARK SHIFT $(\Delta\sigma)_S$ FOR KRYPTON 86–6057 Å LINE

Temperature 63 °K, current density 0·28 Å/cm²

No.	Laboratory	Method	$(\Delta\sigma)_S$ (m^{-1})	Reference
1	NSL	(1)	—0·014	
2	NSL	(2)	—0·015	
3	NSL	(3)	—0·012	
4	NSL	Mean	—0·014	
5	PTB–BIPM	—	—0·019	Engelhard and Terrien (1960)
6	NRC	—	—0·011	Baird and Smith (1959)

The light direction for results 1–3 was CAO and for 6, ACO, but theoretically light direction should have no significance.

(ii) *Effect of Current Density*.—The results are shown in Figure 15. The full curve is the mean of the results of PTB and BIPM individually (Engelhard and Terrien 1960). There is almost exact agreement between this curve and the NSL values for the rate of change of $(\Delta\sigma)_S$ with current density. The difference in absolute values of $(\Delta\sigma)_S$ between NSL and PTB–BIPM (Table 6) is only about 3 parts in 10^9.

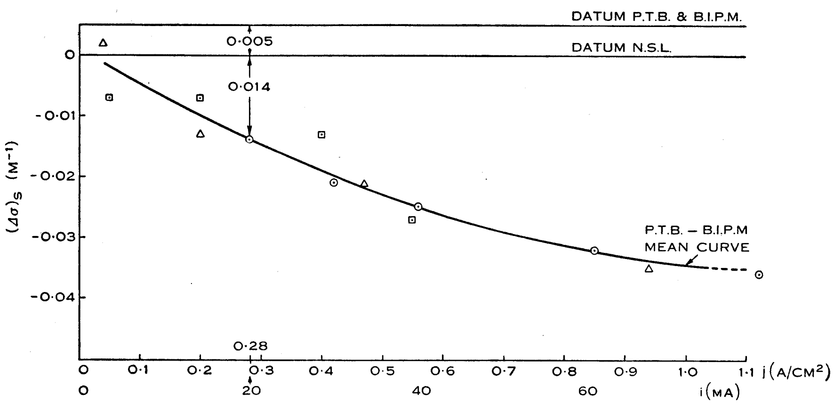

Fig. 15.—Effect of current density j on Stark shift $(\Delta\sigma)_S$ of krypton 86–6057 Å expressed as a variation in the number of waves per metre. Temperature 63 °K. ⊙ NSL, △ PTB, ⊡ BIPM.

(iii) *Effect of Temperature* (*Pressure shift*).—The Stark shift over a temperature range 59–70 °K has been measured and the results are given in Figure 16 for a current density of 0·28 A/cm². Again, comparing the graph of PTB–BIPM (Engelhard and Terrien 1960) with the NSL result (light direction CAO), values from the curves are given in Table 7.

(iv) *Effect of Temperature on the* K *Value*.—The value of K in relation (1) at different temperatures is plotted in Figure 17 and indicates the approximate nature of the relation. The variation appeared to have some correlation with the intensity of the line (see Fig. 9). This point was not pursued further.

VII. Discussion

The photoelectric recording methods described here have measured with high precision the quality of various radiations under vacuum conditions.

Small wavelength changes of 2–3 parts in 10^9 are measurable and, with photomultiplier cells of adequate sensitivity, wavelengths have been compared with the krypton 86 orange-red radiation with an accuracy of better than 1 part in 10^8. These results were obtained with the interferometer in an accurately temperature-controlled room, but without the thermostatically controlled water

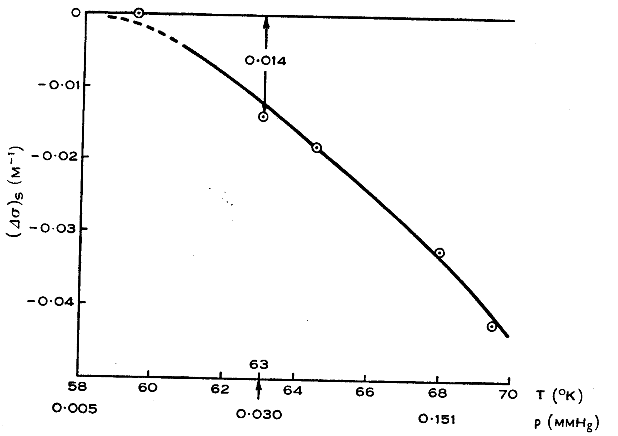

Fig. 16.—Effect of temperature T (pressure shift) on Stark shift $(\Delta\sigma)_S$ of krypton 86–6057 Å, expressed as a variation in the number of waves per metre. Current density 0·28 A/cm².

jacket. The work showed that as a result of the procedures and techniques used, temperature drift effects were extremely small. Nevertheless, with techniques which have been proved capable of measuring displacements to nearly 1 part in 10^9, it is anticipated that improved results will be obtained with more accurate temperature control.

The more serious limitation on higher accuracies in the present experiments was the degree of response of the available photomultiplier tubes to some of the

Table 7

Change in Stark shift with temperature

No.	Laboratory	j (A/cm²)	Change in Stark Shift for Temperature Change from 63 to 62 °K (m⁻¹)	Reference
1	PTB	0·14	+0·004	Engelhard and Terrien (1960)
2	BIPM	0·14	+0·006	Engelhard and Terrien (1960)
3	NSL	0·28	+0·003*	Fig. 16

* Assuming the $j^{2/3}$ law, the NSL result for 0·14 A/cm² would be +0·002 m⁻¹.

less intense lines. The tri-alkali type of tube is excellent for this work, but only one was available, and use had to be made of other types of tubes of much lower efficiency in conjunction with this one, in the two-channel photoelectric system. The magnitude of the standard deviations for different radiations in the wavelength measurements is largely a measure of the efficiency of the photoelectric detector at that radiation. This limitation will shortly be removed and, with highly efficient temperature control, it is considered that wavelength measurements can be made reliable to a few parts in 10^9.

The superiority of the photoelectric detection methods over the conventional photographic methods is shown in Table 3 (*b*). The standard deviations stated can be improved for both the photoelectric and photographic methods, the latter by better temperature control and the former mainly by increased overall

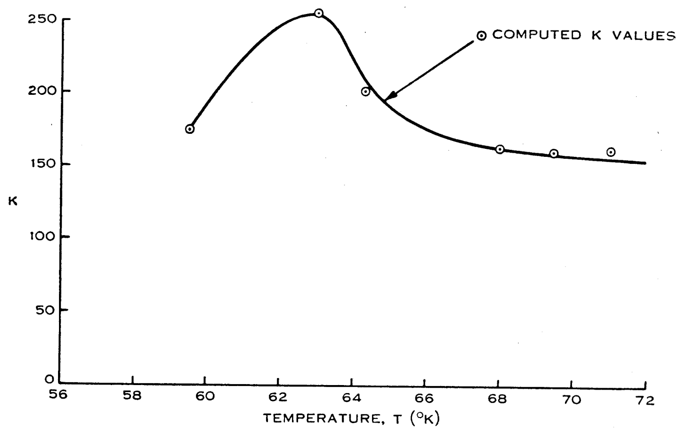

Fig. 17.—Variation of value of Stark shift constant K with temperature T.

photoelectric detection sensitivity. The speed with which fractional orders of interference are observed directly and the removal of the tedium of photographic plate measurement and subsequent reduction of results in the photographic method are major advantages of the photoelectric method.

The krypton 86 radiation $2p_{10}$–$5d_5$, which is recommended for adoption as the primary standard, has been studied in detail. The Engelhard-type lamp performs most reliably, and the pressure and current density at which it operates can be controlled and evaluated with adequate accuracy.

The line profile study of this radiation showed that its half-intensity width is constant to at least $\pm 0\cdot 5$ mK ($0\cdot 0005$ cm^{-1}) if the temperature at the capillary of the lamp is constant to ± 1 °K and if the current density is constant to $\pm 0\cdot 05$ A/cm^2.

The Doppler shift is independent of current density and has the value $+0\cdot 014$ m^{-1} (-50×10^{-6} Å) when the temperature is 63 °K and when the direction of propagation of light is from cathode to anode to observer (CAO).

The Stark shift has the value $-0 \cdot 014\ \mathrm{m}^{-1}$ when the temperature is 63 °K, the current density $0 \cdot 28\ \mathrm{A/cm}^2$, and the light direction CAO. It does not vary by more than $\pm 0 \cdot 004\ \mathrm{m}^{-1}$ if the temperature changes by ± 1 °K or more than $\pm 0 \cdot 003\ \mathrm{m}^{-1}$ if the current density changes by $\pm 0 \cdot 05\ \mathrm{A/cm}^2$.

Thus, if the lamp is operated at a temperature of 63 ± 1 °K (at the surface of the capillary) and a current density of $0 \cdot 28 \pm 0 \cdot 05\ \mathrm{A/cm}^2$, (current of 20 mA for a 3 mm diameter capillary) and viewed so that the direction of propagation of light is from cathode to anode to observer, the wavelength emitted is that for the unperturbed state of the krypton 86 atoms, to 2–3 parts in 10^9. It is this wavelength that would define the metre.

There was evidence of considerable variation in the wavelength values for the mercury lines from various electrodeless lamps. There was, in addition, some evidence of quite short period variation (seconds) in the wavelength of the emitted light from a single lamp. This was particularly noticeable when an attempt was made to use the mercury 198 green radiation as a reference wavelength in studying variations in the wavelength of the krypton 86–6057 Å line for different conditions of lamp operation.

The wavelength values given for the cadmium 114 electrodeless lamp are the least reliable. As explained earlier, the amount of pure cadmium 114 metal in the lamp was inadequate, owing to insufficient supply of the isotope in proper form. Further supplies did not become available in time for other lamps to be made for use in the measurements. From the life tests on identical lamps made with natural cadmium, there is no reason to doubt that the form of electrodeless lamp described here will be satisfactory. The intensity distribution and separation of the cadmium 114 radiations are all extremely suitable for much work in length measurement. It is considered that, though the mercury 198 and cadmium 114 radiations from electrodeless lamps cannot compete with the krypton 86 orange-red radiation in quality as a primary standard, they have many attractive features as secondary working standards. Such radiations may become very strong competitors as a primary standard with the development of atomic beam sources. However, it is also possible that other parts of the electromagnetic spectrum may supply even better primary standards in the future. In all cases the principle of using a natural atomic unit as the primary standard of length is likely to remain. The best existing unit of this kind is the orange-red radiation $2p_{10}$–$5d_5$ of the krypton 86 atom which is reproducible to 2 to 3 parts in 10^9.

VIII. Acknowledgments

The authors are indebted to the following colleagues for assistance with various aspects of the work.

The principle employed in the electromagnetic plate holders was one that was applied by the Division of Electrotechnology of this Laboratory in the design of a calculable capacitor operating under vacuum conditions. Mr. W. K. Clothier of that Division was responsible for the basic design and he has given much valuable advice on electrical aspects of their operation.

Mr. R. S. Fisher constructed the constant current supply and the electrical controller, and Mr. E. Phair made the mercury 198 and cadmium 114 lamps.

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