THE DISTRIBUTION OF METEOR MASSES FOR SPORADIC METEORS AND THREE SHOWERS

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Summary

Distributions of maximum line densities ($\alpha_{\mathrm{max.}}$ electrons/cm) in meteor trails, and hence distributions of meteor masses, are derived from radio-echo data for sporadic and shower meteors. Distributions are obtained in the range $10^{11} < \alpha_{\mathrm{max.}} < 10^{12}$ from comparisons of echo rates, and for $\alpha_{\mathrm{max.}} > 10^{13}$ from durations of persistent echoes. For $\alpha_{\mathrm{max.}} > 10^{11}$ the mass distributions cannot be represented by a simple inverse power law with constant exponent s. For sporadic meteors s increases from $2\cdot 0$ for $10^{11} < \alpha_{\mathrm{max.}} < 10^{12}$ to $2\cdot 5$ for $\alpha_{\mathrm{max.}} > 10^{13}$. For the η-Aquarid, δ-Aquarid, and Geminid showers, $s < 2\cdot 0$ for $\alpha_{\mathrm{max.}} < 5\times 10^{13}$, and $s \geqslant 2\cdot 0$ for $\alpha_{\mathrm{max.}} \geqslant 5\times 10^{13}$. The δ-Aquarids exhibit a concentration of bright meteors to the centre of the stream.

It is suggested that the extent of fragmentation of shower meteors (relative to sporadic meteors) can be estimated from comparisons of the values of s found for the same shower by the echo rate method described in this paper and by the height distribution method developed at Jodrell Bank.

I. Introduction

The variation of the meteor echo rate as a function of the sensitivity of the radio equipment is determined almost entirely by the distribution of meteor masses and the form of the aerial polar diagram. Since measurements of echo rates are easily made, and there is little difficulty in making allowance for the aerial polar diagram, it seems surprising that relative echo rates have been so little exploited in the determination of the distribution of meteor masses, at least amongst the smaller meteors.

It is important to remark that the meteor parameter derived directly from the relative echo rates is not the mass distribution, but rather the distribution of the maximum electron line densities in the trails. The experimental data with which this paper is concerned are interpreted in terms of this latter parameter. However, the conclusions will apply to the mass distributions also, provided that the ionizing efficiency of an evaporated meteor atom is independent of the meteor mass.

The rate of detection of echoes by a radio equipment may be varied in two ways:

(1) By varying the limiting sensitivity of the equipment, that is, by changing the maximum line density of the weakest trail accessible to the equipment. This can be achieved, for a given aerial system, by altering either the transmitter power or the receiver sensitivity. Kaiser (1953) has used experimental values obtained by McKinley (1951), in which the transmitter power was varied, to

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study the distribution of maximum line densities amongst faint sporadic meteors. The results described in the present paper were obtained by varying the receiver gain ; maximum line density distributions are obtained in the range 10^{11}–10^{12} electrons/cm.

(2) With limiting sensitivity held constant, by counting only those echoes whose line densities at the echoing points exceed certain selected values. Here and elsewhere (e.g. Browne *et al.* 1956) the selection has been made on the basis of echo durations. This yields distributions for maximum line densities exceeding 10^{13} electrons/cm.

The range of maximum line densities for which distributions are determined in this paper extends from 10^{11} to a little greater than 10^{14} electrons/cm. Measurements have been made for sporadic meteors and for three showers (Geminids, δ-Aquarids, η-Aquarids) over the period 1957 to 1960. Attempts were also made to derive maximum line density distributions for the June day-time showers (Arietids, ζ-Perseids) and the July Phoenicid shower, but the shower echo rates were too low, in comparison with the sporadic background, to yield useful results.

II. Equipment

During 1957 observations were made with the 67 Mc/s narrow-beam radiant equipment (Weiss 1955, 1960*a*). Calibration for equipment sensitivity using the Geminid shower echo rate gives a maximum line density in the faintest detectable meteor trail of $1 \cdot 3 \times 10^{11}$ electrons/cm (Weiss 1960*a*). A new calibration from the sporadic echo rate, which gives an almost identical result, is described in Section IV.

All the echo durations measured were recorded with this equipment, using a modified recording display whose effective writing speed was increased to 4 in/min by a combination of optical and electrical time bases. The smallest duration which could be measured reliably was $0 \cdot 36$ s. No attempt has been made to exclude echoes whose durations may be affected by distortion of the trail due to turbulent motion of the atmosphere.

For the 1959 and 1960 observations the 67 Mc/s equipment was replaced by a new equipment operating at 61 Mc/s, the transmitter power of 50 kW remaining unchanged. This equipment has separate transmitting and receiving aerials, each with beam axes directed due west at an elevation of 29°. The composite beamwidth is larger than for the 67 Mc/s Yagi array, but the lower aerial gain was almost completely compensated by a decrease in site noise which permitted operation at higher receiver sensitivity. Consequently, the overall equipment sensitivity for the 61 Mc/s observations is little, if any, lower than for the earlier 67 Mc/s observations.

III. Theory of the Echo Rate

(*a*) *The Incident Meteor Flux*

Following previous practice, it will be assumed that the flux of meteors producing trails whose maximum line densities lie in the range $\alpha_{\text{max.}}$ to $\alpha_{\text{max.}}+\mathrm{d}\alpha_{\text{max.}}$ is

$$n\mathrm{d}\alpha_{\text{max.}}=c\alpha_{\text{max.}}^{-s}\mathrm{d}\alpha_{\text{max.}}. \qquad (1)$$

For the theoretical developments it will be assumed that s is a constant. This is known not to be the case for some showers (Browne *et al.* 1956), but the variation of s with $\alpha_{max.}$ is sufficiently slow for the law (1) to constitute a useful definition of the line density distribution.

The definition of radio magnitude in terms of the maximum electron density in the trail and its relation to the visual magnitude scale have been discussed by Browne *et al.*

(*b*) *Echo Characteristics*

The echo is returned from a meteor trail in the vicinity of the specular reflection point, which is not in general at the point of maximum ionization. Two types of echo, depending on the line density α at the reflection point, have been recognized (Kaiser and Closs 1952). When $\alpha < 2\times10^{12}$/cm, the electron volume density is sufficiently low for the incident wave to penetrate right throughout the trail (Lovell-Clegg scattering). The maximum echo amplitude A is proportional to α, and the echo duration τ, defined as the time required for the echo amplitude to fall to 1/e of its initial value, is independent of α. These are referred to as short-duration or decay-type echoes. When $\alpha > 2\times10^{12}$/cm, the incident wave is no longer able to penetrate to the axis of the trail, and the nature of the reflection process changes (persistent scattering). When $\alpha > 10^{13}$/cm, $A \propto \alpha^{1/4}$ and $\tau \propto \alpha$ if the decay is caused only by radial diffusion of the trail; if, in addition, removal of free electrons from the trail by attachment to neutral atmospheric molecules is significant, both A and τ are smaller, the reductions increasing rapidly as α increases. These are known as persistent echoes.

So far as echo durations are concerned, the range of α from 2×10^{12}/cm to 10^{13}/cm is a transition region in which incomplete penetration of the trail occurs, but the exponential decay characteristic of the short-duration echoes occupies an appreciable portion of the total echo duration. Consequently the echo duration is not independent of the echo amplitude and hence of the equipment parameters and of the location of the echoing point within the aerial beam. When discussing echo durations in this paper the limit $\alpha = 10^{13}$/cm will be taken as the lower boundary of the region of persistent echoes.

(*c*) *The Total Echo Rate*

The theory of the radio echo rate has been given by Kaiser (1955*a*). His treatment assumes specular reflection and the evaporation theory for a solid meteor particle in an isothermal atmosphere. Expressions are derived for the instantaneous shower echo rate, when echoes are received only from the echo plane which passes through the observing station and is perpendicular to the direction of the radiant; and also for the echo rate from a distributed source such as sporadic meteors, when echoes are received simultaneously from all elements of the aerial beam.

The shower echo rates discussed in this paper were determined by summing over the whole time of passage of the radiant through the collecting area of the equipment. The second alternative above, the echo rate for a distributed source, is therefore the one to take. With the flux (1), the total echo rate given by Kaiser may be written in the form

$$N = hHI_0 \int_{\alpha_0}^{\infty} W(\alpha)\alpha^{-s} d\alpha, \tag{2}$$

where h=mean height of reflection points, H=atmospheric scale height, and α_0=minimum detectable line density in the direction of the beam axis. In this and subsequent equations the subscript " max." has been dropped. I_0 is an integral whose value depends only on the shape of the ionization curve of the

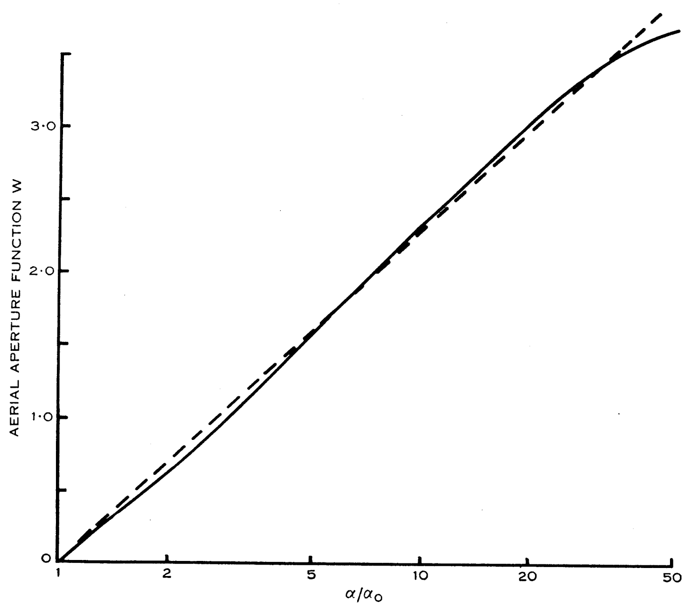

Fig. 1.—Aerial aperture function W for the Adelaide narrow-beam 67 Mc/s radiant equipment (——) compared with $W = \ln(\alpha/\alpha_0)$ (- - - -). The scale of W has been normalized for best fit to the logarithmic law. α_0 is the minimum detectable line density for the equipment.

meteor trails. $W(\alpha)$, which may be described as the aerial aperture function, is a function of α alone. It takes account of the increase, as α increases, in the volume of space within which trail segments of line density α at the echoing points are detectable.

W is determined by the aerial polar diagram and the elevation of the axis of the aerial beam. It has been evaluated numerically for the theoretical polar diagram of the Adelaide narrow-beam aerial system, under the restriction that $\alpha_{\text{max.}} \leqslant 10^{12}$/cm over the greater part of the aerial beam. It is seen from Figure 1 that, apart from a multiplying constant, a very good approximation to W for the range $1 \leqslant \alpha/\alpha_0 \leqslant 30$ is

$$W = \ln(\alpha/\alpha_0), \quad \alpha < \alpha_t. \tag{3a}$$

This law cannot be applied beyond the point of transition, α_t, from Lovell-Clegg to persistent scattering; the appropriate forms are then

$$\left.\begin{aligned} W &= \ln(\alpha_t/\alpha_0)+\tfrac{1}{4}\ln(\alpha/\alpha_t), && \alpha>\alpha_t>\alpha_0 \\ &= \tfrac{1}{4}\ln(\alpha/\alpha_0), && \alpha>\alpha_0>\alpha_t \end{aligned}\right\}. \tag{3b}$$

It will be assumed for the present that (3b) extends into the region of persistent echoes, where $\alpha>10^{13}$/cm; its validity in this region will be examined later. Since the form of the polar diagram for the 61 Mc/s aerial system is similar to that for the 67 Mc/s array, (3a) and (3b) will be taken to apply in this case also.

The expressions (3) for W were derived by considering echo collection from a shower radiant near the zenith. However, collecting areas examined during the course of the computations proved quite insensitive to the zenith angle of the radiant. Expressions (3) will therefore be accepted for sporadic meteors also. This implies neglect of a small correcting factor which depends on the apparent distribution of sporadic radiants. This will introduce a small error into the total echo rate, but the effect on relative echo rates will be negligible.

(d) *Relative Echo Rates and Equipment Sensitivity*

Since we are concerned only with the relative echo rates obtained with different equipment sensitivities, the constants H and I_0 appearing in (2) and the slight change of the mean height h with equipment sensitivity may be ignored. The echo rate will therefore be written

$$N=\int_{\alpha_0}^{\infty} W(\alpha)\alpha^{-s}\mathrm{d}\alpha, \tag{4}$$

with W defined by (3a) or (3b), as appropriate. Carrying out the integrations, the dependence of the total echo rate on the limiting line density α_0 becomes

$$N=\frac{\alpha_0^{1-s}}{(s-1)^2}\left\{1-\tfrac{3}{4}\left(\frac{\alpha_0}{\alpha_t}\right)^{s-1}\right\}, \quad \alpha_0\leqslant\alpha_t, \tag{5a}$$

$$=\tfrac{1}{4}\frac{\alpha_0^{1-s}}{(s-1)^2}, \qquad \alpha_0\geqslant\alpha_t. \tag{5b}$$

These functions, suitably normalized, are sketched in Figure 2 for $s=1\cdot5$ $2\cdot0$, $2\cdot5$. The influence of the transition from persistent to Lovell-Clegg scattering is still noticeable for values of α_0 smaller than $\alpha_t/10$ when $s<2\cdot0$. With $\alpha_t\simeq10^{12}$/cm, this implies that the simple formula which is often used for the echo rate, namely, $N\propto\alpha_0^{1-s}$, is invalid for $10^{11}<\alpha_0<10^{12}$/cm. It is true that this result has been derived only for the special form (3) for the function W, but it is expected to apply to any reasonable aerial system.

It is also important to note that these results are independent of the validity of the evaporation theory. From (2) it is evident that the shape of the ionization curve may profoundly affect the *total* echo rate, but since it does not enter into the computation of W, the *relative* echo rate is unaffected. The shape of the ionization curve will assume importance only if there is a rapid alteration of shape with meteor mass. There is indeed some evidence (Weiss 1960*b*) that short trails are associated with bright meteors, but the change in the length of the trail with meteor brightness is far too slow to affect relative echo rates measured over a comparatively small range of α_0.

(*e*) *The Distribution of Echo Durations*

The theory of the rate of detection of persistent echoes, originally formulated by Kaiser (1955*a*), has been extended to include the effect of removal of free electrons from the trail by attachment to neutral molecules. This severely limits the durations of the most persistent echoes. Only a brief summary of the results will be given here ; full details will be found in Weiss (1961).

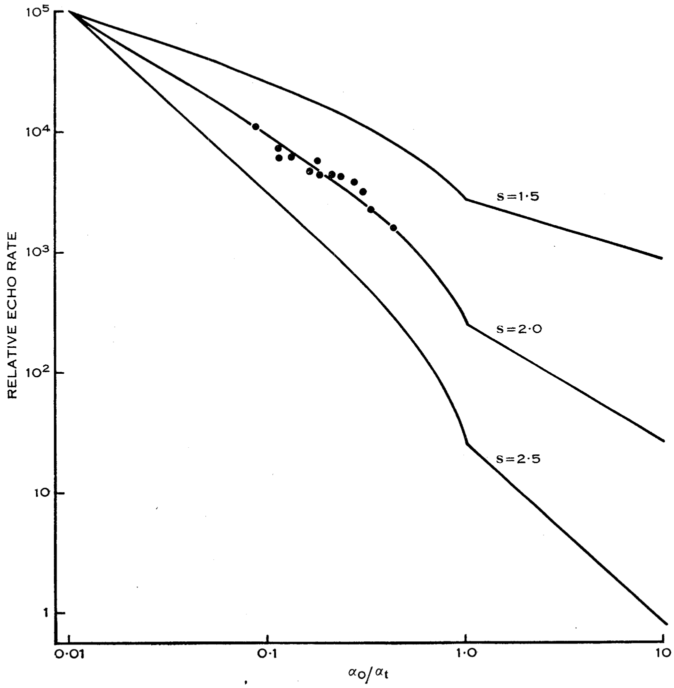

Fig. 2.—The relative echo rate as a function of the limiting line density α_0 and mass distribution parameter s. Full lines are the (normalized) theoretical distributions of equation (5). α_t is the line density corresponding to the point of transition from Lovell-Clegg to persistent scattering. Dots are observations of relative echo rates of sporadic meteors, fitted to the theoretical curve for $s=2\cdot0$.

The effects of attachment are introduced through a parameter $B=\beta_e n_m$. β_e is the attachment coefficient and n_m is the density of the ion-forming molecules. B is a function of height. It is convenient to introduce a specific value of B, namely B_0, the value of B at a datum height of 95 km.

From expression (3b) it is apparent that when $\alpha>10^{13}$/cm, W varies very slowly with both α and s, and it is sufficient to put $W=\text{constant}=\overline{W}_T$. With this substitution, the rate of detection of echoes with durations exceeding T becomes, from (2),

$$N_T = hH\overline{W}_T I_T \alpha_T^{1-s}/(s-1), \tag{6}$$

where α_T is the maximum line density in the weakest trail capable of giving the duration T. The integral I_T is a slowly varying function of T. For the range of s-values and echo durations considered in this paper, $I_0/I_T \simeq \sqrt{2}$. For $\alpha_T \simeq 10^{13}$/cm, when the effects of attachment are small, $T \propto \alpha_T{}^{4/3} v^{-2}$ (v is the meteor velocity). For the most persistent echoes measured, $\alpha_T \simeq 10^{16}$/cm and $T \propto \alpha_T{}^{2/9} v^{2/3} B_0{}^{-2/3}$. The slight change in the mean height h with echo duration may be neglected. For uniform meteor velocity the integral distributions of echo durations, that is, the numbers of echoes N_T whose durations exceed T, in these two limiting cases are accordingly

$$N_T \propto T^{3(1-s)/4}, \tag{7}$$

and

$$N_T \propto T^{9(1-s)/2}. \tag{8}$$

For intermediate echo durations the relation between T and α_T, and hence the distributions of echo durations, are most easily found by numerical means (Weiss 1961).

Values of s may be obtained for shower meteors by direct comparison of the observed distributions of echo durations with these predicted distributions. But, because the dependence of echo duration on meteor velocity varies with the duration, the theoretical distributions for meteors with uniform velocity cannot be used immediately to derive s-values from observational data on the durations of sporadic meteor echoes. It is first necessary to average over all meteor velocities, from 15 to 75 km/s, according to the probabilities of detection. When this is done, it is found, to a good approximation, that the frequency distribution of the durations of sporadic meteor echoes corresponds to that for $v = 40$ km/s.

The errors in s-values derived through these theoretical distributions, arising from neglect of the dependence of I_T and W on T, are unlikely to exceed a few per cent. Further sources of error, of greater importance, are the violations of the basic assumptions that the meteor evaporates as a solid particle and that reflection is specular. Fragmentation and crumbling of the meteor particle, followed by evaporation of the fragments, gives a better description of the ablation process amongst bright meteors than simple evaporation. The failure of specular reflection is brought about by distortion and twisting of the initially straight trail, due to turbulent motion of the atmosphere. This causes an increase, which can be as large as a factor of 4 or 5, in the effective value of W for the more massive meteors. For the narrow-beam aerial system used here, however, the increase in W will not be as large as this.

Considering all sources of error, the uncertainty in values of s derived by comparing observed distributions of echo durations with the theoretical distributions should not exceed 20%. The term " theoretical distributions " is here to be understood to refer to the distributions (7) and (8) and those obtained numerically for values of T intermediate between these two limiting cases of small and large T.

IV. Distribution of Maximum Line Densities, Sporadic Meteors

During 1957 the relative rate of sporadic echoes was determined as a function of limiting line density α_0, by varying the receiver gain. The recordings were made by splitting the receiver video stage into two channels with unequal gains; the sensitivity of one channel was held constant to provide a base rate at the highest sensitivity, and the gain of the other channel was systematically reduced. Relative values of α_0 were found by calibration of the receiver, assuming Lovell-Clegg scattering.

The observations are plotted in Figure 2, where they have been fitted to the theoretical curve for $s=2\cdot0$, which is the accepted value for sporadic meteors (Browne *et al.* 1956). In making this fit, both the echo rate and the value of α_0 at the highest sensitivity were considered as adjustable parameters. From this fit a value of $\alpha_0/\alpha_t=0\cdot09$ is deduced for the minimum detectable line density

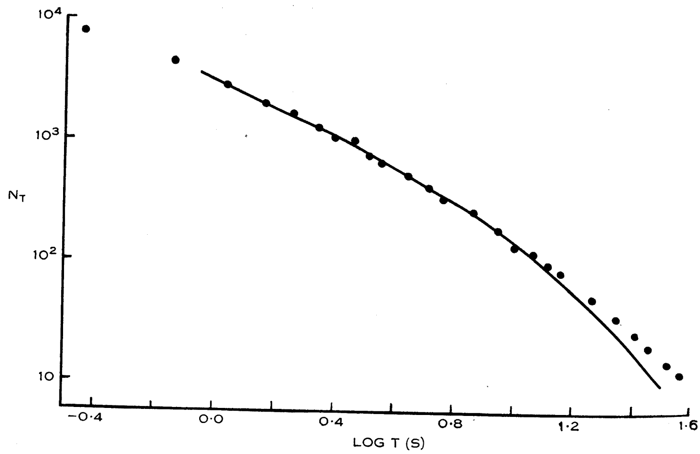

Fig. 3.—Distribution of echo durations at 67 Mc/s for sporadic meteors, 1957. The plot is double-logarithmic and shows the number of echoes N_T with durations $\geqslant T$ s. Observations are shown by dots, and the full line is the theoretical distribution for $s=2\cdot5$, $B_0=0\cdot015$/s.

under normal operating conditions. There is some uncertainty in the value to be taken for α_t. According to the values of the reflection coefficient given in Figure 12 of Kaiser and Closs (1952), $\alpha_t\simeq8\times10^{11}$/cm, which is somewhat lower than the value of $\alpha_t\simeq2\cdot4\times10^{12}$/cm found from the condition that the incident wave should penetrate completely through the trail. Since the transition from Lovell-Clegg to persistent scattering will take place smoothly over a small range of α, rather than abruptly, we will adopt $\alpha_t=1\times10^{12}$/cm. With this value of α_t, $\alpha_0=9\times10^{10}$/cm, which agrees well with the previous determination of $\alpha_0=1\cdot3\times10^{11}$/cm described in Section II. These observations are thus consistent with a value of $s=2\cdot0$ for sporadic meteors in the range of line densities $10^{11}<\alpha_{\max.}<10^{12}$/cm.

The frequency distribution of echo durations leads to a larger value of s for $\alpha_{\max.}>10^{13}$/cm. The numbers of echoes whose durations exceed T s, obtained over an extended period during 1957, are plotted in Figure 3. By comparing these experimental data with theoretical distributions calculated after the manner described in Section III (*e*), we obtain for the conditions of best fit, $s=2\cdot5$, $B_0=0\cdot015/s$. This theoretical distribution is also shown in Figure 3. A value of $s=2\cdot5$ is little larger than the $s=2\cdot34$ found by Hawkins and Upton (1958) from the luminosity function for photographic meteors. The value of $B=0\cdot015/s$ at a height of 95 km is the same as that found by Davis, Greenhow, and Hall (1959) from the relation between echo duration and visual brightness of Perseid and Geminid meteors. However, the shape of the theoretical durations distribution is not particularly sensitive to B_0, and the value of this parameter cannot be stated more precisely than to within a factor of 2.

It may be concluded that the values of *s* for sporadic meteors deduced from the present observations are in agreement with determinations by other workers using different observational methods. The value of *s* increases from $s=2\cdot0$ for $10^{11}<\alpha_{max.}<10^{12}$/cm to $s=2\cdot5$ for $\alpha_{max.}>10^{13}$/cm. With the exception of distributions of echo durations, all observations on shower echo rates will be analysed by comparing them with similar data for sporadic meteors, taking for the latter the above values of *s*.

V. Distributions of Maximum Line Densities, Shower Meteors

(*a*) *Relative Echo Rates*, $10^{11}<\alpha_{max.}<10^{12}/cm$

Total numbers of echoes detected simultaneously at two different equipment sensitivities are listed in Table 1, for three showers. The numbers of echoes refer to totals summed over the dates shown and are not indicative of the relative strengths of the showers. The sporadic echo rates are totals summed over the same periods as the shower echoes. With the assumption $s=2\cdot0$, they provide information on the relative sensitivity of the equipment in the two operating conditions.

Table 1

SHOWER ECHO RATES AND *s*-VALUES FOR MAXIMUM LINE DENSITIES 10^{11}–10^{12} ELECTRONS/CM

Shower	Year	Dates of Observation	Sporadic Echo Rates		Shower Echo Rates		r	R	s_1
			n_0	n_1	N_0	N_1			
η-Aquarids	1960	May 4–9	2672	683	514	215	3·91	2·39	1·61
δ-Aquarids	1959	July 21–Aug. 9	3862	1334	5675	3199	2·90	1·77	1·54
Geminids	1957	Dec. 11–14	210	47	228	108	4·47	2·11	1·50

It will be seen that in no case is the sporadic rate sufficiently low that it may be neglected when estimating the shower echo counts. In fact, the η-Aquarid shower rates are a good deal lower than the sporadic rate and cannot be regarded as free from uncertainty.

If α_0 and α_t were accurately known, the *s*-values for the showers could be determined from graphs such as those illustrated in Figure 2. With $s=2\cdot0$

and α_0 known, the sporadic counts could be used to determine the limiting line density α_1 at the lower sensitivity, and the value of s for the shower could then be read off from the curve which best fitted the shower counts. But since α_0 is not accurately known (the 1959 and 1960 observations were made at 61 Mc/s and the calibrations discussed in Section IV do not apply to this equipment), an alternative procedure has been adopted.

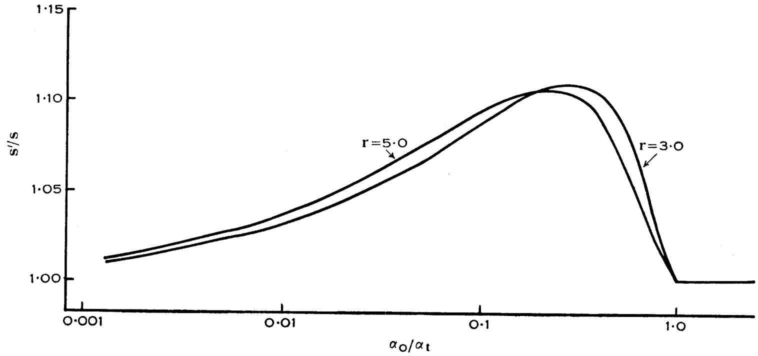

Fig. 4.—The error in the value of s derived from shower echo rates measured at two different equipment sensitivities, when it is improperly assumed that equation (10) is valid in the vicinity of the transition point α_t. s' is the value of s obtained using (10), and the curves are drawn for $s=1\cdot5$. α_0 is the limiting line density of the equipment at the higher sensitivity, r is the ratio of the sporadic meteor echo rates at higher and lower sensitivity.

It will be recalled from equation (5) that when either $\alpha_0 \ll \alpha_t$ or $\alpha_0 \gg \alpha_t$,

$$N \propto \alpha_0^{1-s}. \tag{9}$$

Denoting the counts at higher and lower sensitivity by the subscripts 0 and 1 respectively, and putting $R=N_0/N_1$ for showers and $r=n_0/n_1$ for sporadics, it follows that within the range of validity of (9) and with $s=2\cdot0$ for sporadics, the values of s for showers are given by

$$s=1+(\log R)/(\log r). \tag{10}$$

In view of the general similarity of the shapes of the curves of Figure 2 for different values of s, it is reasonable to expect that (10) will not be seriously in error even in the vicinity of the transition point α_t. Values of s found using (10) are listed in Table 1.

It remains to assess the errors introduced by the assumption that (10) holds over the range of line densities to which these determinations apply. This can be done in the following manner. Assume an r and an α_0/α_t. Calculate α_1/α_t from (5) with $s=2\cdot0$. Evaluate the ratio R corresponding to these values of α_0/α_t and α_1/α_t when $s\neq2\cdot0$. With this R and the given r, use (10) to find the value of s, which we call s', which results from the improper use of (10). Two sets of calculations, with $s=1\cdot5$ and $r=3\cdot0$ and $5\cdot0$, suffice to indicate the errors in the s-values given in Table 1. The results are plotted in Figure 4.

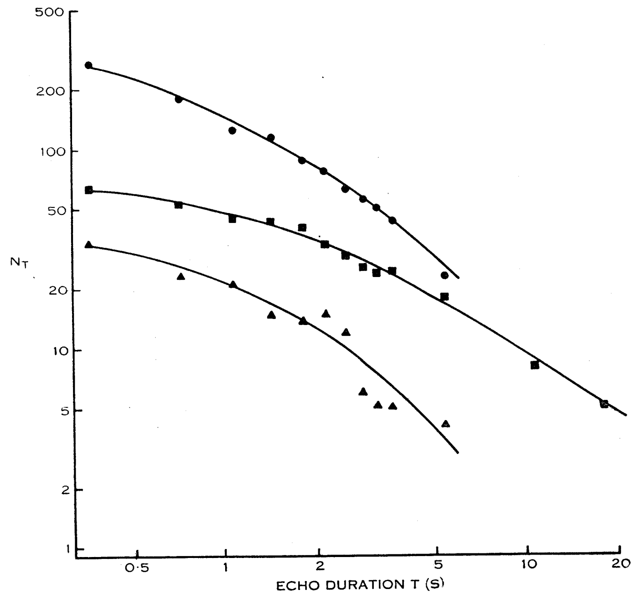

Fig. 5.—Distributions of shower echo durations at 67 Mc/s. The plot is double-logarithmic and shows the numbers of echoes N_T with durations $\geqslant T$ s. ■ η-Aquarids ; ● δ-Aquarids ; ▲ Geminids.

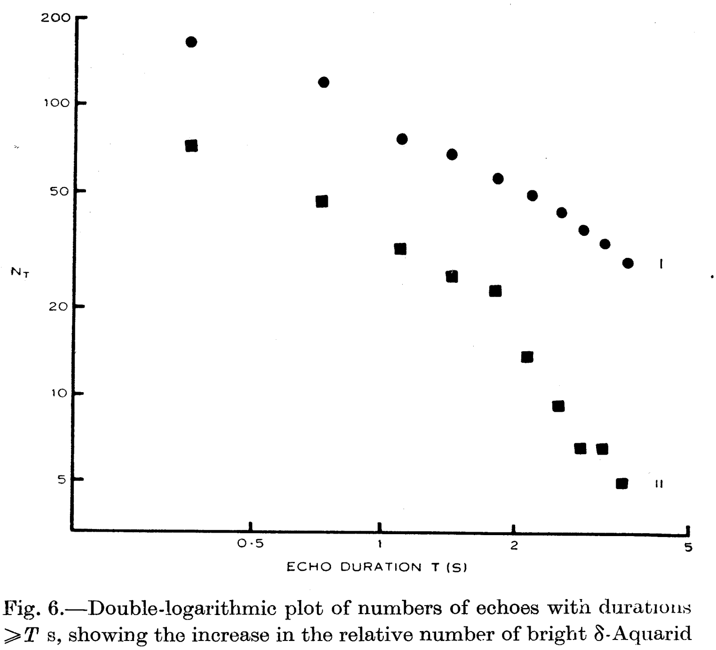

Fig. 6.—Double-logarithmic plot of numbers of echoes with durations $\geqslant T$ s, showing the increase in the relative number of bright δ-Aquarid meteors when the shower rate is a maximum. I : echoes observed over period of peak activity, July 28–31, 1957. II : July 27 and August 1–3, 1957.

It is seen that for the showers under consideration and with probable values of α_0 not much larger than $0 \cdot 1\alpha_t$, the values of s listed in Table 1 are not likely to be in error by more than 10%. They are also upper limits.

(b) *Durations Distributions,* $\alpha_{max.} > 10^{13}/cm$

Distributions of echo durations are given for the three showers in Figure 5. They have been obtained by subtraction of the estimated sporadic components from the distributions for all echoes (shower plus sporadic) over the periods of activities of the showers. Unlike that for the sporadic meteors, these distributions cannot be fitted, over the whole range of durations from $\frac{1}{3}$ to 10 s, by theoretical curves with constant values of s, unless the attachment parameter B_0 takes on impossibly large values. The values of s have therefore been determined using the same value of B_0 as was found from sporadic meteors, namely, $B_0 = 0 \cdot 015$/s. They are listed in Table 2. The measurements for the η-Aquarids apply to slightly higher line densities than for the other two showers because of the higher velocity of the former meteors.

In each case there is a large increase in s as $\alpha_{max.}$ increases from 10^{13} to 10^{14}/cm.

TABLE 2

s-VALUES FOR MAXIMUM LINE DENSITIES $>10^{13}$ ELECTRONS/CM FROM DISTRIBUTIONS OF ECHO DURATIONS, 1957

Shower	Dates of Observation	s_2	Limits of $\alpha_{max.}$ (cm^{-1})	s_3	Limits of $\alpha_{max.}$ (cm^{-1})
η-Aquarids	May 3–10	1·4	$2–6 \times 10^{13}$	2·0	$6 \times 10^{13}–5 \times 10^{14}$
δ-Aquarids	July 23–Aug. 7	1·9	$1–4 \times 10^{13}$	2·7	$4 \times 10^{13}–1 \times 10^{14}$
Geminids	Dec. 9–14	1·7	$1–4 \times 10^{13}$	2·7	$4 \times 10^{13}–1 \times 10^{14}$

(c) *Changes in* s *across the δ-Aquarid Stream*

The δ-Aquarid activity is sufficiently high that day-by-day changes in the distribution of maximum line densities can be traced, at least qualitatively, during the greater portion of the 1957 activity. In Figure 6 the distributions of echo durations for July 27 and August 1–3 are compared with those for July 28–31; the latter four days cover the period of peak activity. The two curves do more than merely indicate the average behaviour over the two periods; without exception, the distribution for each individual day conforms to the shape characteristic of the period to which it belongs. Figure 7 (*a*) gives the proportion of echoes with duration $\geqslant 0 \cdot 36$ s. These two diagrams taken together imply an increase in the number of bright meteors, relative to that of the fainter meteors, over the period of peak activity. This increase, which is most marked amongst the brightest meteors ($\alpha_{max.} > 4 \times 10^{13}$/cm), extends down to meteors at least as faint as $\alpha_{max.} = 10^{13}$/cm.

Values of s, determined for individual days in 1959 from comparison of echo rates at different equipment sensitivities, are plotted in Figure 7 (*b*); this applies to meteors with $\alpha_{max.} < 10^{12}$/cm. Unlike the brighter meteors, there is here no evidence for a change in s at the time of peak activity. Although the relative echo rate and durations data apply to different years, this constancy of s amongst the fainter meteors is consistent with the above-noted decrease in s for the brighter meteors at the time of peak activity.

A similar concentration of bright meteors to the centre of the stream has been noted by Browne *et al.* for the Geminids. The Perseids, however, show a contrary behaviour, with a rise in the relative number of fainter meteors when the shower rate is a maximum.

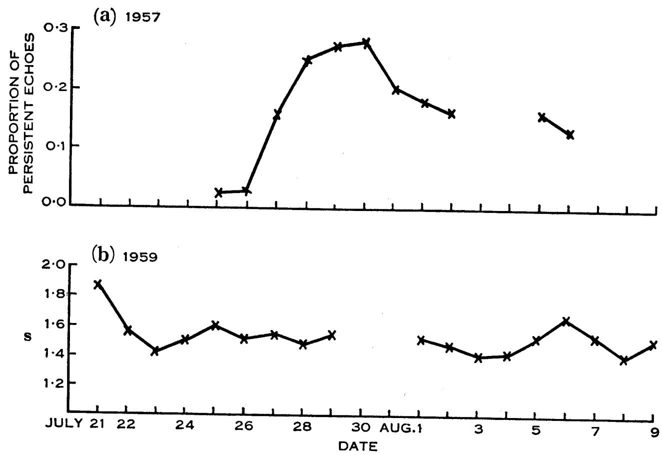

Fig. 7.—The increase in the proportion of bright meteors near maximum activity, δ-Aquarids. (*a*) Proportion of persistent echoes with durations $\geqslant 0\cdot 36$ s, $\alpha_{max.} \geqslant 10^{13}$/cm ; (*b*) s-values from relative echo rates, using equation (10), $\alpha_{max.} < 10^{12}$/cm.

VI. The Rate of Persistent Echoes

The form (3b) for the aerial aperture function W when $\alpha_{max.} > \alpha_t$ has been used in interpreting relative echo rates, and also echo durations distributions. It will be recalled that this involves an extrapolation of the numerical evaluation illustrated in Figure 1, which is itself an empirical derivation. Experimental verification of the form adopted for the function W is therefore desirable. This will be achieved by *a posteriori* arguments if it can now be shown that the proportion of persistent to all echoes is consistent with the expressions (3) and the s-values which have been derived from them through (5a) and (6).

Echo counts giving the proportion which echoes with durations $\geqslant 0\cdot 36$ s bear to the total numbers of echoes are listed in Table 3. Counts for the showers relate to the same dates as the data in Table 2. Sporadic counts were made in March, May, July, and December 1957.

As has already been pointed out in Section III (*e*), if expression (3b) holds then W is a slowly varying function of $\alpha_{max.}$ when $\alpha_0 \ll \alpha_i$ and $\alpha_{max.} > 10^{13}$/cm. This behaviour, arising as it does essentially from the weak dependence of echo amplitude upon line density amongst persistent echoes, is expected to be

TABLE 3

PROPORTIONS OF PERSISTENT TO TOTAL ECHOES

Source of Meteors	Total Echoes Detected	Persistent Echoes Detected	Proportion of Persistent Echoes	Effective Aerial Aperture Function $\overline{W}_T$
η-Aquarids	361	63	0·17	8·2
δ-Aquarids	1,177	269	0·23	11
Geminids	268	34	0·13	4·4
Sporadics	72,500	7,825	0·11	23

characteristic of any reasonable aerial polar diagram. It is therefore sufficient to put W=constant. This constant value of W, which has been denoted in (6) by $\overline{W}_T$, represents some average value of W for $\alpha_{\max.}>10^{13}$/cm. It may be calculated from the experimental data of Tables 1–3 in the following manner.

Let the theoretical number of echoes from meteors for which $\alpha_0 \leqslant \alpha_{\max.} \leqslant \alpha_T$ be denoted by ν_1. Writing s_1 for the value of s for this range of $\alpha_{\max.}$, and subtracting from (5a) the number of echoes from meteors with $\alpha_{\max.} \geqslant \alpha_T$ when $s=s_1$, we obtain

$$\nu_1=\frac{I_0\alpha_0^{1-s_1}}{(s_1-1)^2}\left\{1-\frac{3}{4}\left(\frac{\alpha_0}{\alpha_t}\right)^{s_1-1}\right\}-\frac{W_1 I_0 \alpha_T^{1-s_1}}{s_1-1}, \tag{11}$$

where

$$W_1=\ln(\alpha_t/\alpha_0)+\tfrac{1}{4}\ln(\alpha_T/\alpha_t)+1/4(s_1-1). \tag{12}$$

Now let ν_2 be the theoretical number of echoes from meteors for which $\alpha_{\max.} \geqslant \alpha_T$. In calculating ν_2 it is necessary to take into account the relative values of the ionization curve integrals I_0 and I_T, and also the changes in the s-values (Tables 1 and 2). For convenience it has been assumed that the changes in s take place discontinuously at α_T and α_2, which are the lower and upper limits of $\alpha_{\max.}$ in the fourth column of Table 2. For $\alpha_T \leqslant \alpha_{\max.} \leqslant \alpha_2$ we put $s=s_2$, and for $\alpha_{\max.} \geqslant \alpha_2$, we put $s=s_3$. In order to ensure continuity in the differential distribution (1), the value of the constant c must be adjusted at each change in s-value. The fitting conditions are

$$c_1\alpha_T^{-s_1}=c_2\alpha_T^{-s_2},$$
$$c_2\alpha_2^{-s_2}=c_3\alpha_2^{-s_3},$$

and with $c_1=1$ these become

$$c_2=\alpha_T^{s_2-s_1},$$
$$c_3=c_2\alpha_2^{s_3-s_2}.$$

Then

$$\nu_2=\overline{W}_T I_T \alpha_T^{s_2-s_1}\left\{\frac{1}{s_2-1}(\alpha_T^{1-s_2}-\alpha_2^{1-s_2})+\frac{1}{s_3-1}\alpha_2^{1-s_2}\right\}. \tag{13}$$

Denoting the observed proportion of persistent echoes by x, we have

$$\nu_1/\nu_2=(1-x)/x. \tag{14}$$

$\overline{W}_T$ is the only unknown occurring on the right-hand side of (11) and (13). It can therefore be calculated from (14) using the values of s_1, s_2, s_3, and α_T, α_2 already given in Tables 1 and 2, and x from Table 3. Values of $\overline{W}_T$ obtained with $\alpha_0=10^{11}$/cm, $\alpha_t=10^{12}$/cm, and $I_0/I_T=\sqrt{2}$ are listed in the last column of Table 3.

These values of $\overline{W}_T$ are to be compared with the theoretical values, slightly larger than 3, which are obtained from (12) with s_1 replaced by s_2. For showers, $\overline{W}_T$ exceeds the theoretical value by factors of from $1 \cdot 5$ to $3 \cdot 5$. This is not unexpected. As already mentioned, the values of s_1 given in Table 1 are upper limits. Further, trail distortion and consequent detection of trails not formed in the specularly reflecting condition will result in an increase in the effective aerial aperture for persistent echoes. Thus even if W is correctly represented by expressions (3a), (3b), experimental values of $\overline{W}_T$ should still be too high. In addition, any misalignment of, or errors in phasing of, the elements of the aerial arrays will widen the main lobe of the aerial beam, particularly at the fringes, and so still further increase the relative aperture for persistent echoes.

In the case of sporadic echoes, however, the excess of $\overline{W}_T$ over the theoretical value is uncomfortably large. None of the factors mentioned in the previous paragraph as increasing $\overline{W}_T$ for showers should be more effective for sporadic meteors. The values of s_1 for showers were obtained by comparison of shower with sporadic echo rates, assuming $s_1=2 \cdot 0$ for the latter. From the point of view adopted here, this value is exact (and well verified experimentally) and this source of error is eliminated. The incidence of trail distortion is identical amongst sporadic and shower meteors. The proportion of persistent echoes showing recognizable signs of amplitude or range disturbance is remarkably constant, being $0 \cdot 35$ for sporadics and δ-Aquarids, and $0 \cdot 36$ for Geminids and η-Aquarids. Finally, the aerial polar diagram is of course independent of the source of the meteors, and it is impossible that $\overline{W}_T$ should increase as s increases.

The only feasible explanation seems to be that persistent sporadic echoes are collected in minor aerial lobes which are not effective for shower echoes. Echo detection in minor lobes of low gain could markedly increase the proportion of persistent echoes without much affecting the total echo rate. Counts of persistent shower echoes were confined to intervals of time which extended little beyond the times of passage of the radiants through the main collecting areas of the equipments. Any minor lobes sufficiently removed in azimuth from the axis of the major lobe thus make no contribution to the persistent echo rates for showers, but are fully effective for sporadics where no time selection is involved. Collection in minor lobes in elevation is precluded by echo range discrimination. The theoretical polar diagram of the array predicts only two azimuthal minor lobes in which detection of persistent echoes could occur, but no measurements of the aerial polar diagram have been made at very low levels of gain. Examination of the diurnal pattern in the incidence of persistent echoes during the peak activity of the δ-Aquarid shower has established that there are no minor lobes comparable in gain with the main lobe, but this does not rule out the possibility that weaker minor lobes are present.

It is worth while pointing out that if, despite the arguments advanced above, it is assumed that (3b) gives the correct value of $\overline{W}_T$, the observed rate of persistent sporadic echoes implies a limiting line density for the equipment under normal operating conditions of $\alpha_0 \simeq 8 \times 10^{11}$/cm. This is quite incompatible with the $\alpha_0 \simeq 1 \times 10^{11}$/cm deduced from the calibrations already described.

The preceding discussion suggests that the experimental values of $\overrightarrow{W}_T$ are not excessively large. It follows that within the limits imposed by specular reflection, by neglect of trail distortion, and by an aerial pattern conforming to the theoretical polar diagram, the logarithmic form (3) is a satisfactory representation of the aerial aperture function. Amongst persistent echoes these restrictive conditions are relaxed and (3) undoubtedly understates the value of this function. Whilst this is of some consequence to the rate of persistent echoes, it exerts much less influence on the total echo rate if $\alpha_0 \ll \alpha_t$. The values of s given in Table 1 are therefore considered to be reliable.

VII. Discussion

It has been shown that, provided proper precautions are taken when interpreting rates of persistent echoes, relative echo rates measured at different equipment sensitivities can be used to obtain reliable determinations of the distribution of maximum electron line densities in meteor trails, and hence the mass distribution, over a wide range of brightness. The accuracy of these determinations depends on how well the aerial polar diagram is known. Results of acceptable accuracy are possible even if detailed knowledge of the aerial system is lacking.

Values of s for the Geminid shower agree well with determinations made over the same range of brightness by Browne *et al.* (1956) at Jodrell Bank, using different techniques. Sporadic meteors and the Geminid and δ-Aquarid showers all show increases in s as $\alpha_{max.}$ increases from 10^{11}/cm to 5×10^{13}/cm; the η-Aquarids show no increase in s over this range of $\alpha_{max.}$. For the first two showers the increase in s for $\alpha_{max.} > 5 \times 10^{13}$/cm is very steep indeed, so steep that it amounts effectively to a cut-off of meteors larger than the mass corresponding to $\alpha_{max.} \simeq 10^{14}$/cm. Sporadic meteors, on the other hand, do not appear to be subject to such a physical cut-off, and the value of $s=2 \cdot 5$ still applies for $\alpha_{max.} > 10^{15}$/cm, which corresponds to a radio magnitude of -3, according to the definition given by Kaiser (1955*b*).

One attractive feature of the present method is that it is quite insensitive to the ionization curve of the meteor trails, and so is largely independent of the incidence of fragmentation amongst the meteors. The only other method of measuring s for fainter meteors which has received adequate theoretical treatment is the height distribution method (Kaiser 1954; Weiss 1959). This is at a disadvantage in measuring distributions of meteor masses, as it is sensitive to the shape of the ionization curve and the dependence of the height of the trails on maximum line density. The agreement between the s-values for fainter Geminid meteors found from echo rates (this paper) and from height distributions (Browne *et al.*), both of which were obtained by comparison with $s=2 \cdot 0$ for sporadic meteors, can be taken to mean that Geminid meteors are subject to fragmentation to the same extent as sporadic meteors.

Similar comparisons of values of s derived for identical ranges of brightness from echo rates and from height distributions, for the same shower and preferably simultaneously, promise an easy means of estimating the incidence of fragmentation amongst shower meteors. Since the values of s are obtained by either method by comparison of shower data with similar data for sporadic meteors, it is the extent of the fragmentation relative to that for sporadic meteors which is really measured. Apart from the Geminids, s-values have been derived from echo rates for the δ-Aquarids and the η-Aquarids (this paper), and from height distributions for the Quadrantids, Perseids, and day-time Arietids (Browne *et al.* 1956). Measurements of s-values by the complementary methods for these five showers would complete the specification of the extent of the fragmentation amongst the fainter meteors of the more prominent permanent meteor showers. Owing to lack of contrast with the sporadic background it is doubtful whether useful results could be obtained for any of the remaining, weaker, permanent showers.

Of the six showers for which a value of s has been obtained, the day-time Arietids is the only one for which $s>2\cdot0$ for $\alpha_{max.}<10^{12}$/cm. According to Browne *et al.* (1956), for this shower $s=2\cdot7$ from the height distribution method. A possible interpretation of this result is that the distribution of maximum line densities is much the same as for the other showers, but that extreme fragmentation so narrows the height distribution that the anomalous s-value is found. The echo rate measurements needed to settle this point were attempted as part of the present programme, but were frustrated by low echo rates arising from the small elevation of the radiant at transit. It would be profitable to repeat these measurements in the northern hemisphere, where the radiant is much more favourably situated.

Another shower which would repay study is the periodic Giacobinid shower. The photographic measurements of Jacchia, Kopal, and Millman (1950) have shown that the brightest meteors (visual magnitudes $<+1$) belonging to this stream suffer an unusually large amount of fragmentation during their flight through the atmosphere. It would be interesting to ascertain whether such extreme fragmentation is also characteristic of the fainter meteors of this stream. Because of the high declination ($+55°$) of this radiant, the shower is invisible from Adelaide.

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