JOULE HEATING OF THE UPPER ATMOSPHERE

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[*Manuscript received July* 13, 1961]

Summary

The joule heating and motion of uniform ionized gas is discussed, on the assumption that uniform electric and mechanical force fields are orthogonal to the (homogeneous) magnetic field. Application to the ionosphere during geomagnetic disturbance reveals

(i) Joule heating at a rate of order 10^{-5} erg cm^{-3} sec^{-1} in the region 100 to 200 km altitude at the auroral zone is common during geomagnetic disturbance.

(ii) Scale heights and temperatures at altitudes above about 100 km increase with geomagnetic disturbance.

(iii) The energy of the process causing geomagnetic disturbance is in general not measureable by the energy of geomagnetic disturbance.

(iv) A horizontal gradient of pressure of size 10^{-10} dyne cm^{-3} may be maintained at heights above 150 km in the auroral zone during geomagnetic disturbance.

(v) Wind speeds in the polar ionosphere increase with geomagnetic disturbance.

I. Introduction

It is the purpose of the present paper to demonstrate the importance of joule heating in the upper atmosphere. The heating arises from those electric currents flowing in the ionosphere which are responsible for geomagnetic disturbances.

II. Theory

(*a*) *Fields in a Uniform Ionized Gas*

The ionosphere is far from uniform as regards structure and distribution of electrical and mechanical fields of force. However, theory of the inter-relationship of uniform electrical and mechanical force fields and velocity field in a uniform ionized gas will be cited and implications as regards the ionosphere will be developed.

The relevant theory is to be found in a paper by Piddington (1954) and the references contained therein. In rectangular right-handed coordinates an electrostatic field $\mathbf{E}_s$ $(E_{sx}, E_{sy}, 0)$ is applied at time $t=0$ orthogonal to resident homogeneous magnetic field $\mathbf{H}$ $(0, 0, H)$ as is the mechanical force $\mathbf{F}$ $(F, 0, 0)$. Velocity $\mathbf{v}$ $(v_x, v_y, 0)$ is induced. The fields are assumed uniform.

The electric current $\mathbf{j}$ $(j_x, j_y, 0)$ flowing in the gas is given by

$$\mathbf{j}=\sigma_1\mathbf{E}'+\sigma_2\frac{\mathbf{H}\times\mathbf{E}'}{H}, \tag{1}$$

where $\mathbf{E}'=\mathbf{E}_s+\mathbf{v}\times\mathbf{H}$. The equation of motion of the gas is

$$\rho\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t}=\mathbf{j}\times\mathbf{H}+\mathbf{F}, \tag{2}$$

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where ρ is the mass density of the gas. It follows from equation (1) that at all times the magnitudes of the electric current and field are related by

$$j=(\sigma_1\sigma_3)^{\frac{1}{2}}E', \tag{3}$$

where $\sigma_3=(\sigma_1+\sigma_2^2/\sigma_1)$, the Cowling conductivity. The joule heating Q $(=\mathbf{j}\cdot\mathbf{E}')$ is given by

$$Q=\sigma_1 E'^2, \tag{4}$$

or (using eqn. (3))

$$Q=j^2/\sigma_3. \tag{5}$$

In the steady state,

$$\mathbf{j}=\frac{\mathbf{F}\times\mathbf{H}}{H^2}=\sigma_1\mathbf{E}'+\sigma_2\frac{\mathbf{H}\times\mathbf{E}'}{H}, \tag{6}$$

whence it follows that in the steady state,

$$v_x=E_{sy}/H+F/H^2\sigma_3, \qquad v_y=-E_{sx}/H-F\sigma_2/H^2\sigma_1\sigma_3, \tag{7}$$

$$j_x=0, \qquad j_y=-F/H. \tag{8}$$

Piddington shows that in reaching this steady state the velocity and current have both a steady and oscillatory component. The latter has period $\beta^{-1}=2\pi\rho/\sigma_2 H^2$ and an amplitude which decreases with characteristic time $\alpha^{-1}=\rho/\sigma_1 H^2$.

Now equations (7) and (8) represent the relationship between $\mathbf{E}_s$, $\mathbf{v}$, $\mathbf{j}$, and $\mathbf{F}$ in a steady state independently of the manner in which any of these quantities varies or is forced to vary at a prior time. (The parameters α and β are more specific and pertain to the initial conditions and manner of arriving at the steady state.)

Even though the ionosphere is not uniform we assume that the above theory is applicable to it at any given level. The theory appears to be best applicable in the polar regions where the magnetic field is vertical and therefore we need not be concerned in the first instance with gradients of density and conductivity in the direction of $\mathbf{E}'$.

(*b*) *The Ionosphere*

Figure 1 shows α^{-1} and β^{-1} plotted as functions of height using Chapman's (1956) model atmosphere for conductivity (his model *h*) and also in an atmosphere in which the electron density is 10 times that of Chapman's (to simulate disturbed conditions in the auroral regions). For this calculation densities used are those of the Rocket Panel (cf. Newell 1960, Table II).

For the purpose of discussion of geomagnetic disturbance it is assumed that the electric field is orthogonal to the geomagnetic field and the latter assumed vertical. Consider now the following sequence of events. In a still ionosphere in which no prior perturbing mechanical force acts, an electrostatic field E is suddenly applied ($t=0$). Note that the geomagnetic field lines are virtually equipotentials of this field because of the good conductivity in their direction. Joule heating will give rise to a horizontal gradient of pressure (∇p). It is assumed that viscous forces are negligible compared to ∇p, thus $F=\nabla p$. This assumption is seen to be justifiable in Section IV (a). If a steady state can exist it will take a certain time to be established. This time is the larger of the times α^{-1} or the characteristic time involved in establishing a steady ∇p (if such is possible); thus it will be at least α^{-1} (see Fig. 1). Figure 2 shows $(\sigma_1\sigma_3)^{\frac{1}{2}}$ (proportional to j) and σ_1 (proportional to Q) plotted as a function of height in Chapman's (1956) h-model atmosphere. It is apparent that whereas the model ionosphere appears to be most conducting (according to σ_3) in a narrow height range about 100 km height, neither the maximum current nor the maximum heating is found at these heights. The current (proportional to $(\sigma_1\sigma_3)^{\frac{1}{2}}$) centres at about 130 km height and its broad peak has a width at half-peak current of 70 km; the joule heating peak is at about 150 km height and has a width at half-peak heating of about 50–60 km. Thus it is seen that a simple physical picture is conveyed by σ_1 and $(\sigma_1\sigma_3)^{\frac{1}{2}}$, but not by σ_3.

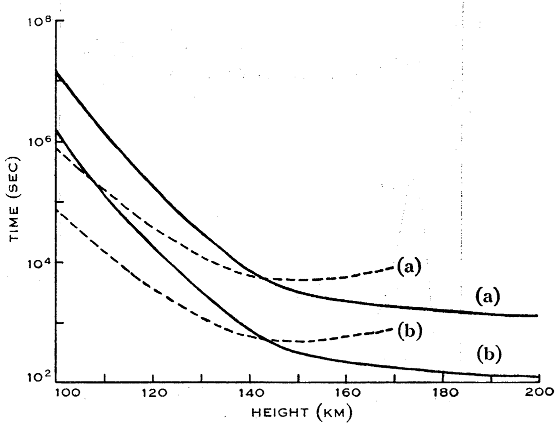

Fig. 1.—Full line, graph of α^{-1} $(=\rho/\sigma_1 H^2)$; dashed line, graph of β^{-1} $(=\rho/\sigma_2 H^2)$; curves (a) refer to Chapman's h-model atmosphere (see Fig. 2) and curves (b) to a disturbed atmosphere with electron density 10 times the former.

In the steady state (eqns. (7) and (8)) $E'=\nabla p/H(\sigma_1\sigma_3)^{\frac{1}{2}}$. Since, in the model discussed, $\mathbf{v}$ is uniform, and $\mathbf{H}$ homogeneous, therefore $\mathbf{v}\times\mathbf{H}$ is a potential field, and the geomagnetic field lines are equipotentials of the total field $\mathbf{E}'=\mathbf{E}_s+\mathbf{v}\times\mathbf{H}$. (*Note that* $\mathbf{H}$ *represents the total geomagnetic field and not its horizontal component.*) Thus in the steady state

$$\nabla p=(\sigma_1\sigma_3)^{\frac{1}{2}}E'H=jH. \tag{12}$$

∇p varies in height in the same way as j (see Fig. 2).

In Section IV it will be seen how a gradient of pressure can arise naturally out of joule heating of the ionosphere. Consider the following sequence of events. An electrostatic field in the ionosphere is " switched on " as the result say of a solar wind blowing on the outer geomagnetic field (Cole 1961). Joule heating is rapid (see below) and a quasi-steady state may soon be established comprising E', ∇p, and j given by equations (7) and (8). Later the applied electrostatic field is switched off. A new steady regime is established given by equations (7) and (8) with $E_s=0$. In this state then

$$v=\nabla p/\{H^2(\sigma_1\sigma_3)^{\frac{1}{2}}\}=j\ /\{(\sigma_1\sigma_3)^{\frac{1}{2}}H\}. \tag{13}$$

At this stage $\mathbf{v}$ makes an angle of $\cot^{-1}(\sigma_2/\sigma_1)$ with $\mathbf{j}$ (see Piddington 1954, Fig. 3). Equation (13) demonstrates the direct connection of polar ionospheric winds with geomagnetic disturbance—a dependence inferred by the author (Cole 1959, 1960*a*) from a different standpoint.

On the assumption that $E_s=0$ at all heights it follows from the constancy of E' that v would be constant at all heights in a steady state. However, reference to Figure 1 (curves of α^{-1}) shows that a steady state would take about 10^6 sec

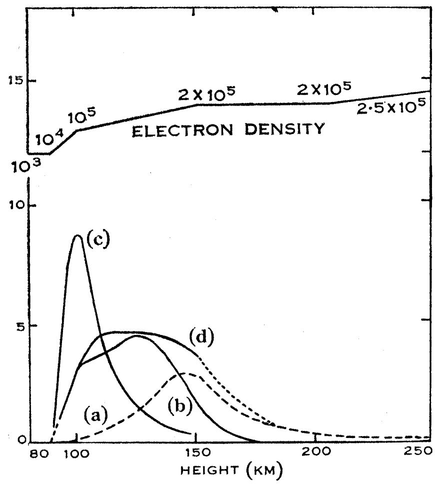

Fig. 2.—Curves (*a*) $\sigma_1\times10^{15}$ e.m.u.; (*b*) $\sigma_2\times10^{15}$ e.m.u.; (*c*) $\sigma_3\times10^{14}$ e.m.u. taken from Figure 2 of Chapman's (1960) *h*-model; (*d*) $(\sigma_1\sigma_3)^{\frac{1}{2}}\times10^{15}$ e.m.u. calculated therefrom. The dotted portion is a suggested interpolation. The electron density distribution for this model is shown at the top of the diagram.

(12 days) to be established at 100 km. This is an unreal situation. An upper limit to the time to reach a steady state is provided by a characteristic time of geomagnetic disturbance, i.e. a few hours (10^4 sec). Reference to Figure 1 shows that a steady state could be reached down to 130 km altitude in a disturbed ionosphere. Below this height a steady state would seem highly improbable. The flow of current below this height would at all times be governed by a combination of $\mathbf{E}_s$ and $\mathbf{v}\times\mathbf{H}$ components of electric field.

The graphs of β^{-1} (Fig. 1) may bear some relevance to the jagged nature of magnetic records. Electric currents would have a component which would fluctuate with a period characteristic of their height. It seems significant that periods (β^{-1}) between 10^4 and 10^3 s are characteristic of that portion of the atmosphere where the bulk of disturbance current flows (see Fig. 2) and these same periods are prominent in magnetic records.

(c) *Winds in a Disturbed Ionosphere*

The geomagnetic control of winds in a disturbed ionosphere is noted in equation (13) above. Using Fukushima's average S_D system, a likely value of j in the auroral zone is 3×10^{-10} e.m.u. A likely value of $(\sigma_1\sigma_3)^{\frac{1}{2}}$ is 10 times that at its peak in Figure 2, i.e. $\sim 5 \times 10^{-14}$ e.m.u. It follows that an " average maximum value of v " is 100 m s^{-1}. Winds of such speed are observed in the polar dynamo region (Elford 1959).

The term " average maximum value of v " is used to delineate those times when the electrostatic field E_s is negligible. It is clear that a lesser value of v would exist when some of the electrostatic field E_s is operating. Nevertheless this substantiates a conclusion reached elsewhere (Cole 1959, 1960*a*) that winds in the polar dynamo regions are significantly correlated with geomagnetic disturbance. The correlation is best when the applied electrostatic field is zero. These winds are dependent on that flux of energy from the Sun which causes geomagnetic disturbance (Cole 1960*a*, 1960*b*) as distinct from winds in the lower atmosphere which depend on flux associated with the solar constant.

(d) *Geomagnetic Disturbance*

The energy (ε) of the processes within the ionosphere, exosphere, and the Earth causing geomagnetic disturbance manifests itself in numerous ways, e.g. (i) the production of heat and light in the upper and outer atmosphere; (ii) the production of heat in the ground and sea; (iii) the acceleration of charged particles in the outer atmosphere; (iv) the energy of electric and magnetic field; (v) the kinetic energy of mass movement in the upper and outer atmosphere. Eventually all the energy of geomagnetic disturbance is converted to heat locally except for any electromagnetic energy emitted by the Earth or particles ejected outwards from the magnetosphere. This heat is a meaningful indicator of the energy of the process causing geomagnetic disturbance, whereas the change of geomagnetic energy during disturbance, in general, is not.

Let us simplify the discussion by reference to the situation in which geomagnetic disturbance is caused only by steady electric currents driven by an externally applied e.m.f. in the ionosphere. Let the only dissipation be by joule heating (Q). Then, at time t after switching on the e.m.f., the energy (ε) expended is (neglecting the energy radiated during switching)

$$\varepsilon(t) = t\int_V Q\mathrm{d}V + \int_V \frac{(H+\Delta H)^2 - H^2}{8\pi}\mathrm{d}V, \tag{8}$$

where V is a volume surrounding the current and its magnetic field and ΔH the perturbation in the geomagnetic field due to the current. The last term is constant under the conditions assumed. *It follows that the energy of the process causing geomagnetic disturbance is in general not measurable by the energy of geomagnetic disturbance.* Of course, if the e.m.f. is switched off, then during the free decay energy of geomagnetic disturbance is converted into an equal amount of heat. It will be of interest later to compare this heat with the joule heating in equation (8).

Since $Q = j^2/\sigma_3$ and since j determines the magnetic disturbance, the power input (for given magnetic disturbance) is inversely proportional to the conductivity of the ionosphere. An estimate of Q during average magnetic disturbance S_D can be derived as follows: Fukushima and Oguti (1953) have produced an equivalent current system of the S_D field for the equinoctial season where it is assumed that two-thirds of the observed geomagnetic variation is caused by overhead electric current flow. From their Figure 1 it follows that, near the auroral zones, the intensity of this current is approximately $1 \cdot 5 \times 10^5$ A per degree of latitude. Assuming that the current flows in a sheet 50 km thick (cf. Fig. 1) it follows that $j \approx 3 \times 10^{-10}$ e.m.u. Suppose (with Akasofu 1960*a*) that σ_3 is increased during disturbance so that $\int \sigma_3 \mathrm{d}h \approx 3 \times 10^{-7}$ e.m.u. Then the average σ_3 throughout the 50 km height range would be 6×10^{-14} e.m.u. Then the average Q throughout this region would be $1 \cdot 5 \times 10^{-6}$ erg cm^{-3} sec^{-1}. It is to be noted that if the disturbance currents are driven by an electric field whose equipotentials are the geomagnetic lines of force then since σ_3 at 150 km altitude is an order of magnitude less than the average whilst j is practically undiminished there (Fig. 1), Q may be an order of magnitude greater at 150 km altitude than the above average, i.e. of order 10^{-5} erg cm^{-3} sec^{-1}. Note in passing that if the assumption of increased conductivity is not made then Q becomes larger still.

A second estimate of Q during a moderate disturbance can be made as follows. Akasofu (1960*a*) deduces a value of $E' = 2 \times 10^4$ e.m.u. for the driving electric field of geomagnetic disturbance in a particular instance. He assumed an enhanced average conductivity 20 times that of Fejer whose model is comparable to Chapman's (1956) model. Evidence in favour of enhanced electron density (of order 10^6 cm^{-3}) over wide regions in the auroral E-region may be found in the work of Heppner, Byrne, and Belon (1952) and Knecht (1956), and in auroral forms in the work of Seaton (1954) and Ohmolt (1954, 1961).

Let us take $n_e \approx 2 \times 10^6$ at 150 km (i.e. 10 times Chapman's value). It follows that at this height $\sigma_1 \approx 3 \times 10^{-14}$ e.m.u. Whence from equation (4), at 150 km altitude $Q_0 \approx 10^{-5}$ erg cm^{-3} sec^{-1}. The larger value of the heating obtained in this second estimate is due to the fact that the current density involved is 2 times larger, viz. 6×10^{-10} e.m.u. Moreover the disturbance reported by Akasofu was only moderate, i.e. 770γ at the ground. In the auroral zone, disturbances (and therefore ionospheric currents) of three times this value are not uncommon. This suggests that provided the conductivity is not further enhanced, the joule heating would be an order of magnitude higher still in limited regions, viz. of order 10^{-4} erg cm^{-3} sec^{-1}.

These calculations strongly support the conclusion that with moderate magnetic disturbance in the auroral zones the rate of input of heat at 150 km due to joule heating is of order 10^{-5} erg cm^{-3} sec^{-1}. This makes joule heating by those ionospheric electric currents causing even moderate geomagnetic disturbance (which exists for more than 30% of the time) a significant factor in the heat balance of the disturbed ionosphere.

Chapman (1918) suggested that the recovery phase of a magnetic storm was one of free decay of the magnetic energy of disturbance established in an earlier phase, i.e. no external e.m.f. is applied during this phase. It is of interest now to try to estimate the joule heating during such decay of a large magnetic storm of magnetic energy of disturbance of order 10^{22} ergs. The rate of decay (assumed approximately linear) of such field is of order 10^{18} ergs sec^{-1}. Chapman (cf. Chapman and Bartels 1951, p. 892) considers two models for the source of energy of a magnetic storm: (i) currents on a sphere concentric with the Earth; (ii) an equatorial ring current. Consider the former. Assuming this dissipation to take place in a layer 50 km thick of the ionosphere the average rate of joule heating is 10^{-7} erg cm^{-3} sec^{-1}. Now the rate of heating just following peak disturbance will be at least three times as large as this. This estimate, based on free decay in a large magnetic storm, is comparable with that for only average S_D disturbance in temperate latitudes. This suggests that free decay is a more rapid process than Chapman suggests and that therefore an externally applied e.m.f. acts even during this phase of the storm. However, this tentative conclusion is dogged by lack of knowledge of the distribution of ionization and current density during this phase of the storm. The distribution of Q with height would be similar to curve (*a*) of Figure 2.

Of course the rate of joule heating in the polar ionosphere must range over many orders of magnitude, if we can use the range of geomagnetic disturbance merely as a criterion. At times of the largest magnetic storms it is likely to be an order or two of magnitude larger still.

Referring again to Figure 1 of Fukushima and Oguti (1953) one sees that at middle latitude the current in the ionosphere is about 1/10 of that in auroral latitudes. Unfortunately the conductivity of the ionosphere during the same period is unknown. If it is supposed that the conductivity in these latitudes is that of a quiet ionosphere (i.e. $\approx 1/20$ of that in auroral zone) then Q is only 1/5 of that in the auroral zone, i.e. about 3×10^{-7} erg cm^{-3} sec^{-1}. This is not inconsiderable. So the phenomenon of joule heating during geomagnetic disturbance must be considered as a world-wide phenomenon with concentration in auroral latitudes. However, if the conductivity is increased at these times then the estimate of Q may be greatly reduced.

III. Two Models

To estimate the temperature increases in order of magnitude two models are cited. The first is a cylindrically symmetric model to simulate the heating in a long (of order 1000 km) auroral electrojet. The second is a spherically symmetric model of heating to simulate (i) the heating over the large area of the auroral zone, or (ii) the global heating due to (uniform) geomagnetic disturbance.

(*a*) *Cylindrically Symmetric Model*

Magnetic records suggest that ionospheric electric currents in the auroral zone sometimes flow in horizontal filaments (cf. Cole 1960*a*). Let us consider then a model in which the heating has cylindrical symmetry and decreases from the centre outwards. We calculate the temperature profile produced in an originally unheated infinite atmosphere of uniform conductivity. Convection is neglected.

Let R be the radius measured from the axis of the cylinder. In a steady state all the heat produced inside the cylinder of radius R is transported outwards across its surface as the flux F. Hence

$$F=-2\pi RAT^{\frac{5}{2}}\frac{\mathrm{d}T}{\mathrm{d}R}=\int_0^R 2\pi RQ\,\mathrm{d}R, \tag{9}$$

where A is as defined in equation (12). Thus

$$T^{3/2}=T_1^{3/2}-\frac{3}{2A}\int_{R_1}^{R}\left[\frac{1}{R}\int_0^R RQ\,\mathrm{d}R\right]\mathrm{d}R. \tag{10}$$

Assume $Q=Q_0\exp(-R^2/h^2)$. When $R/h<1$, equation (10) yields, after integration in series,

$$T^{3/2}=T_1^{3/2}-\frac{3h^2Q_0}{4A}\left[\frac{R^2-R_1^2}{h^2}-\frac{R^4-R_1^4}{8h^4}+\frac{R^6-R_1^6}{36h^6}\cdots\right], \tag{11}$$

when $|R-R_1|<h$ the first term in the series equation (11) dominates the other terms in the series. Thus T may be well-approximated in this region by

$$T^{3/2}=T_1^{3/2}+\frac{3Q_0}{4A}(R_1^2-R^2).$$

Thus the temperature at the axis is greater than $(3Q_0R_1^2/4A)^{2/3}$ for R_1/h sufficiently less than unity. Table 1 shows this value calculated assuming $R_1/h=\frac{1}{2}$ and for R_1 values of 1, 10, 100 km and Q_0 values of 10^{-6}, 10^{-5}, 10^{-4} erg cm^{-3} sec^{-1}. Blank spaces correspond to rather improbable model parameters.

TABLE 1

Q_0 \ R_1	1 km	10 km	100 km
10^{-6}	12°	260	5600
10^{-5}	56°	1200	—
10^{-4}	260°	5600	—

(b) *Spherically Symmetric Model*

In Chapman's h-model atmosphere σ_1 and therefore Q is roughly symmetrical about the 150 km level (r_0). Therefore in order to investigate the temperatures that may be established by the above heating process we adopt the following model for Q in an originally isothermal atmosphere

$$Q=Q_0\exp\{(r-r_0)/h\},\quad r\geqslant r_0, \tag{11a}$$

$$Q=Q_0\exp\{(r_0-r)/h\},\quad r\leqslant r_0. \tag{11b}$$

r, r_0 are distances (cm) from the centre of the Earth. Spherical symmetry is assumed.

In the steady state neglecting convection, after Nicolet (1960),

$$F = 4\pi r^2 \lambda_c \frac{\mathrm{d}T}{\mathrm{d}r}, \tag{12}$$

where F is the heat flow through a concentric sphere of radius r and λ_c $(=AT^{\frac{1}{2}})$ is the thermal conductivity. Thus for $r < r_0$

$$\frac{\mathrm{d}}{\mathrm{d}r}\left(4\pi r^2 A T^{\frac{1}{2}} \frac{\mathrm{d}T}{\mathrm{d}r}\right) = 4\pi r^2 Q_0 \exp\{(r-r_0)/h\}.$$

Provided that $h/r \ll 1$, which is likely to be the case,

$$\begin{aligned} T^{3/2} = T_2^{3/2} &- \frac{3}{2}\left(\frac{1}{r} - \frac{1}{r_2}\right) r_1^2 T_1^{\frac{1}{2}} \frac{\mathrm{d}T_1}{\mathrm{d}r_1} \\ &+ \frac{3}{2}\frac{Q_0 h^2}{A}[\exp\{(r-r_0)/h\} - \exp\{(r_1-r_0)/h\}] \\ &+ \frac{3}{2}\left(\frac{1}{r} - \frac{1}{r_2}\right) r_1^2 \frac{Q_0 h}{A} \exp\{(r_1-r_0)/h\}, \end{aligned} \tag{13}$$

where subscripts 1, 2 represent reference levels for temperature and its gradient. Whilst for $r > r_0$

$$\begin{aligned} T^{3/2} = T_4^{3/2} &- \frac{3}{2}\left(\frac{1}{r} - \frac{1}{r_4}\right) r_3^2 T_3^{\frac{1}{2}} \frac{\mathrm{d}T_3}{\mathrm{d}r_3} \\ &- \frac{3}{2}\frac{Q_0 h^2}{A}[\exp\{(r_0-r)/h\} - \exp\{(r_0-r_4)/h\}] \\ &+ \frac{3}{2}\left(\frac{1}{r} - \frac{1}{r_4}\right) r_3^3 \frac{Q_0 h}{A} \exp\{(r_0-r_3)/h\}. \end{aligned} \tag{14}$$

As an example, let us suppose that reference levels 1, 2 both refer to 100 km height. Assume $T_{1,2} = 225$ °K, $\mathrm{d}T_1/\mathrm{d}r_1 \approx 0$, $h = 25$ km, and $A = 180$ (Nicolet 1960). Table 2 shows values of the temperature at the 150 km (0) level for a range of values of input Q_0 at the level of maximum heating if a steady state were possible. The heat flux a long way above and below the level r_0 is $Q_0 h$. Thus for $Q_0 = 10^{-5}$ erg cm^{-3} sec^{-1} and $h = 25$ km it is $2\cdot5$ ergs cm^{-2} sec^{-1}.

TABLE 2

TEMPERATURE AT 150 KM LEVEL

Q (erg cm^{-3} sec^{-1}) ..	10^{-8}	10^{-7}	10^{-6}	10^{-5}
T (°K)	240	370	1180	5270

It is of interest to convert Q values into equivalent temperature (δ) rises per second in the event of no heat loss by conduction or convection. Thus $\delta = MQ/\rho k$, where M is the molecular weight and k is Boltzmann's constant. Putting $Q_0 = 10^{-5}$, $\delta = 10^{-1}, 1, 10$ °K sec^{-1} at 130, 145, 160 km altitude respectively. Thus the scale height would tend, in the absence of heat loss, to double in a matter of hours, minutes, and seconds, respectively at these heights.

These times represent the times during which significant change would take place and give an indication of the time required to reach a steady thermal state.

(c) *Discussion of Models*

The crude models cited above are satisfactory to demonstrate the order of magnitude of the temperature rises to be expected. Auroral electrojets may be of various widths of more or less rectangular cross section. Electric current in such structures has been examined previously (Cole 1960*a*). The smallest in width may be well approximated (as regards temperature increase) by a cylinder, but the largest would be intermediate between a large cylinder and portion of a spherical shell about the Earth. Tables 1 and 2 show that given an auroral electrojet of moderate size in which $Q=10^{-5}$ erg cm^{-3} sec^{-1} (a moderate value) changes in ionosphere temperature (at and above 150 km) of significance ($\sim$1000 °K) may be expected. Available data on ionization densities and magnetic disturbance, though crude and not necessarily simultaneous, are adequate to demonstrate the importance of the phenomenon of joule heating. Dessler (1959) and Akasofu (1960*b*) have considered ionospheric heating due to hydromagnetic waves. Dessler derived a peak heating of order only 10^{-8} erg cm^{-3} (see his Fig. 2).

IV. Implications

(a) *Latitude Variation of Upper Air Temperatures and Pressures*

Johnson (1960) has suggested that there should be global pressure and temperature equalization at 200 km altitude. However, he did not take into account the Lorentz forces on the upper air due to those electric currents flowing in the ionosphere which cause geomagnetic disturbance. Joule heating of the air by those currents will create regions of higher than normal pressures. Such pressures may be balanced by the Lorentz forces $\mathbf{j}\times\mathbf{H}$ acting on the gas, in a steady state, cf. equation (12). Whence the horizontal gradient of pressure $\nabla p=(\sigma_1\sigma_3)^{\frac{1}{2}}E\mathbf{H}\sin I$, where I is the magnetic dip. Let us consider the height of 200 km. $(\sigma_1\sigma_3)^{\frac{1}{2}}\approx\sigma_1$ here. Put $n_e=4\times10^6$ (disturbed auroral ionosphere), $\sigma_1/n=2\times10^{-21}$ (Chapman 1956). A moderate value of $E'=2\times10^4$ e.m.u. (Akasofu 1960*a*), $H=0\cdot6$, $\sin I\approx1$. Then $\nabla p\approx10^{-10}$ dyne cm^{-3}. Thus across an auroral belt of size 800 km (at moderate disturbance) a pressure difference (Δp) at 200 km altitude of size 10^{-2} dyne cm^{-2} may be maintained. Pressure differences of this size are measured near 140 km altitude above New Mexico. This force Δp is seen to be very much larger than likely viscous stress. Johnson (1960) puts the viscous stress per unit area at about 10^{-6} dyne cm^{-2} for a wind of 150 m sec^{-1} at 200 km altitude. Of course the higher one goes presumably the less becomes $\mathbf{j}\times\mathbf{H}$ and the larger becomes the coefficient of viscosity (proportional to $T^{\frac{1}{2}}$), so that at some height above the present region of interest viscosity may not be negligible. Portion of this increase of pressure is due to increase of temperature and portion due to increase of density following increase of scale height. The above theory is qualitatively and quantitatively in accord with the difference observed between densities at Fort Churchill and White Sands at the same height (Fig. 6, Newell 1960). However this agreement is shadowed by the fact that the observations were not simultaneous.

By reference to Fukushima and Oguti's S_D average current system one would expect on the average a high pressure just outside the auroral zones in the 22 hr to 10 hr sector a low pressure replacing it in the 10 hr to 22 hr sector. Inside the zone one would expect a high pressure region at the centre of the evening cells and a low at the centre of the morning cells. This would be roughly true in the region 100–150 km for Fukushima and Oguti's average system would give bias to the (strong) current at this height. The direction of the current at higher levels would be different and this would change the direction of ∇p at these heights.

It is of interest to calculate the change in pressure (Δp) across the auroral zone at various heights (h), and compare it with the pressure $p_{\text{W.S.}}$ at White Sands at the same altitude. For this calculation it is assumed that $n_e = 10^6$ at all heights (a disturbed auroral ionosphere), $E' = 2 \times 10^4$ e.m.u., and the auroral zone width is 1000 km. One finds that

$$\Delta p / p_{\text{W.S.}} > 1, \text{ when } h > 110 \text{ km (approx.)}.$$

Present day analyses of the quiet ionosphere above 100 km suggest a temperature gradient ($\mathrm{d}T/\mathrm{d}r$) of a few degrees km^{-1} indicating a source of high temperature T_S outside the atmosphere. Above an auroral electrojet the temperature profile must change so that there would be no region above 150 km (0 level) with $T < T_0$ provided $T_0 < T_S$. Until more is known about the simultaneous ionization and current density in other latitudes it is not possible to say whether a similar (smaller) change attributable to magnetic disturbance should appear in the temperature profile at other latitudes.

(*b*) *Ionospheric Disturbances in the* F *Region*

Martyn (1953) and Sinno (1953) distinguish components of F_2 region storms related to local time (S_D component) and to time measured from the time of commencement of a magnetic storm (D_{St} component). Maeda and Sato (1959) conclude that these disturbances can be interpreted fairly satisfactorily by the theory of ionization drift in the ionosphere (cf. Martyn 1953). However, they note that according to the vertical drift theory f_0F_2 in the zone of the polar cap must not vary much. This is in contradiction to the observational results of Knecht (1959), who finds marked variations at the South Geographic Pole. Sato (1957) has applied vertical drift theory in an attempt to match observations of magnetic and F_2 region disturbance at the auroral zone. He claims that the main features of disturbance are explicable thereby. However, he states that since in an individual storm F_2 records are sparse it is difficult to compare fully the calculated results with observations. Obayashi (1958) examines an instance of F_2 layer disturbance which according to him cannot be explained by conventional electron drift theory.

Of course, if significant joule heating takes place in the E region, then a lifting of the atmosphere at E and F region heights may take place together with a decrease of ionization there due to increased recombination (King and Roach 1961), giving enhanced emission of the 6300 A line of atomic oxygen. Let us refer to this as process A. It is suggested that three processes at least, namely the ionization drift, process A, and corpuscular bombardment, are required to explain the whole of F region disturbance. The importance of process A may be judged by the intensity of 6300 A produced and is expected to increase with proximity to the auroral regions, as would corpuscular bombardment.

It was seen above that an average low and middle latitude value of Q_0 may not exceed 10^{-7} erg cm^{-3} sec^{-1}. This may cause negligible F region disturbance compared to vertical drift of ionization under electrodynamic forces. At the auroral zone, however, or at times of exceptionally large magnetic disturbance, Q_0 may reach 10^{-5} erg cm^{-3} sec^{-1}. In this situation process A must work in conjunction with vertical drift. Under these conditions 6300 A may be intense enough to be visible as auroral light. This appears to have happened in the instance reported by King and Roach (1961). These authors associated a red auroral arc with a region of decreased ionization density in the F_2 region. They suggested that corpuscular bombardment of the F region brought about a change in temperature sufficient to cause considerably enhanced recombination which in turn produced the visible 6300 A auroral arc. However, at the time of their observation a magnetic disturbance of size 300γ was in progress. This is moderate magnetic disturbance and would give a Q_0 value of 10^{-6}–10^{-5} erg cm^{-3} sec^{-1}. From Table 2 it is seen that this is sufficient to reverse the normal temperature gradient between the E and F region as required by King and Roach. Between these two extreme situations the importance of process A would increase with increase of geomagnetic disturbance. This would account for the increase with K_p of 6300 A intensity as monitored at a station well equator-ward of the auroral zone (Sandford 1959; and others). This would also account for the variation of satellite orbital acceleration with geomagnetic disturbance (Jacchia 1959*a*, 1959*b*) due to increase in scale height of the neutral particles—an effect which eludes electrodynamic lift theory.

Tandberg-Hanssen (1958) has shown that the F_1 layer rises during geomagnetic storms but that there is no significant change in height of the E layer. He suggests there is a heat source between 100 and 200 km altitude at these times. This is consistent with the above theory.

It is suggested, then, that whilst electrodynamic lift theory accounts extremely well for the changes in some F region ionization parameters with geomagnetic disturbance at low and moderate latitudes, the additional effect of joule heating allows theory to embrace numerous associated disturbance effects noted above.

V. Discussion

The above theory pertained to a uniform medium and many of the conclusions were based on consideration of a steady state. Neither of these conditions is expected to be realized in the ionosphere. However, it is considered that the relationships used indicate the order of the interpreted effect. As an exploratory study in this field this is adequate at present. Some confidence is gained from the way the crude theory matches the crude observations at numerous points with no notable disagreement. Refinement of the theory should proceed hand in hand with the collection of more suitable working data.

It is considered that ground based observations of magnetic disturbance and reasonable inferences about the structure of the ionosphere during such disturbance firmly establish that joule heating is commonly of order 10^{-5} erg cm^{-3} sec^{-1} at 100–200 km altitude over the auroral zone. Of course this phenomenon ranges over several orders of magnitude even in high latitudes. In steady state heating of an auroral ionospheric belt of width 500 km a horizontal flux of heat of order 100 ergs cm^{-2} sec^{-1} may therefore be expected. A downward flux of several ergs cm^{-2} sec^{-1} can also be expected. The horizontal flux could exist in the form of wind of air at temperature of order 1000 °K with speed of order 10^4 cm sec^{-1} at 200 km altitude. This heating could reverse the temperature gradient between the E and F region ionospheres. The theory can best be tested by simultaneous in situ measurements of ionization, current, and air density up to 200 km altitude in the ionosphere during geomagnetic disturbance.

Finally, it is important to realize that the energy of the process causing geomagnetic disturbance is not fully determined by the energy of geomagnetic disturbance and requires at least one more parameter to specify it, viz. the conductivity of the ionosphere.

VI. References

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