A PHENOMENOLOGICAL MODEL FOR HYPERNUCLEAR BINDING ENERGIES*

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The form of the dependence of the binding energy of the Λ-particle in hypernuclei on the mass number A is of interest in obtaining empirical information about the hyperon-nucleon interaction. As an introductory calculation we considered the simple model in which the total Λ-nucleon interaction is replaced by a potential well $V(r)$ in which the Λ moves and in which the only effect of varying A is to vary the radius but not the depth of the well. The binding energy of the Λ, B_Λ, is then given by the ground state energy of a particle in this well. The aim of our calculations was to determine whether the present experimental values of B_Λ defined a unique well shape.

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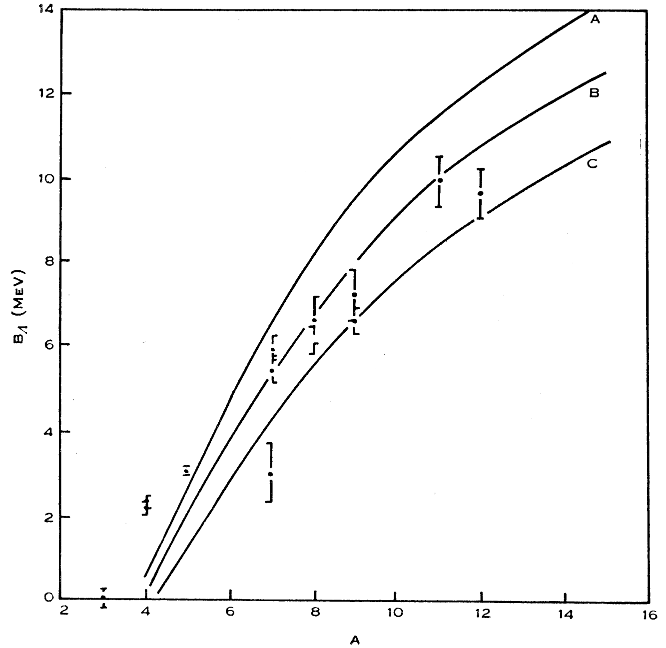

Fig. 1.—Square well, $r_0=1\cdot 1$ fermi. Curve A : $V_0=29$ MeV ; curve B : $V_0=27$ MeV ; curve C : $V_0=25$ MeV.

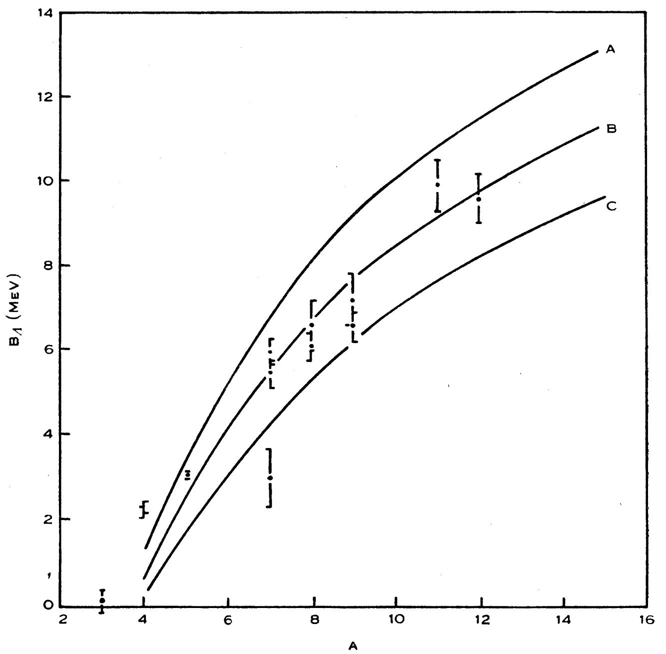

Fig. 2.—Square well, $r_0=1\cdot 3$ fermi. Curve A : $V_0=24$ MeV ; curve B : $V_0=22$ MeV ; curve C : $V_0=20$ MeV.

Various authors, for example, Ivanenko and Kolesnikov (1956) and Walecka (1960) have considered this model taking

$$\begin{aligned} V(r) &= -V_0 \quad \text{for } r<R=r_0(A-1)^{\frac{1}{3}}, \\ &= 0 \quad \text{for } r>R, \end{aligned}$$

where r_0 is a radius parameter. We shall refer to this as well (a). We considered this well and also the well (well (b)) given by

$$\begin{aligned} V(r) &= -V_0 && \text{for } r<R, \\ V(r) &= -V_0 \exp\{-(r-R)/0\cdot 7\} && \text{for } r>R, \end{aligned}$$

where we measure r, R in units of 1 fermi$=10^{-13}$ cm.

For well (a) we may find $B_\Lambda(V_0, A)$ by solving (e.g. Walecka 1960)

$$s=\frac{1}{\sqrt{(1-x)}}\cot^{-1}\left(-\sqrt{\frac{x}{1-x}}\right), \tag{1}$$

where

$$\begin{aligned} s &= (A-1)^{\frac{1}{3}}\left(\frac{1}{1+1\cdot 18/(A-1)}\right)^{\frac{1}{2}} r_0^2\left(\frac{2mV_0}{\hbar^2}\right)^{\frac{1}{2}}, \\ x &= B_\Lambda(V_0, A)/V_0. \end{aligned}$$

In passing it may be noted that the solution is found directly from tables if we rewrite (1) as

$$\frac{\sin\{s\sqrt{(1-x)}\}}{s\sqrt{(1-x)}}=\frac{1}{s}.$$

While well (b) is a more realistic well than well (a), $B_\Lambda(V_0, A)$ can only be found by numerical integration of the Schroedinger equation. This was done using the computer SILLIAC.

The results are plotted in Figures 1–4 and are presented here for general interest as a comparison and extension of Walecka's results. The experimental values are those of Ammar *et al.* (1960), which are approximately $0\cdot 36$ MeV higher than those used by Walecka, due to a different value of Q_Λ, the energy release in the π^--decay mode of the free Λ, and which are much more accurate than those of Ivanenko and Kolesnikov. Evans, Jones, and Zakrzewski (1959) have also reported a ${}_\Lambda B^{11}$ with $B_\Lambda=11\cdot 7\pm 0\cdot 8$ MeV, where to obtain this value the same Q_Λ as in Ammar *et al.* (1960) is used.

It is seen that the curve of best fit, Figure 4, is not oversensitive to variations in r_0 or to a change in the shape of the well. The situation would be improved if there were some experimental values of B_Λ with $A>12$, but some general, though tentative, observations can still be made.

The best fit to the experimental points is obtained with

$$\begin{aligned} \text{well } (a): &\quad V_0=27 \text{ MeV} \quad (r_0=1\cdot 1 \text{ fermi}), \\ &\quad V_0=22 \text{ MeV} \quad (r_0=1\cdot 3 \text{ fermi});* \\ \text{well } (b): &\quad V_0=21 \text{ MeV} \quad (r_0=1\cdot 1 \text{ fermi}). \end{aligned}$$

* Walecka obtains $V_0=21\cdot 7$ MeV with $r_0=1\cdot 3$ fermi, but this uses different experimental values and is equivalent to our $V_0=22$ MeV.

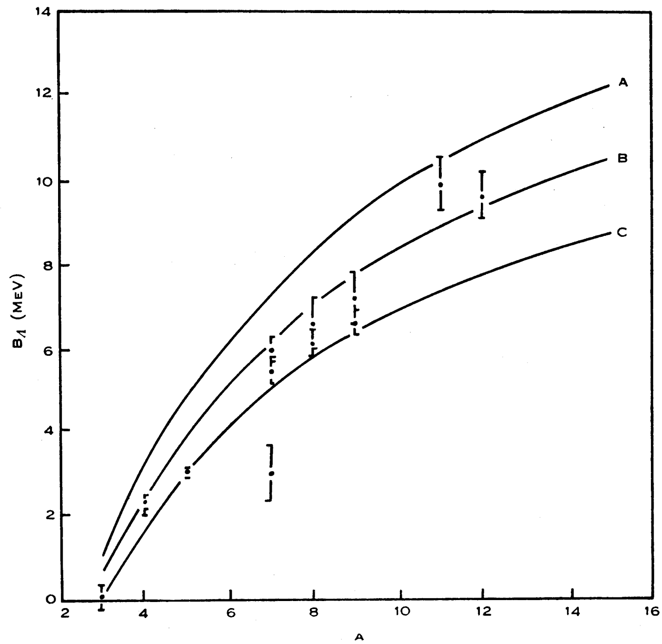

Fig. 3.—" Square " well with exponential sides, $r_0=1\cdot1$ fermi. Curve A : $V_0=23$ MeV ; curve B : $V_0=21$ MeV ; curve C : $V_0=19$ MeV.

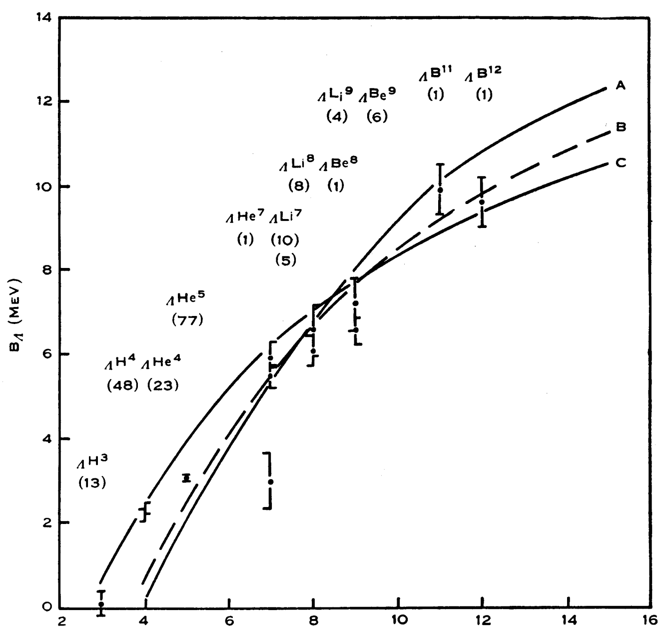

Fig. 4.—Comparison of curves of best fit. Curve A : square well, $r_0=1\cdot1$ fermi, $V_0=27$ MeV ; curve B : square well, $r_0=1\cdot3$ fermi, $V_0=22$ MeV : curve C : " square " well with exponential sides, $r_0=1\cdot1$ fermi, $V_0=21$ MeV. The number of events for each hyperfragment is shown.

It is seen that saturation is reached very slowly, e.g. for the square well with $r_0=1\cdot3$ fermi, $V_0=22$ MeV, $B_\Lambda=16\cdot6$ MeV for $A=50$ and $18\cdot5$ MeV for $A=100$.

In reality V_0 would be expected to decrease with decreasing A but to be almost constant for $A>7$, hence B_Λ will have been overestimated for small A, i.e. for small A the experimental values should lie below the curve. This indeed is the case for well (*b*), although here the predicted values for $A=11$, 12 may be too low. For well (*a*) no curve which gives a reasonable fit for higher A passes above the experimental points for small A. This is to be expected, since well (*a*) is a poor approximation to $V(r)$ for small A. It is also known that for nuclei of small A the radius parameter is approximately $1\cdot5$ fermi (cf. our values $1\cdot1$ and $1\cdot3$ fermi).

It would appear that the best fit is given by a well like well (*b*) but with a less gradual tail, although, as mentioned earlier, this depends on further experimental results. It can be seen that this model gives a surprisingly accurate agreement with all B_Λ except the exceptional case of $_\Lambda He^7$.

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