POLARIZATION IN NUCLEAR COLLISIONS WITH CHANNEL COUPLING

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Summary

The effect of the coupling between elastically and inelastically scattered waves on the polarization of particles scattered in nuclear collisions is considered for the simplest case of E0 excitations of the nucleus. The angular distribution of the polarization for elastic and inelastic collisions is found to be sensitive to the magnitude of the coupling parameter (the transition potential).

I. Introduction

With direct interaction established as the main mechanism operating in nuclear reactions at high energies, i.e. above the region where individual compound nucleus resonances play an independent part, there have been many calculations of differential scattering cross sections using the optical model and the Born approximation, either with simple plane waves or with distorted waves. With the inclusion of spin-orbit forces the work has been extended to the calculation of polarization produced in nuclear collisions.

Taking account of coupling between the elastically and inelastically scattered waves, however, causes appreciable changes in angular distributions (Mohr 1959; referred to later as paper I), and this throws doubt on the accuracy of nuclear parameters derived by fitting experimental angular distributions with curves calculated on the Born approximation.

In the present paper the channel coupling theory is extended to give the polarization in both elastic and inelastic collisions, for the simplest case of E0 excitations of the nucleus.

II. Theory

We use the obvious notation of paper I, suitably extended below, and the same simplifying assumptions to make the problem tractable, namely,

(i) the incident particles uncharged and the target nucleus spinless,

(ii) the excitation energy small compared with the incident energy E,

(iii) the nuclear potential $-V_{11}$ for the inelastically scattered particles the same as the nuclear potential $-V_{00}$ for the incident and elastically scattered particles,

(iv) the transition potential $-V_{01}$ spherically symmetrical (E0 excitations).

We introduce a spin-orbit term of the usual Thomas type to give the nuclear potential

$$-V_{00}+\gamma(h/2\pi mc)^2 r^{-1}(\mathrm{d}V_{00}/\mathrm{d}r)\mathbf{l}\cdot\mathbf{s},$$

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where m is the mass of the pion, and $\mathbf{l}\cdot\mathbf{s}=l$ and $-(l+1)$ for $j=l+\frac{1}{2}$ and $l-\frac{1}{2}$ respectively. We take for $-V_{00}$ a square well of radius R, and then the spin-orbit term has the form of a Dirac δ function at $r=R$.

If we disregard channel coupling and spin-orbit coupling, the interior solution $F_l(r)$ of the radial wave equation for the incident and elastically scattered waves of order l will be a wave acted on by the attractive potential $-V_{00}$ and having at infinity a phase shift η_l.

With channel coupling, the coupled equations for the elastically and inelastically scattered waves $F_l^0(r)$ and $F_l^1(r)$ respectively have interior solutions $F_l^+(r)$ and $F_l^-(r)$ corresponding to waves in potential wells of depth $V_{00}+V_{01}$ and $V_{00}-V_{01}$ respectively. If these two solutions are extended beyond the nuclear boundary to large distances, they will have phase shifts η_l^+ and η_l^- respectively. Then the general solutions are (paper I, equations (13) and (15)) :

$$F_l^0=LF_l^+ + MF_l^-, \tag{1a}$$
$$F_l^1=LF_l^+ - MF_l^-, \tag{1b}$$

where L and M are numerical coefficients.

If the spin-orbit potential, of δ function form at $r=R$, is now allowed to act on the two coupled interior waves as they reach the distance R, the slopes of each wave are changed at this point by amounts proportional to l and $-(l+1)$ for $j=l+\frac{1}{2}$ and $l-\frac{1}{2}$ respectively. If the four resulting waves F_l^{++}, F_l^{+-}, F_l^{-+}, and F_l^{--} are now integrated out to infinity, they will have phase shifts η_l^{++}, η_l^{+-}, η_l^{-+}, and η_l^{--}, where the second suffix $+$ or $-$ refers to the case of $j=l+\frac{1}{2}$ or $l-\frac{1}{2}$ respectively. These phases are most readily obtained in the usual way by fitting each of the four interior solutions at $r=R$ smoothly to the general exterior form

$$\cos\eta_l j_l(kr)+(-)^l \sin\eta_l n_l(kr),$$

and then

$$F_l \sim (kr)^{-1}\sin(kr-\tfrac{1}{2}l\pi+\eta_l). \tag{2}$$

Taking into account the angular dependence of the lth order wave functions, the four interior solutions have to be combined in the following proportions (Mott and Massey 1949) :

$$\left.\begin{array}{ll}(l+1)F_l^{++}P_l(\cos\theta), & lF_l^{+-}P_l(\cos\theta)\\ (l+1)F_l^{-+}P_l(\cos\theta), & lF_l^{--}P_l(\cos\theta)\end{array}\right\}\text{ for }\psi_{3el}, \tag{3a}$$

$$\left.\begin{array}{ll}-F_l^{++}P_l^1(\cos\theta)\exp i\varphi, & F_l^{+-}P_l^1(\cos\theta)\exp i\varphi\\ -F_l^{-+}P_l^1(\cos\theta)\exp i\varphi, & F_l^{--}P_l^1(\cos\theta)\exp i\varphi\end{array}\right\}\text{ for }\psi_{4el}, \tag{3b}$$

and a similar combination for ψ_{3in} and ψ_{4in}, with opposite signs for the terms in F_l^{-+} and F_l^{--} on account of (1).

ψ_3 and ψ_4 are the " large components " of the four-component Dirac wave function describing the elastically or inelastically scattered waves, denoted respectively by the suffixes " *el* " and " *in* ". They have the asymptotic form :

$$\psi_{3el}\sim e^{ikz}+r^{-1}e^{ikr}f_{el}(\theta), \tag{4a}$$
$$\psi_{4el}\sim r^{-1}e^{ikr}g_{el}(\theta), \tag{4b}$$
$$\psi_{3in}\sim r^{-1}e^{ikr}f_{in}(\theta)\exp i\varphi, \tag{4c}$$
$$\psi_{4in}\sim r^{-1}e^{ikr}g_{in}(\theta)\exp i\varphi, \tag{4d}$$

with the z axis taken along the incident beam, where

$$f_{el}=\Sigma_l k^{-1}(2l+1)\alpha_l P_l\,(\cos\theta),\qquad f_{in}=\Sigma_l k^{-1}(2l+1)\beta_l P_l\,(\cos\theta), \tag{5a}$$
$$g_{el}=\Sigma_l k^{-1}(2l+1)\alpha_l^s P_l^1\,(\cos\theta),\qquad g_{in}=\Sigma_l k^{-1}(2l+1)\beta_l^s P_l^1\,(\cos\theta). \tag{5b}$$

Taking linear combinations of the four interior solutions (3) for each of ψ_{3el}, ψ_{4el}, ψ_{3in}, and ψ_{4in} in their asymptotic form (2), and equating them to the corresponding asymptotic form (5) of the exterior solutions (4), we have

$$a(l+1)F_l^{++}+blF_l^{+-}+c(l+1)F_l^{-+}+dlF_l^{--}=\mathrm{i}^l(2l+1)\{\sin(kr-\tfrac{1}{2}l\pi)+\alpha_l\exp\mathrm{i}kr\}, \tag{6a}$$
$$-aF_l^{++}+bF_l^{+-}-cF_l^{-+}+dF_l^{--}=\mathrm{i}^l(2l+1)\alpha_l^s\exp\mathrm{i}kr, \tag{6b}$$
$$a(l+1)F_l^{++}+blF_l^{+-}-c(l+1)F_l^{-+}-dlF_l^{--}=\mathrm{i}^l(2l+1)\beta_l\exp\mathrm{i}kr, \tag{6c}$$
$$-aF_l^{++}+bF_l^{+-}+cF_l^{-+}-dF_l^{--}=\mathrm{i}^l(2l+1)\beta_l^s\exp\mathrm{i}kr, \tag{6d}$$

where, for brevity of notation, the F's here denote their asymptotic forms (2), and a, b, c, and d are arbitrary (complex) coefficients. The correct combination of signs before the F's in (6a) and (6c) is indicated by (3a) and (1), and the combination of signs in (6b) and (6d) follows from (3b); but this result may be confirmed by employing the alternative method of projection operators used in the weak coupling case (Lepore 1950). A further set of four similar equations may be obtained by equating the *slopes* of the wave functions at the nuclear boundary.

We thus have eight equations to determine the eight unknown constants a, b, c, d, α_l, α_l^s, β_l, and β_l^s, though only the last four interest us. The equations may be solved without difficulty, using the properties of determinants, to give:

$$\alpha_l=\{(l+1)(E^{++}-1)+l(E^{+-}-1)+(l+1)(E^{-+}-1)+l(E^{--}-1)\}/4\mathrm{i}(2l+1), \tag{7a}$$
$$\beta_l=\{(l+1)(E^{++}-1)+l(E^{+-}-1)-(l+1)(E^{-+}-1)-l(E^{--}-1)\}/4\mathrm{i}(2l+1), \tag{7b}$$
$$\alpha_l^s=(-E^{++}+E^{+-}-E^{-+}+E^{--})/4\mathrm{i}(2l+1), \tag{7c}$$
$$\beta_l^s=(-E^{++}+E^{+-}+E^{-+}-E^{--})/4\mathrm{i}(2l+1), \tag{7d}$$

where $E^{++}=\exp 2\mathrm{i}\eta_l^{++}$, etc.

The polarization P for either elastic or inelastic collisions is given by

$$P=(fg^*-f^*g)/\mathrm{i}(ff^*+gg^*), \tag{8}$$

where the values of f and g are given by (5a) and (5b) with (7).

In the limiting case of zero coupling between the two channels ($V_{01}=0$), we have $\eta_l^{++}=\eta_l^{-+}=\eta_l^{0+}$ (say), $\eta_l^{+-}=\eta_l^{--}=\eta_l^{0-}$ (say), and (7) reduce to the well-known results (Mott and Massey 1949):

$$\alpha_l=\{(l+1)(\exp 2\mathrm{i}\eta_l^{0+}-1)+l(\exp 2\mathrm{i}\eta_l^{0-}-1)\}/2\mathrm{i}(2l+1), \tag{9a}$$
$$\alpha_l^s=(-\exp 2\mathrm{i}\eta_l^{0+}+\exp 2\mathrm{i}\eta_l^{0-})/2\mathrm{i}(2l+1), \tag{9b}$$
$$\beta_l=0=\beta_l^s. \tag{9c}$$

In the limiting case of zero spin-orbit coupling, $\eta_l^{++}=\eta_l^{+-}=\eta_l^{+0}$ (say), $\eta_l^{-+}=\eta_l^{--}=\eta_l^{-0}$ (say), and (7) reduce to

$$\alpha_l=(\exp 2\mathrm{i}\eta_l^{+0}+\exp 2\mathrm{i}\eta_l^{-0}-2)/2\mathrm{i}, \tag{10a}$$
$$\beta_l=(\exp 2\mathrm{i}\eta_l^{+0}-\exp 2\mathrm{i}\eta_l^{-0})/2\mathrm{i}, \tag{10b}$$
$$\alpha_l^s=0=\beta_l^s. \tag{10c}$$

Some remarks are necessary about the possibility of eliminating the simplifying assumptions made in the above theory. The elimination of the assumptions $k_1=k$ and $V_{11}=V_{00}$ still allows the general method of paper I to be extended to the problem of polarization, but the determinants obtained in the solution of (6) no longer reduce to simple expressions. The use of rounded-off potential wells for V_{00} and V_{01} causes the spin-orbit potential to overlap the potentials V_{00} and V_{01}, and a clean-cut separation of the effects of channel coupling and spin-orbit coupling on the interior wave functions is no longer possible: spin-orbit terms must be included in the coupled equations themselves, and this

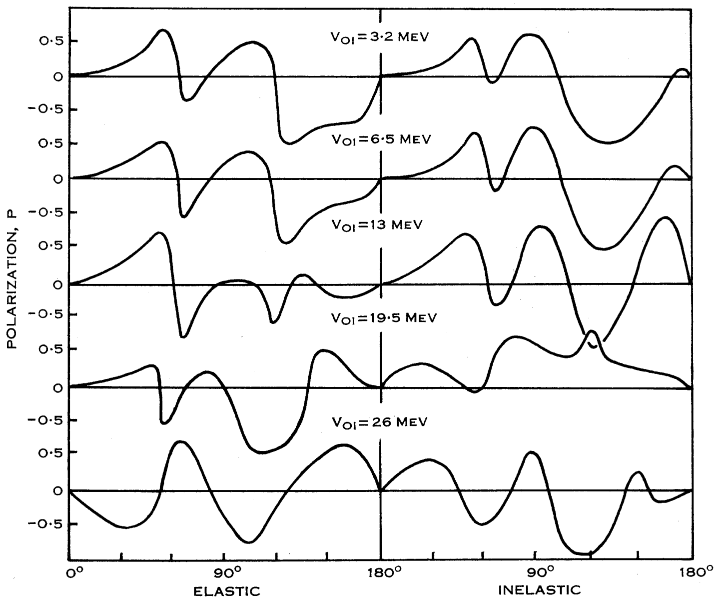

Fig. 1.—Angular distribution of the polarization of 18 MeV neutrons after elastic or inelastic collision with a light nucleus represented by a square well of radius $R=2\cdot9$ fermi and depth $V_{00}=47$ MeV, and spin-orbit coupling parameter $\gamma=0\cdot17$, for various values of the spherically symmetrical transition potential V_{01} indicated on the curves.

complicates the analysis. The consideration of excitations of higher multipolarity than E0 adds very greatly to the complexity of the problem, even with the neglect of spin-orbit forces (Mohr 1961), while the introduction of target nucleus spin will introduce further difficulty.

In view of all these difficulties and the fact that cases of strong E0 transitions are not found in the experiments, no direct comparison with experiment of theoretical angular distributions of polarization calculated with channel coupling can be attempted at the present time. Our purpose has been to estimate the magnitude of the effect of channel coupling on polarization, and to show that in general the effect is too large to be disregarded.

III. Results and Discussion

The parameters chosen for numerical calculations were $R=2\cdot9$ fermi (corresponding to carbon), $V_{00}=V_{11}=47$ MeV, incident energy $E=18$ MeV, as in paper I, in order to permit comparison of the effect of channel coupling on the polarization and on the scattering cross section. In paper I it was found that the magnitudes of β_0, β_1, β_2, and β_3 reached their maximum value of $0\cdot5$ for values of V_{01} of 15, 23, 25, and 5 MeV respectively, so in this work we took values of V_{01} between 0 and 26 MeV. The values chosen for the spin-orbit parameter were taken in the range of values used in Born approximation calculations for a light nucleus like carbon.

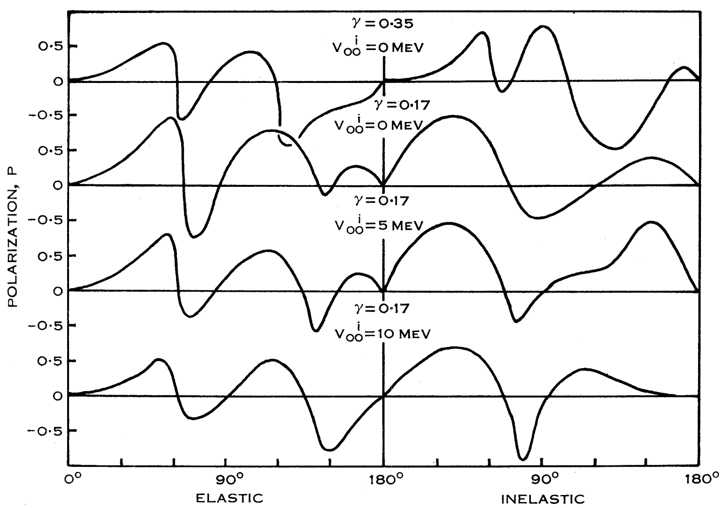

Fig. 2.—Angular distribution of the polarization of 18 MeV neutrons, for the same square well as for Figure 1. The top two curves show the effect of altering the spin-orbit coupling parameter γ. The bottom three curves show the effect of introducing increasing amounts of an imaginary component V^i_{00} and V^i_{11} into the nuclear potentials V_{00} and V_{11}.

The effect of varying the channel coupling is shown in Figure 1. The smallest two values of the transition potential V_{01} give practically the same angular distribution of the polarization, and this is not unexpected, since the form of the distribution will tend more and more closely to that given by the Born approximation as V_{01} tends to zero. The curves for $V_{01}=19\cdot5$ and 26 MeV are considerably different, however, especially at the larger angles.

The effect of increasing the spin-orbit coupling parameter γ from $0\cdot17$ to $0\cdot35$ is shown by the top two curves in Figure 2, a marked change being produced in the inelastic polarization. The effect of introducing an imaginary component $V^i_{00}=V^i_{11}$ into the nuclear potentials V_{00} and V_{11} to allow for nuclear absorption of the incident particles is shown in the bottom three curves of Figure 2. The inelastic polarization is seen to be very sensitive to the value of V^i_{00}, the elastic polarization much less so.

It appears, then, that the polarization is as sensitive to the channel coupling as the scattering, for E0 excitations. For E2 excitations, however, the scattering is even more sensitive (Mohr 1961), and it is likely that the polarization will similarly be more sensitive.

Channel coupling therefore cannot be ignored, and it introduces still another variable into this work. Any attempt to use a less crude model for channel coupling than that considered here, and to alter the many adjustable nuclear parameters to obtain a unique fit with experimental curves would seem to require a prohibitive amount of work.

IV. References

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