CONSTRUCTION OF THE EQUIVALENT ELECTRIC CIRCUIT FROM A FORCE CIRCUIT DIAGRAM BY A SIMPLE GRAPHICAL METHOD*

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It has been shown elsewhere (Tschoegl 1961) that mechanical problems may be formulated by constructing a force (or torque) circuit diagram setting forth the essential characteristics of the mechanical system by a set of conventional symbols. The mechanical problem may then be solved directly by the operational methods used in electric circuit analysis. Similarly, rheological problems may be formulated and solved by the aid of analogous stress circuit diagrams. The solution of mechanical and rheological problems via the equivalent electric circuit is thus completely by-passed. However, the equivalent circuit may still be needed for purposes of simulation. It is the aim of this communication to show how the equivalent electric circuit may be derived from a given force, torque, or stress circuit diagram by a simple graphical method. This is also the simplest way of setting up an equivalent circuit for a given mechanical problem since the force or torque circuit diagram is readily found by inspection.

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Consider the damped vibration absorber (Churchill 1958) represented in Figure 1. By inspection, simply tracing the actual connection of the elements and bearing in mind that force and mass are represented by 2-terminal symbols and that one terminal of the latter must always be referred to ground (Tschoegl 1961), we find the force circuit diagram as shown in Figure 2, disregarding the

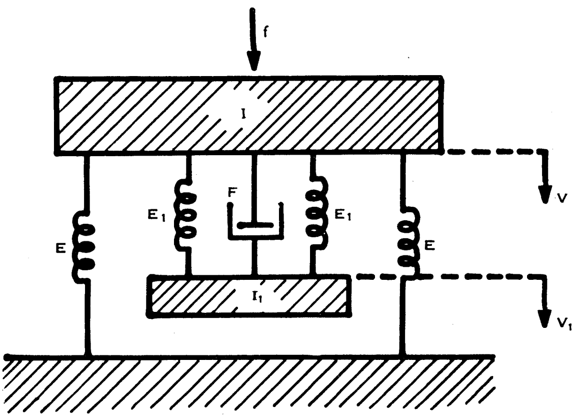

Fig. 1.—Damped vibration absorber.

dotted lines. $f(s)$ and $v(s)$ are the Laplace transforms of force and velocity, while Is, F, E/s, etc. are the mechanical impedance operators. The symbols are the usual representation of the inertance I by a mass, the frictance (mechanical, or viscous, resistance) F by a dashpot, and the elastance E (the reciprocal of the compliance J) by a spring. The circle represents applied force and the small

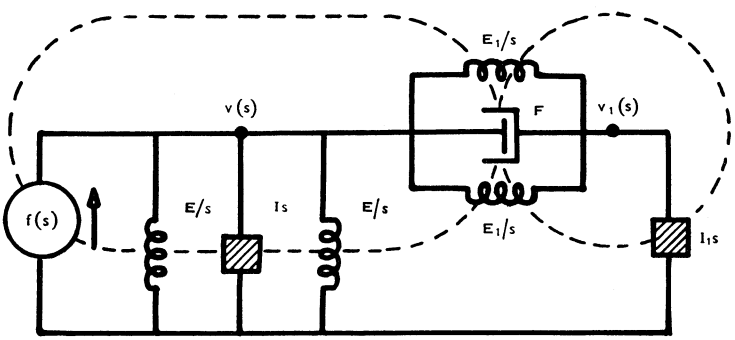

Fig. 2.—Force circuit diagram for damped vibration absorber.

full circles velocity nodes. The ground or reference node is indicated by the base line. The nodes shown therefore actually represent node-pairs. From the force circuit diagram we obtain by " node-pair analysis " the Laplace transformations of the equations of motion (for zero initial conditions) as follows:

$$[Is+2E/s+F+2E_1/s]v(s)-[F+2E_1/s]v_1(s)=f(s),$$
$$[I_1s+F+2E_1/s]v_1(s)-[F+2E_1/s]v(s)=0.$$

We now draw circles, as shown by the dotted lines in Figure 2, around each node shown, connecting all circuit elements attached to that node. Each circle represents a mesh of the equivalent electric circuit. Elements through which two circles pass are common to two meshes. The equivalent electric circuit can now be redrawn (Fig. 3) making the following substitutions:

Force Circuit	*Equivalent Electrical Circuit*
Inertance I	Inductance L
Frictance F	Resistance R
Elastance E	Reciprocal Capacitance $1/C$
Force f	Voltage V
Velocity v	Current i

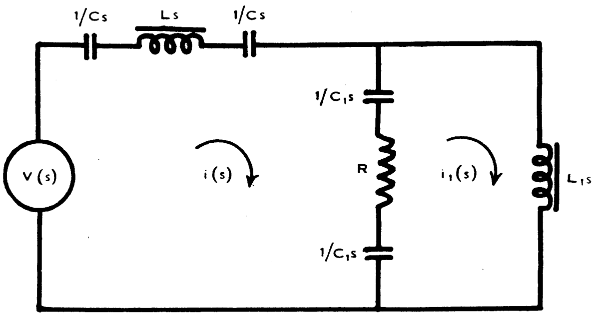

Fig. 3.—Equivalent electric circuit for damped vibration absorber.

From Figure 3 we find by the ordinary methods of " mesh-analysis ":

$$[Ls+2/Cs+R+2/C_1s]i(s)-[R+2/C_1s]i_1(s)=V(s),$$
$$[L_1s+R2/C_1s]i_1(s)-[R+2/C_1s]i(s)=0.$$

The method just outlined is related to the method for the graphical construction of the dual of an electric circuit (Bloch 1945; Gardner and Barnes 1953), originally proposed by Cauer (1934). In Cauer's method a dot is placed inside each mesh, another is placed outside the circuit, and the dots are connected by lines passing through the circuit elements. The dots represent the nodes of the dual circuit, the outside dot being the reference or ground node. Although it is formally possible to derive the original from the dual (and similarly the equivalent circuit from the force circuit diagram) by the same procedure, the method proposed here is the correct topological dual of Cauer's method.

References

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