SYSTEMS FOR SIMULTANEOUS IMAGE FORMATION WITH RADIO TELESCOPES

By D. J. McLean* and J. P. Wild*

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Summary

The large arrays of aerials often necessary for high resolution in radio astronomy are generally used to observe a brightness distribution in an area of the sky by scanning the aerial beam, point by point, across the area. In this paper we investigate the possibility of observing the complete distribution simultaneously over a given area. Two possible analogue systems capable in principle of forming such images, are described; in one, the analogue image is formed optically, in the other with ultrasonic methods. The problems and limitations of the two systems are examined and it is concluded that further technical developments are required before practical systems with useful sensitivity and resolution can be achieved.

I. Introduction

The recording of radio brightness distributions in the sky is the ultimate observational aim of many radio astronomical investigations. For a variety of reasons it may be desirable to make these observations rapidly—for instance to avoid the effects, important at metre wavelengths, due to variations in the ionosphere; to observe variable sources, such as solar or Jovian bursts; or simply to collect a large amount of data in a reasonable period of time. In the present paper we aim to investigate a number of possible rapid systems, especially as they might be applied to the observation of solar bursts. However, the generalization to other situations is obvious.

More precisely, we aim to find a system capable of observing the Sun, quiet or active, at metre wavelengths in a time comparable with the time scale of the fastest observed bursts. We can take this time to be about one second. The resolution required to observe details of the Sun at metre wavelengths makes a full aperture system, such as a steerable paraboloid reflector or even a circle filled with closely spaced dipoles, quite impracticable (2 km would be a typical diameter). Instead we must use what we term a *dilute* aerial array, such as the Mills Cross (Mills and Little 1953) or circular-array (Wild 1961). With both aerial systems, sharp " pencil " beams emerge as the result of taking the difference between two directivity patterns, both possessing widespread side lobes but only one having appreciable response in the central beam. In the course of the subtraction process, the mean *power* of signals received in the widespread side lobes cancel one another but the noise *fluctuations* do not. Hence with neither system is it possible to improve the signal-to-noise ratio beyond a certain limit fixed by radiation from the surrounding Sun and cosmic background entering the side lobes. The result is that the observations must be integrated,

* Division of Radiophysics, C.S.I.R.O., University Grounds, Chippendale, N.S.W.

at each point, over a period which ranges from about $0 \cdot 1$ s to several seconds depending on the bandwidth of the associated receivers and the image quality required. This is of the same order as the total time available. It appears therefore that the most satisfactory solution to our problem would be a system which produces and integrates an image of the whole distribution simultaneously, as a lens and film do for the optical astronomer. The requirements are similar to those which led to the development of the iconoscope for television.

In what follows we shall confine our discussion largely to the simple case of a circular (i.e. annular) array. Most of the results can be applied in principle to any other suitable array.

II. Conceivable Systems

One way to scan the beam of an aerial array across a source is to vary the lengths of line connecting the different elements of the array to the receiver. In the same way it is possible to observe in several different directions with the same array by means of several receivers joined to the elements of the array through appropriate lengths of line. In principle this leads to a possible solution of the problem : all points of the distribution can be observed simultaneously by having a separate receiver and detector to observe each point. However, since a useful picture requires observations at each of several thousand independent points and hence several thousand receivers, this system is scarcely practicable as it stands.

It is possible, however, to use a simpler system consisting of a much smaller number of receivers, the beams of which are swept together along parallel lines to observe the distribution in a number of simultaneous parallel scans. Under certain circumstances, this may represent an adequate compromise between rapidity of observation and complexity of equipment.

Again, in principle, the intensity distribution could be calculated with a digital computer from the instantaneous amplitudes and phases of the signals at the various aerials. However, for an array of 10^2 elements and a bandwidth of 10^6 c/s, this computer would have to accept about 10^8 amplitudes per second, calculate about 10^6 intensity distributions per second (each distribution consisting of 10^4 independent points, a total of 10^{10} calculated values of intensity per second), and then average these 10^6 intensity distributions to supply an output of 10^4 values of intensity per second.

These numbers are so great that we must surely reject the idea of a digital computer and seek some sort of analogue computer which will not be daunted by the enormous rate at which information must be handled. The analogy which interests us most is the obvious one of an optical telescope and a photographic plate. This is developed in the next section.

III. An Optical Analogue System

We wish to observe the intensity distribution which would be observed by scanning the beam of the circular array (shown in Fig. 1 (*a*)) across the source. This distribution is the same as that produced at the image plane of the hypothetical arrangement shown in Figure 1 (*b*). Basically the arrangement consists of a lens which focuses the radio waves on an image plane. To make the image identical* with the distribution observed by scanning the source with the annular array of discrete aerials, an opaque screen is placed in front of the lens with holes cut in the screen at the positions occupied by the aerials of the array.

The system of image formation depicted in Figure 1 (*b*) is possible at optical wavelengths but becomes quite impracticable at radio wavelengths of several metres. Figure 2 shows a proposed equivalent system, in which we convert the received radio signals to light waves, which are then focused to form a complete image in monochromatic light of the radio brightness distribution. In Figure 2

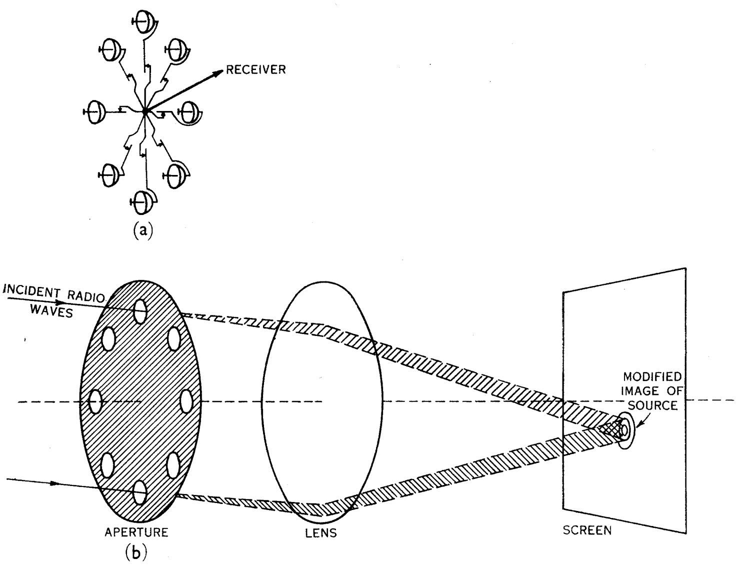

Fig. 1 (*a*).—Circular array of radio aerials, capable of giving an image by point-by-point scanning.
Fig. 1 (*b*).—Hypothetical system for focusing radio waves to yield an image identical with that of (*a*).

the undetected radio-frequency signals from each aerial are used to modulate the transmission and phase of an optical aperture or *pupil* in step with the amplitude and phase of the radio signals. The pupils are arranged on the otherwise opaque screen in the same pattern as the corresponding aerials.

The *pupil array* is illuminated with a parallel beam of light (from a point source and collimating lens) and the light which passes through the pupil array is focused on the image plane. Since the light, after passing through the pupil array, has the same amplitude and relative phase as the radio waves incident on the aerials, the image formed is the same as that formed in the hypothetical system of Figure 1 (*b*) as desired. The only effect of varying the scale of the pupil array is to vary the scale of the image.

* The proof of the identity of the two image distributions in the case of a point source follows at once from the well-known equivalence of the polar diagram of an aerial system and the diffraction pattern of its aperture.

It is possible at least in principle to build up a pupil which will produce the desired effect. One such pupil, using interference between beams reflected from piezo-electric crystals, driven by the radio signals, is shown in Figure 3. It should be noted that a few of the critical dimensions, especially those in the pupils, must be maintained accurately, to a small fraction of a wavelength. Also, the maximum bandwidth of the radio-frequency circuits is limited (but not severely) by dispersion effects in the system. On the other hand, it is permissible to use frequency changing between the aerials and the pupils, provided only that the new frequency is greater than the bandwidth.

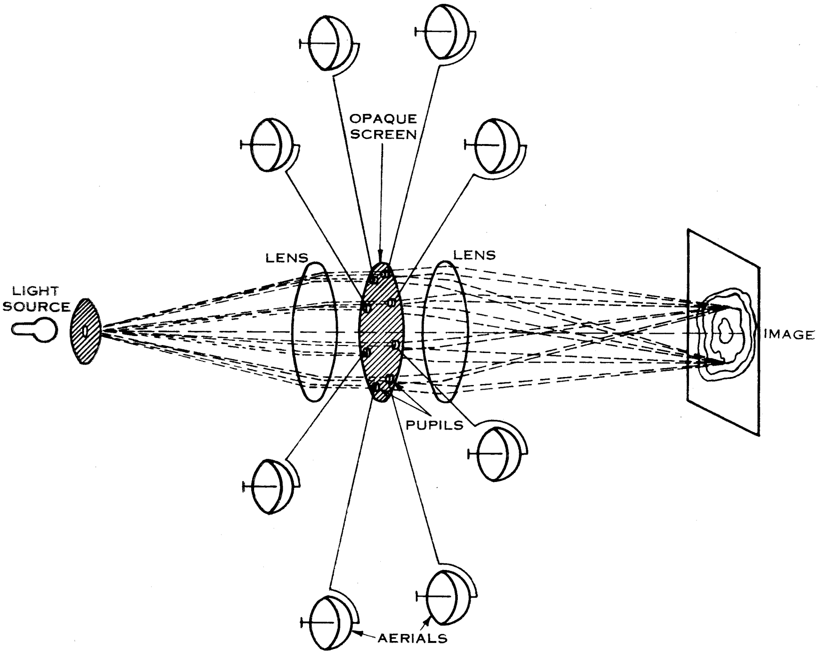

Fig. 2.—Principle of the optical system of forming an image of radio brightness. Note the analogy with Figure 1.

It is interesting to note the analogy between the optical system of Figure 2 and the system containing a large number of receivers or detectors fed through different lengths of feeder (see Section II). In the optical system the detectors are replaced by small areas of photographic emulsion and the lengths of feeders by the optical paths from the pupils to the different points of the image plane.

Now the image yielded directly by a circular array or annular aperture is distorted and requires modification or *correction* to suppress unwanted diffraction or side-lobe effects. The correction process proposed by Wild (1961) consists of cross-correlating the recorded image with a special two-dimensional correction function, which has been obtained analytically; the corrected image is then identical with that which would be given by a uniform aperture having the same radius as the annular aperture used. The correction function assumes both positive and negative values and so the process involves more than simple smoothing.

The process of correction is desirably incorporated in the observing instrument so that the recorded images are fully corrected. Fortunately, the optical system described above can be simply adapted to function in this way, since to cross-correlate the image with any given two-dimensional function G, one merely replaces the point source of light by an extended light source whose intensity distribution is given by G. Essentially this arrangement has been employed by McLean (1961) to demonstrate the feasibility of the correction process.

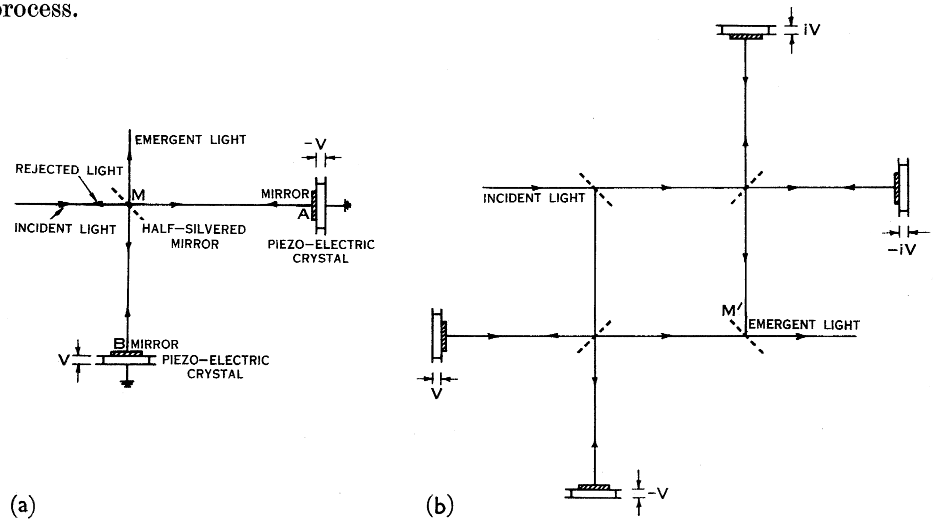

Fig. 3.—Illustrating one possible design for a pupil by which the phase and amplitude of an applied radio-frequency signal is imposed on a coherent beam of monochromatic light. The complete unit is shown in (*b*) ; it consists of two amplitude modulators of the type (*a*) operated in phase quadrature. In (*a*), the rays reflected from A and B interfere at the half-silvered mirror with phases which are varied by displacements of the mirrors A and B. These displacements are controlled by applying radio-frequency voltages across the pair of piezo-electric crystals. The two crystals are fed in anti-phase and, with proper adjustment (i.e. paths for zero volts differing by $\lambda/2$), the amplitude of the emergent beam is proportional to the instantaneous voltage (if not too large) while the phase is constant. In (*b*), the two pairs of crystals are fed in phase quadrature and it may easily be shown that, if the two modulated light beams are also combined in phase quadrature, the emergent beam possesses both the relative amplitude and phase of the applied signal.

In Figure 4 we show one complete arrangement which includes the correction process in the optical system. It is the same as Figure 2 except that the point source of light is replaced by two widely-separated extended sources, in one of which the brightness distribution is given by the positive part of the correction function, in the other by the negative part. These produce two non-overlapping image distributions ; the difference between these two distributions is the required distribution. The difference is obtained by obscuring alternate strips of both images with coarse gratings, then combining the two images optically so that the unobscured parts of one image fall on the obscured parts of the other, and vice versa. If this interlaced image is now scanned at right-angles to the grating by a TV image tube, the output will contain an alternating component proportional to the required difference.

Unfortunately, this optical system falls short of ideal owing to the noise introduced by the system itself. It can be seen, for instance, that most of the available light falls on the opaque screen of Figure 4, while the light which passes through the pupil array must be spread over an area considerably larger than that of the useful image. The image is therefore very weak and so fluctuations in the number of quanta striking a given area will introduce noise. This effect is enhanced by the correction process, in which the small difference between two

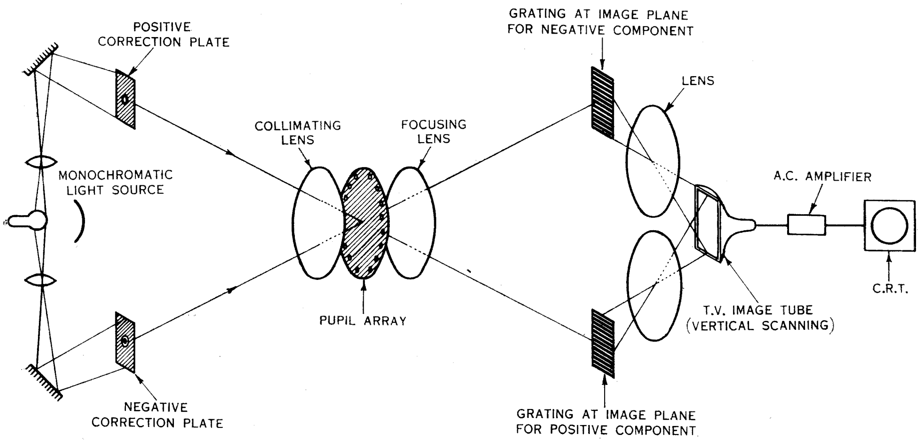

Fig. 4.—The complete optical system with the correction process included.

almost equal intensities is taken. We have calculated that the signal-to-noise ratio of the system expected for the final image is given by*

$$\frac{\text{signal}}{\text{noise}} \simeq \left(\frac{\alpha k \gamma \lambda^3 \overline{B}\, T}{hcn^5} \right)^{\frac{1}{2}}, \tag{1}$$

where k is the efficiency of a pupil (estimated to be $k \simeq 0 \cdot 2$),

γ is the efficiency of all other optical parts ($\gamma \simeq 0 \cdot 5$),

α is the quantum efficiency of the optical detector ($\alpha \simeq 0 \cdot 1$ for a sensitive TV image tube),

λ is the wavelength of light to be used ($\lambda \simeq 5 \times 10^{-5}$ cm),

$\overline{B}$ is the mean brightness of the positive part of the correction plate ($\overline{B} \simeq 0 \cdot 75 \times 10^8$ erg cm^{-2} steradian^{-1} ($=0 \cdot 5 \times 10^4$ candles cm^{-2}) for one line of a bright mercury arc),

T is the integration time in seconds,

n is the number of array elements or pupils,

c is the velocity of light,

h is Planck's constant.

* This formula assumes that, for a given diameter, d, of the complete pupil array, the individual pupils are made as large as possible. It is interesting to note that the result is independent of the magnitude of d. This is because an increase in d (and hence the area of the aperture) demands that the solid angle subtended by the correction plate at the aperture must be correspondingly reduced.

Substituting the numerical values suggested above yields

$$\frac{\text{signal}}{\text{noise}} \simeq 2 \cdot 4 \times 10^4 T^{\frac{1}{2}} n^{-5/2}. \tag{2}$$

It has been shown (Wild 1961) that the maximum resolution with which a given area of the sky can be observed is proportional to n.* Equation (2) therefore indicates the compromise which must be made between resolution and the effects of noise introduced by the optical imaging system when the brightest available light source is used. For instance, if we require a signal-to-noise ratio of 30 and an integration time of 1 s, then we must have $n < 15$, which implies severely restricted resolution. Without introducing further elaborate complexities into the optical system, we believe that this limitation can only be overcome by the use of much brighter light sources than are currently available. It is possible that the development of light amplifiers such as optical masers could provide the answer.

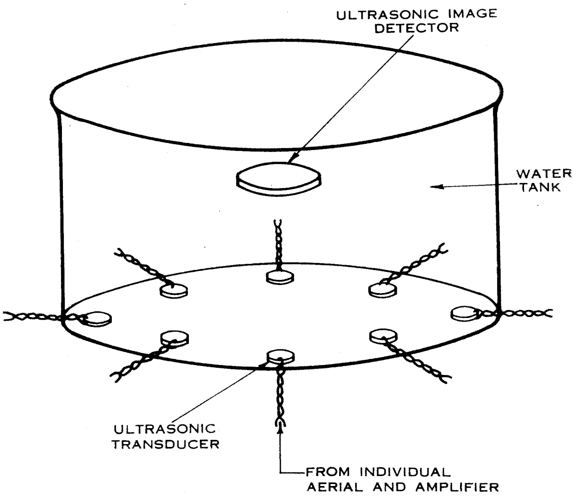

Fig. 5.—Showing the basic arrangement of an ultrasonic system.

IV. An Ultrasonic System

We have seen that with currently available techniques the optical system has two main drawbacks ; firstly, the poor signal-to-noise ratio of the image, which is essentially the result of the inefficient use of the light flux generated by the source ; and secondly, the great technical difficulties to be expected in the construction of the pupils—a result of the extremely short wavelength of light. The first of these problems could be overcome if the point source and pupil array

* This is because a ring of n discrete aerials introduces a set of side lobes into the polar diagram which are not present in the polar diagram of a continuous annulus. If these side lobes are not to introduce highly undesirable effects the diameter of the aerial array must be small enough to keep the secondary side lobes outside the area in which it is expected to observe strong sources. This of course restricts the available resolution when a field of given angular area is viewed with a system containing a given number of aerials.

were replaced by an array of coherent, modulated sources; the emitted waves may be of any physical kind provided their interference pattern is capable of detection. Nearly all the radiated intensity can then be used in the formation of the image, and so the efficiency is enormously improved. However, the abandonment of the system of Figure 4 would mean that a different means of aerial correction would have to be incorporated, presumably at a later stage of the system. The second problem may be directly overcome by the choice of waves of convenient wavelength (e.g. 1 mm).

Systems incorporating both the above modifications are conceivable with both microwaves and ultrasonic waves, although in both cases the problem of image detection is clumsy or requires techniques which have not yet been properly developed. Of the two, the ultrasonic system seems simpler and the remaining discussion will be restricted to it.

Figure 5 shows a possible arrangement using ultrasonic waves. In place of a pupil array we have an array of quartz transducers. After amplification and coherent frequency changing* (but before detection), the signals from the aerials are used to drive the transducers. Here we see one of the advantages of a circular array: instead of using a lens, it is only necessary to tilt the crystals towards the centre of the desired image. The image is then formed as a distribution of ultrasonic intensity across a plane in the medium (e.g. water) surrounding the crystals.

There remains the problem of detecting or rendering visible such a distribution in a suitable manner. A detection system is required to fulfil the same function as the photographic plate or TV pick-up tube of the optical system: the system must be one in which the image is integrated simultaneously over the whole picture, it must be quantitatively faithful in reproduction, and it must be rapid enough to register in periods of the order of one second. The various methods which have so far been developed for observing ultrasonic images scarcely satisfy these requirements.

While the ultrasonic system must await the possible development of a suitable detector, its potential advantages over the optical system are very great. Firstly, no fundamental limitations in detection sensitivity exist, so that the corresponding restrictions on resolution are removed; secondly, with wavelengths of $\sim$1 mm the constructional problems are much less exacting; and thirdly, the transducer elements are simple in comparison with the intricate pupil elements of the optical system.

V. References

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* It is desirable to change the frequency down to about 10 Mc/s, or lower because the attenuation of ultrasonic waves is excessive at higher frequencies. However, the procedure of frequency changing can be shown to impose restrictions on the usable frequency bandwidth of the system owing to dispersion effects in the analogue image. The greater the fractional frequency change, the less the allowable bandwidth. Hence an ultrasonic frequency of 10 Mc/s may be regarded as an acceptable compromise. At this frequency the wavelength in water is 0·14 mm.