SINGULARITIES IN DAILY RAINFALL*

By G. O'MAHONY†

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Summary

Investigations are made of the observed singularities in January daily rainfall means at a number of Australian stations. Simple graphical methods throw doubts on the random nature of a number of the series, and these doubts are strengthened by results of a significance test involving the parameters of the statistical distribution of daily rainfall. Further work will be necessary to decide whether the observed peaks are due to other than chance, because (i) in the January data the apparent effects were not observed at all the stations studied, and (ii) in a brief survey of some December and February data the series appeared to be completely random.

I. Introduction

During the last 10 years a considerable amount of meteorological literature has been devoted to investigations into apparent singularities in daily rainfall amounts and the related problem of deciding whether there are preferred dates for the occurrence of heavy falls.

The suggestion was made by Bowen (1953) that the observed peaks in amounts of daily rainfall over a period are not due to random causes but result from the presence of increased amounts of meteoritic dust, the increases occurring on or near particular dates. The hypothesis is that the extraterrestrial dust provides nuclei which are rain-producing, and that the dates of peak rainfall occur approximately 30 days after the Earth enters a major meteor stream.

The problem of deciding whether the observed peaks in daily rainfall are random or not is extraordinarily complex. The difficulty arises from the extreme skewness of the statistical distribution. The skewness is positive, and at most stations in Australia, medians and modal values of daily rainfalls are zero for every day of the year. In examples such as these it is apparent that the quoting of the standard deviation has very little meaning, and the standard errors of means do not provide adequate tests for investigating the observed differences between samples. This has been expressed quantitatively by Hannan (1955) when he considered daily rainfalls and showed that observations 16·5 standard deviations from the mean occur with a probability of 10^{-4}, while if the distribution had been normal with the same mean and standard deviation, the occurrence of an observation more than 6·1 standard deviations from the mean would occur with a probability of 10^{-9}.

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† Bureau of Meteorology, Melbourne.

Because of the difficulties introduced by the extreme non-normality of the parent data most of the investigations which have been undertaken in this field have been based on distribution-free methods (e.g. see Brier 1961). Most of this paper, which must be classed as a preliminary investigation, also relies on non-parametric methods, but in a later section the statistical distribution of daily rainfall has been derived and it is shown that a very good fit to the particular data can be obtained using the Γ-distribution. This was demonstrated by Das (1955).

This study is directed, not towards investigating the possible activity of meteoritic dust particles as rain-forming nuclei, but rather towards deciding whether the observed peaks in daily rainfall are due to random causes.

II. The Data

The selection of data for the investigation was governed by the ready availability of long and reliable rainfall records which had already been committed to punch cards.

The analysis extends over the months of December, January, and February for Sydney, Melbourne, and Adelaide, and includes also January data for Port Macquarie in New South Wales, for Brisbane and Rockhampton in Queensland, and for Port Hedland in Western Australia.

In a later more complete investigation it is hoped to extend the study to stations throughout Australia and covering the complete year.

III. Graphical Analysis of January Data

Graphical analysis of January data is now considered. In his original paper Bowen (1953) referred for the most part to Sydney daily rainfall and in subsequent investigations (Bowen 1956*a*, 1956*b*) he pooled data from Australian and overseas stations. In the present work, data from individual stations are treated separately and no corrections have been made for the occurrence of leap years. It is realized that the first restriction does away with desirable interstation smoothing, but a study of daily rainfall records shows that pronounced peaks at one or two stations may " carry " a number of other series which are apparently random, and so convey the impression that the occurrence of the peaks is spread over a much greater area than it is in fact. Also, weighting systems introduce considerable subjectivity and so are suspect (cf. Jenkinson 1960). Since the data consist of results of observations made once each day (at 9 a.m.) a correction for the leap year is a refinement which is hardly justified.

The January mean daily rainfalls for Sydney, Melbourne, Adelaide, and Hobart are shown in Figure 1, the first three curves being for the 100 years 1861–1960, and the Hobart graph covering the period 1907–60. It will be seen that while the scale is constant over the four curves, the origin is not at zero in all cases.

As found by Bowen (1953), using slightly different period lengths, there are very pronounced peaks in the Sydney data for January 12 and 23. These do not show up in the curves for the other cities, although at Melbourne there is a small peak on January 22. A comparison between the four curves does not suggest an influence which is common to all.

Bowen's hypothesis has been criticized (Martyn 1954 ; Swinbank 1954), part of the criticism is based on the grounds that the variability of daily rainfall is so great that if a few years records are added to or taken from the data being investigated, an entirely new set of peaks can be produced. Also, if very few

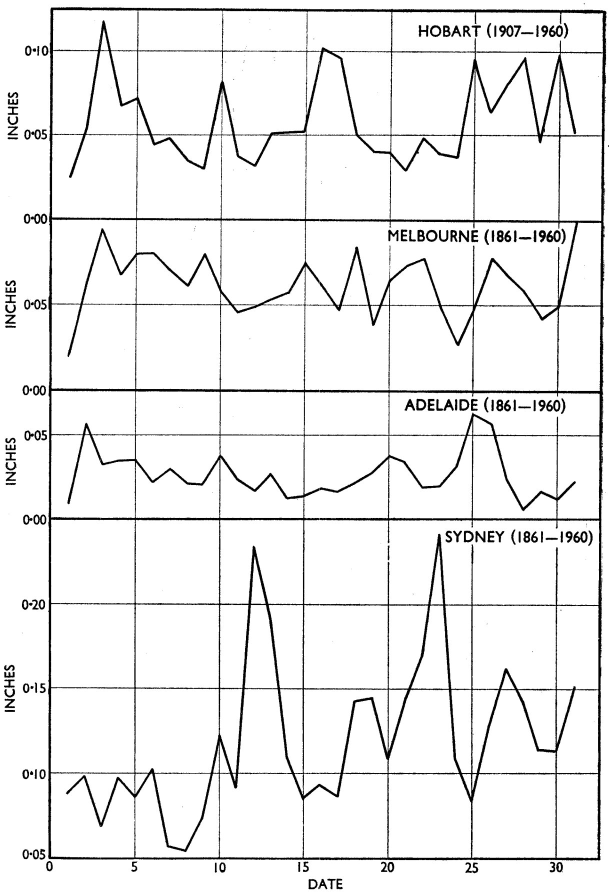

Fig. 1.—Mean daily rainfall for January at Melbourne, Adelaide, Sydney, and Hobart.

extreme values are omitted the peaks disappear. The validity of these arguments can be seen by referring to the Appendix, which sets out in tabular form the 100 years (1861–1960) of the January rainfall at Sydney, the units being recorded in hundredths of an inch. The bottom row shows the total for each day for the complete period.

The total for January 23 is 24·09 in. and it will be seen that in the 100 years, rainfalls of more than 1 in. have occurred on this date only six times. The total for January 12 is 23·99 in., and on only seven of these dates have the daily totals

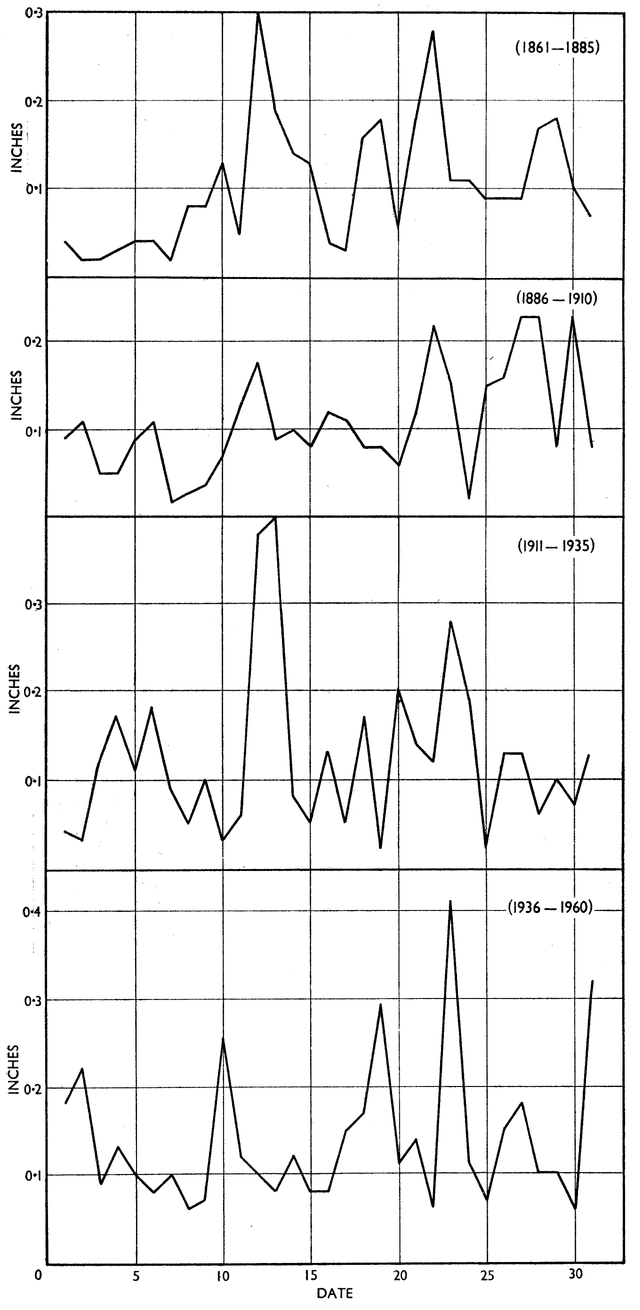

Fig. 2.—Mean daily rainfall for January at Sydney, four 25-year curves.

exceeded 1 in., the last occurrence being in 1918 when a fall of 6·53 in. was recorded. From these values it is apparent that even if the occurrence of singularities can be shown to be non-random, their use as aids to prediction must be very limited.

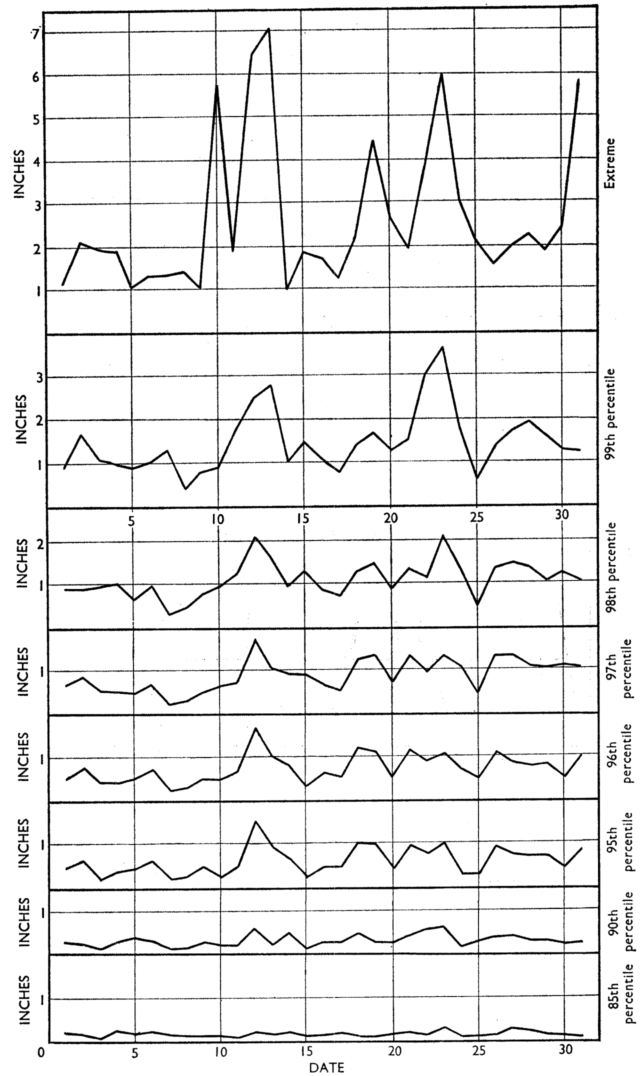

Fig. 3.—Sydney daily rainfall for January, percentiles.

From the data set out in the Appendix, the total number of falls greater than 1 in. is 96. If a Poissonian distribution is fitted to the variate represented by the number of occurrences of falls greater than an inch on any one date, and acting on the null hypothesis that such falls are equally likely on any day, then the seven occurrences on January 12 constitute the only value which is significant at the 5% level. However, reliance cannot be placed on a test of significance based on the Poissonian distribution since the dates in question were all selected because of their associated rainfall peaks.

Following on the criticisms mentioned above, the investigation by graphical methods was continued under two headings:

(i) The division of the total records into subseries.

(ii) The plotting of percentile values of daily falls.

In the first of these, the 100-year records were subdivided into four consecutive 25-year series. This subseries length was selected arbitrarily and without reference to the parent data. Figure 2 contains the January data for Sydney set out in the form of the four 25-year graphs, and it will be seen that in all of the curves a peak has occurred on January 22 or 23 while in three of them a peak appears on January 12 or 13.

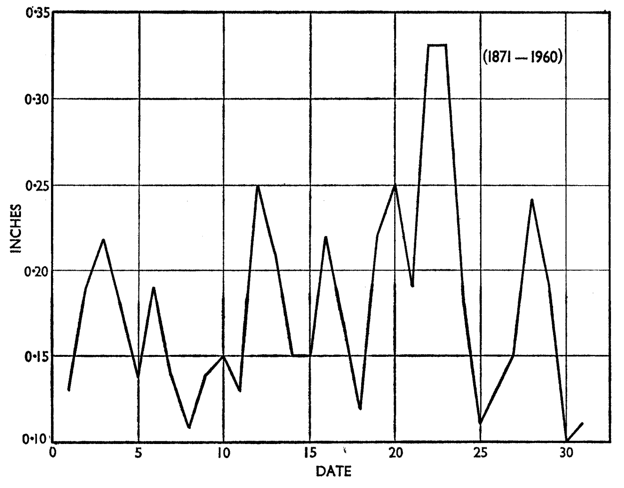

Fig. 4.—Mean daily rainfall for January at Port Macquarie.

The degree of uniformity of these peaks in the individual subseries appears sufficient to raise doubts concerning the random nature of the generating process, and the doubts are further strengthened by studying the percentile curves.

Figure 3 shows a number of percentile graphs of the same data. The first curve represents extreme value rainfall at Sydney on each January date over the 100 years, the second shows the 99th percentile, that is, the second highest rainfall on each date, and then the process has been continued in unit steps down to the 90th percentile. The 90th and 85th percentile values are also plotted, and the ordinate scale in each case has been kept constant to emphasize the rapid decrease in amount when relatively few high values are omitted.

Because of the degree of skewness of the data, it was to be expected that dates showing peaks in mean values would coincide with those of peaks in extreme values, and this is borne out by the first curve in Figure 3, in which the peaks on January 12–13 and 23 are prominent. What would not be expected in a random series is the manner in which these peaks, and particularly that of January 12–13, persist down to the 90th percentile.

An interesting point about these data is that there is not a significant correlation between the total rainfall on any date and the number of wet days.

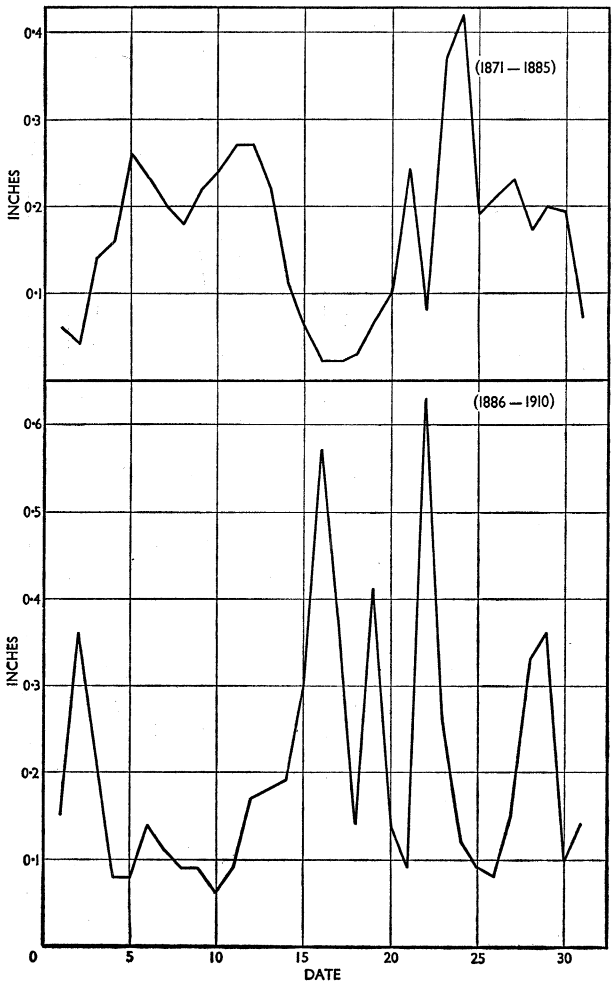

Fig. 5 (*a*).—Mean daily rainfall for January at Port Macquarie 1871–1885 and 1886–1910.

Using the 100 years of January rainfall at Sydney, the sample correlation between these two variates was found to be $+0\cdot14$. Here

$$\begin{aligned} t &= r(1-r^2)^{-\frac{1}{2}}(n-2)^{\frac{1}{2}} \\ &= 0\cdot14(1-0\cdot14^2)(29)^{\frac{1}{2}} \\ &= 0\cdot78, \end{aligned}$$

and with 29 degrees of freedom, a t-value of $2\cdot04$ would be required for significance at the 5% level. This effect had already been noted by Bowen (1953).

The graphical analysis of January data was extended to other stations to determine whether peaks also occurred about these two dates, Figure 4 shows the graph of daily rainfall for the period 1871–1960 for Port Macquarie (31° 28′ S., 152° 55′ E.). There are well-defined peaks on January 12, and 22, and 23. This second peak survived in each subseries based on the periods 1871–85, 1886–1910, 1911–35, and 1936–60, these graphs are presented in Figure 5. The peak on January 12 is not so pronounced in the smaller series.

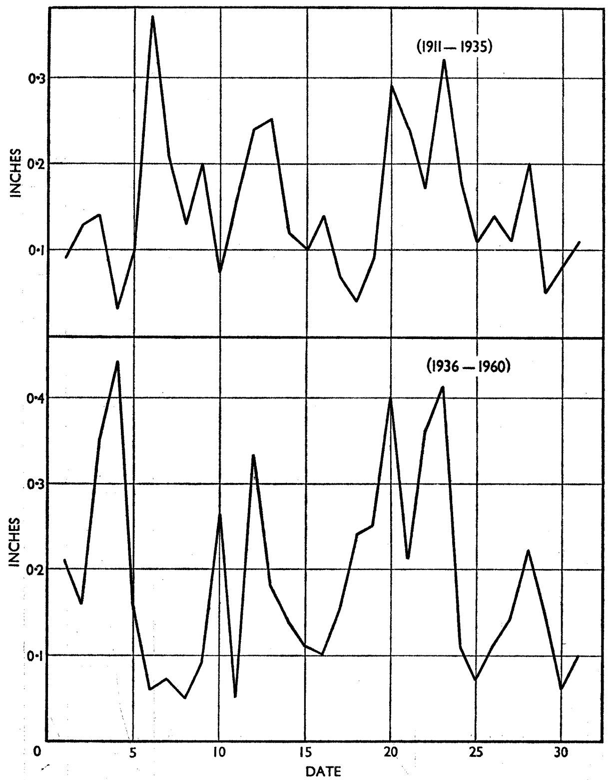

Fig. 5 (*b*).—Mean daily rainfall for January at Port Macquarie 1911–1935 and 1936–1960.

The fact that the peak on January 22–23 is found in all four subseries is not put forward as proof of the non-random nature of the occurrence since heavy rainfalls at Sydney and Port Macquarie are usually associated with the same meteorological situations. Thus if a rare event has occurred by chance at Sydney, then because of the widespread effect of the pressure systems, the same general results will be recorded at Port Macquarie. However, the fact that the existence of the peaks has been noted at both places does reduce the probability of them being due in some way to observational irregularities.

Figure 6 contains three graphs of daily rainfall means for January at Brisbane (27° 28′ S., 153° 02′ E.), the first being for the "total" period 1909–60, and the others being for the two subseries 1911–35 and 1936–60. The long period curve shows trend characteristics, and superimposed on these are pronounced peaks including those of the 13th and 24th. When compared with the subseries graphs there is not the degree of consistency which appeared in the curves for Sydney.

Figure 7 shows the graph of daily rainfall for January at Rockhampton (23° 45′ S., 150° 30′ E.) for the period 1908–60 and contains a large peak on January 21–23 and a smaller peak on January 10–11. These peaks persist in curves of the subseries data, 1911–35 and 1936–60, as shown in Figure 8.

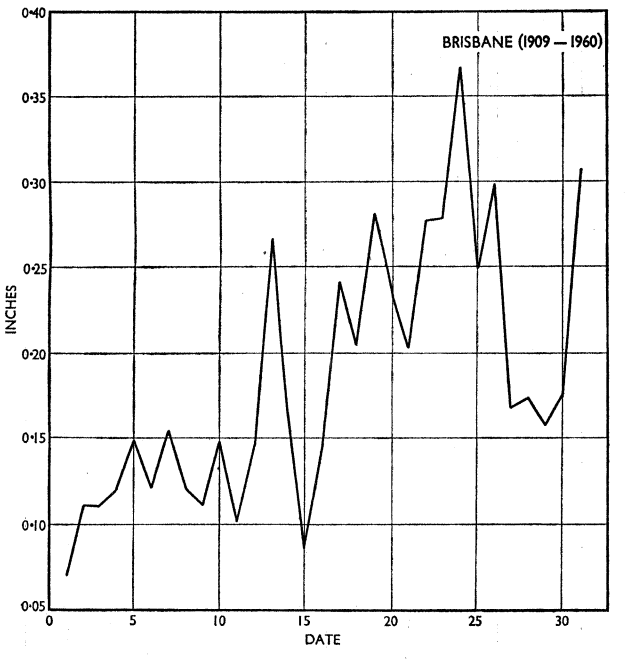

Fig. 6 (*a*).—Mean daily rainfall for January at Brisbane.

The Rockhampton data show that the major January rainfall peaks are located on or about the same dates as those of Sydney. While it is true that summer rainfall at each place occurs for the most part with the southward movement of tropical cyclones or of northern troughs, the distance between the stations (approx. 700 miles) is such that it is unusual for the one meteorological situation to cause heavy rainfall at both simultaneously.

The extent of this meteorological dissimilarity is illustrated in Table 1. It contains, for the years 1888–1960, the rainfall recorded at both places on the 3 days January 21, 22, and 23 in any year in which at least one inch fell at either station on one of these days.

It can be seen that in 22 of the 73 years being considered an inch of rain fell on at least one of the three dates at either place, but that falls of an inch or more a day at both stations in the same period and year occurred on only five occasions. These were 1895, 1922, 1927, 1933, and 1955.

A study of the relevant synoptic charts illustrates the different types of meteorological situations leading to the results set out in Table 1. In 1918, when only 0·03 in. of rain fell in Sydney over the 3-day period, Rockhampton received more than 20 in. A cyclonic depression with centre below 996 mb crossed the coast near Rockhampton on January 22, and on 23 it was located at 23° S., 145° E. Sydney was covered by a ridge of high pressure during the period.

In 1924, Sydney recorded more than 3 in. of rain during the relevant 3 days, while Rockhampton reported only 0·11 in. The surface charts for the period showed two cyclonic centres each below 998 mb located off the east coast of

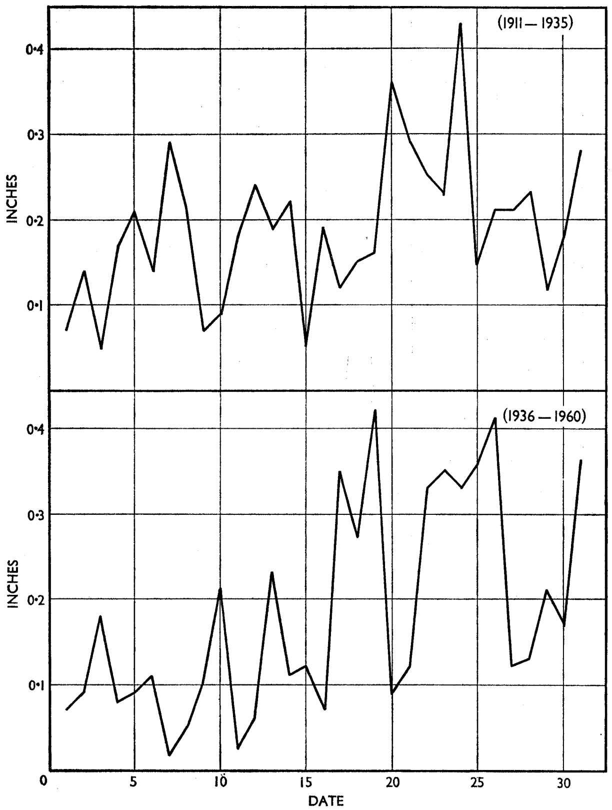

Fig. 6 (*b*).—Mean daily rainfall for January at Brisbane.

Australia. One of these was near 20° S., 152° E. and the other near 30° S., 157° E. during most of the 3-day period. The rainfall at Sydney occurred in the resulting strong on-shore winds, while Rockhampton was practically rainless because of the area of divergence about it.

These two cases provide the general pattern for the cause of rain at only one of the stations. The situation for 1955 is exceptional. Reference to Table 1 shows the similarity of the rainfall amounts at both places during the period. The surface synoptic charts for the 3 days contained a low pressure system extending from 16° S., 133° E. to 21° S., 145° E. and from there to the east coast near Sydney. Heavy rainfall was recorded in most of the Northern Territory, Queensland, and New South Wales during the 3 days being considered.

In the following sections, after deriving the statistical distribution of the daily rainfalls, a test is attempted on the significance of the observed peaks. The method used is to select peak dates in Sydney and then test the Rockhampton rainfalls on these dates, the validity of the test resting on the assumption that

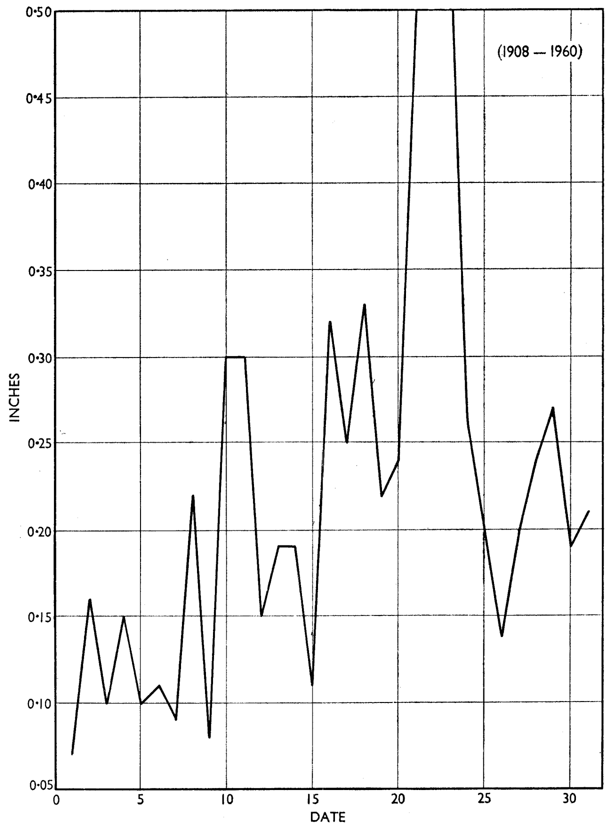

Fig. 7.—Mean daily rainfall for January at Rockhampton.

the Rockhampton and Sydney peaks are independent of each other. From the discussion of the 1955 data this is obviously not strictly true, but from an overall study of Table 1 and the relevant synoptic charts it appears that the approximation is reasonable.

Before passing on to the examination of the statistical distribution, January data of Port Hedland (20° 19′ S., 118° 24′ E.) were analysed graphically. This station was selected because it is on the north-west coast of the Continent and so not subject to the same meteorological systems as the places previously examined. Figure 9 shows the mean daily rainfall for Port Hedland for the period 1913–60 and also the curves for two subperiods 1913–35 and 1936–60.

The total period curve shows maximum peak values on January 30, 12, and 21, while the last two only have survived in both of the smaller period curves.

From these graphical surveys of daily rainfall for January there does appear evidence that peaks observed at Sydney persist beyond the geographic boundaries of Sydney's climatic region, and as indicated above, the following section is devoted to examining the effect quantitatively.

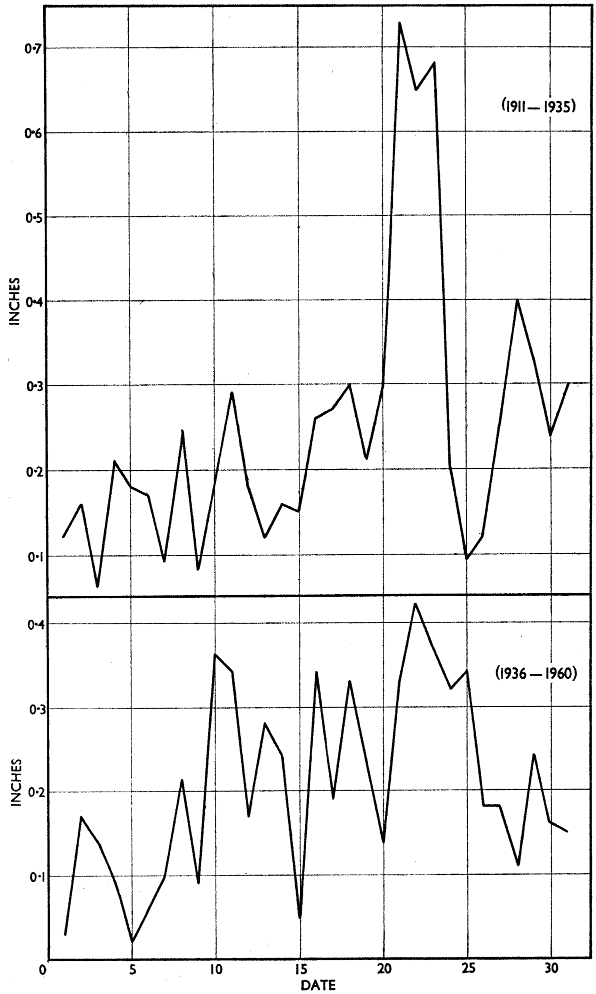

Fig. 8.—Mean daily rainfall for January at Rockhampton, two 25-year series.

IV. The Distribution of Daily Rainfall

Daily rainfall has a skew distribution with lower limit zero and it approximates the Γ-distribution. Das (1955) suggested a truncation technique whereby it is possible, using an approximation, to take into account the number of zeros and small values below an arbitrarily selected truncation point.

TABLE 1

DAILY RAINFALL (IN.)

Year	Sydney			Rockhampton		
	Jan. 21	22	23	Jan. 21	22	23
1890	0	0	0	0·03	0·02	1·05
1893	0	0	1·05	0	0	0
1894	0·01	0	0	0	0	2·49
1895	0·32	0·80	2·13	1·74	0·55	0·87
1899	0	0·01	0	0	0·05	2·22
1901	0	3·06	0·17	0	0	0
1918	0	0·03	0	4·56	9·69	7·53
1922	1·15	0·06	0	0	1·30	0
1924	1·34	0·70	1·30	0	0·09	0·02
1927	0	1·15	0·07	0·71	0·33	1·49
1929	0	0	0	2·02	0·39	0·05
1930	0	0	0	3·33	2·26	5·32
1931	0·07	0	0	1·02	0·02	0
1933	0·21	0·71	3·72	3·20	0·67	2·14
1946	0·21	0	0·02	3·46	0·13	0·09
1951	1·31	0	0·12	0	0	0
1952	0	0	0	1·00	0·11	0·03
1953	0	0	0·82	0·76	0·03	1·23
1955	0	0·32	5·98	0	0·28	6·75
1956	0·97	0	0	0·98	6·67	0·45
1957	0	0·10	1·07	0	0	0
1960	0	0·14	0·59	1·46	1·80	0

The gamma distribution has the form

$$f(x)=\frac{1}{\beta^{\gamma}\Gamma(\gamma)}e^{-x/\beta}x^{\gamma-1}. \tag{1}$$

The di-gamma function is given by

$$\psi(\gamma)=\frac{d}{d\gamma}\ln\Gamma(\gamma). \tag{2}$$

The maximum likelihood equations are

$$\frac{1}{N}\,\frac{\partial L}{\partial(1/\beta)}=\gamma\beta-\frac{1}{N}\sum_{i=1}^{m}x_i=0, \tag{3}$$

and

$$\frac{1}{N}\,\frac{\partial L}{\partial\gamma}=\ln\left(\frac{1}{\beta}\right)-\frac{d}{d\gamma}\ln\Gamma(\gamma)+\frac{n}{N}\ln\delta-\frac{n}{N\gamma}+\frac{1}{N}\sum_{i=1}^{m}\ln x_i=0, \tag{4}$$

and these can be shown to reduce to

$$\psi(\gamma+1)-\ln\gamma-\frac{m}{N\gamma}-\ln\frac{1}{\bar{x}}-\frac{n}{N}\cdot\ln\delta-\frac{1}{N}\sum_{i=1}^{m}\ln x_i=0, \tag{5}$$

where δ is the truncation point (in this analysis it has been selected as 0·05 in.),
N is total number of observations,
n is number $\leqslant$ 0·05 in.,
m is number $>$ 0·05 in.

The derived formula is approximate in that it involves writing δ^γ/γ for

$$\int_0^\delta e^{-x/\beta}x^{\gamma-1}dx.$$

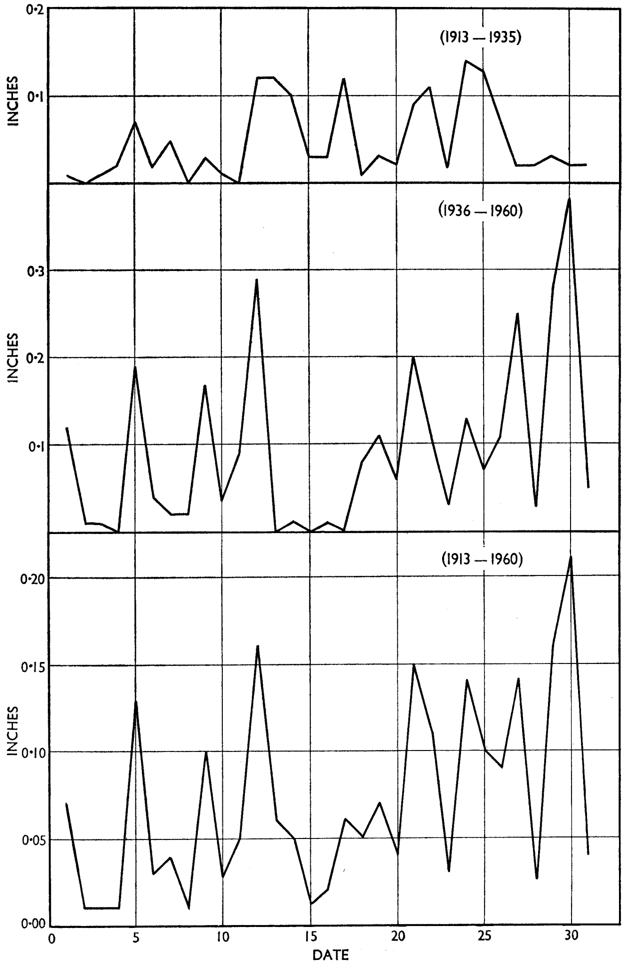

Fig. 9.—Mean daily rainfall for January at Port Hedland.

In applying the theory to the Sydney data for January for the 100-year period, these two values were calculated and were found to be 11·37 and 11·16 respectively, so little precision was lost by the approximation.

The calculation of the parameter values was carried out by means of tables (Davis 1933) and the computations of the expected frequencies were then made using Pearson's (1951) incomplete gamma tables.

TABLE 2

TRUNCATED GAMMA DISTRIBUTION

Sydney Daily Rainfall January (1861–1960)

Class Interval (in.)	Observed Frequency		Expected Frequency	
	f_0	Σf_0	f_e	Σf_e
0 –0·10	2507	2507	2518·4	2518·4
0·11–0·20	180	2687	155·6	2674·0
0·21–0·30	112	2799	106·9	2780·9
0·31–0·40	55	2854	63·5	2844·4
0·41–0·50	43	2897	46·2	2890·6
0·51–0·60	30	2927	45·6	2936·2
0·61–0·70	24	2951	14·9	2951·1
0·71–0·80	18	2969	24·1	2975·2
0·81–0·90	20	2989	18·4	2993·6
0·91–1·00	14	3003	14·6	3008·2
1·01–1·50	56	3059	46·2	3054·4
1·51–2·00	20	3079	21·4	3075·8
2·01–3·00	11	3090	16·5	3092·3
3·01–5·00	5	3095	6·8	3099·1
5·01–7·10	5	3100	0·9	3100·0

TABLE 3

TRUNCATED GAMMA DISTRIBUTION

Rockhampton Daily Rainfall January (1908–60)

Class Interval (in.)	Observed Frequency		Expected Frequency	
	f_0	Σf_0	f_e	Σf_e
0 –0·10	1262	1262	1259·4	1259·4
0·11–0·20	84	1346	102·7	1362·1
0·21–0·30	46	1392	39·9	1402·0
0·31–0·40	32	1424	35·7	1437·7
0·41–0·50	30	1454	26·6	1464·3
0·51–0·60	17	1471	21·5	1485·8
0·61–0·70	20	1491	18·4	1504·2
0·71–0·80	18	1509	12·6	1516·8
0·81–0·90	13	1522	14·0	1530·8
0·91–1·00	8	1530	11·3	1542·1
1·01–1·50	46	1576	37·3	1579·4
1·51–2·00	25	1601	21·5	1600·9
2·01–3·00	20	1621	22·7	1623·6
3·01–4·00	11	1632	10·2	1633·8
4·01–5·00	4	1636	6·2	1640·0
5·01–6·00	3	1639	2·6	1642·6
6·01–7·00	2	1641	1·4	1644·0
>7·00	2	1643	1·8	1645·8

Table 2 contains the observed and theoretical frequencies and cumulated frequencies within class intervals of the January daily rainfalls for Sydney over the 100 years.

The parameter values for the theoretical distribution were

$$\gamma=0\cdot104,$$
$$\beta=1\cdot11.$$

The fit is obviously very good and the χ^2-test showed no significant difference between the observed and theoretical distributions.

TABLE 4

TRUNCATED GAMMA DISTRIBUTION

Rockhampton Daily Rainfall January 21, 22, and 23 (1908–60)

Class Interval (in.)	Observed Frequency		Expected Frequency	
	f_0	Σf_0	f_e	Σf_e
0 –0·10	114	114	74·3	74·3
0·11–0·20	5	119	45·7	120·0
0·21–0·30	8	127	5·1	125·1
0·31–0·40	2	129	2·6	127·7
0·41–0·50	4	133	3·8	131·5
0·51–0·60	0	133	2·0	133·5
0·61–0·70	2	135	2·2	135·7
0·71–0·80	3	138	1·4	137·1
0·81–0·90	0	138	1·6	138·7
0·91–1·00	1	139	1·1	139·8
1·01–1·50	6	145	5·2	145·0
1·51–2·00	2	147	2·4	147·4
2·01–3·00	3	150	3·9	151·3
3·01–4·00	3	153	2·3	153·6
4·01–5·00	1	154	1·5	155·1
5·01–6·00	1	155	1·1	156·2
6·01–7·00	2	157	0·7	156·9
(>7·00)	2	159	2·1	159·0

The test is approximate when applied to data in this form (see Watson 1957) but it can be applied here with reasonable confidence because of the large expected frequency in each cell and because of the number of degrees of freedom.

The χ^2-value was found to be 20·70 and for 12 degrees of freedom (grouping the last three rows of Table 2) this is not significant.

A theoretical distribution was fitted to the Sydney data for the peak date January 22 and the parameters were found to be

$$\gamma=0\cdot107,$$
$$\beta=5\cdot75.$$

The theory was next applied to the complete January data for Rockhampton (1908–60) and the expected and observed frequencies are contained in Table 3.

Again the fit is very good and the χ^2-value of $12 \cdot 66$ for 14 degrees of freedom (combining the last four rows of Table 3) is not significant.

The relevant parameter values were

$$\gamma = 0 \cdot 100,$$
$$\beta = 2 \cdot 19.$$

To test whether rainfall on peak dates at Rockhampton is significantly different from the rainfall on other days, it is obviously pointless to select the extreme values and then test them. However, if, following on Table 1 above

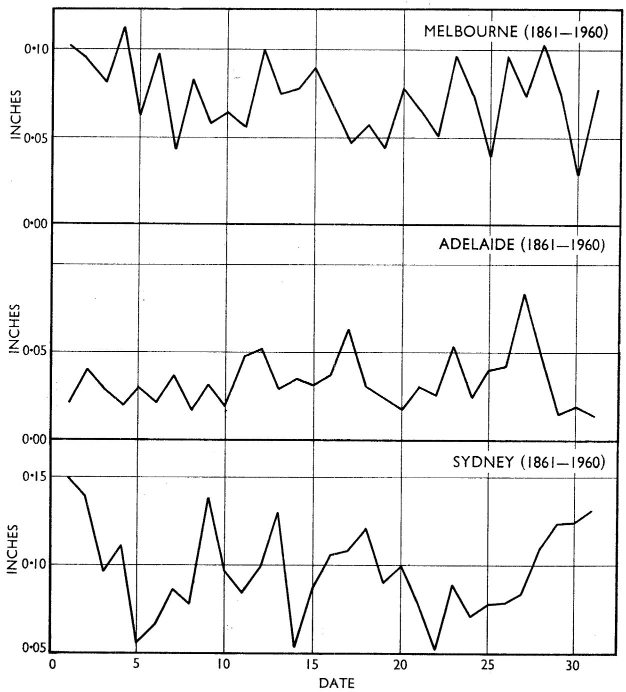

Fig. 10.—Mean daily rainfall for December at Melbourne, Adelaide, and Sydney.

and the relevant discussion, we can assume that large rainfalls at Sydney and Rockhampton are independent of each other, it is quite logical to test the rainfalls at Rockhampton on dates selected because they showed peaks at Sydney.

Acting on this assumption, a truncated Γ-distribution was fitted to the Rockhampton rainfall for January 21, 22, and 23 for the period 1908–60, these dates being selected because January 22 contained a major peak in Sydney data. The expected and observed frequencies are shown in Table 4.

A χ^2-value has not been computed here, since from inspection it is apparent that above the $0 \cdot 10$ in. interval the fit is very good and below that value it is extremely poor. This may be due in part to the difficulty in interpolating accurately in the lower portion of Pearson's (1951) incomplete Γ-tables.

The relevant parameter values were found to be

$$\gamma = 0\cdot097,$$
$$\beta = 5\cdot10.$$

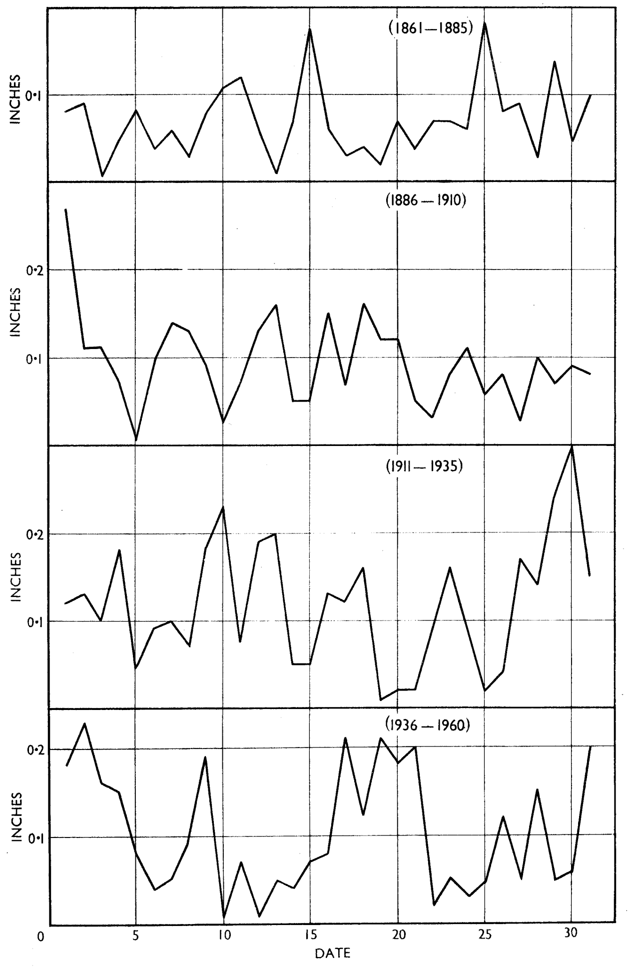

Fig. 11.—Mean daily rainfall for December at Sydney, four 25-year curves.

It is apparent that all four γ-values are almost identical, but the β-values for January 22 in Sydney and for the 3 days, January 21, 22, and 23 in Rockhampton are considerably greater than the corresponding values for the complete month.

The standard errors of the estimates of the values of the parameters γ and $1/\beta$ can be obtained by calculating the second derivatives of the maximum likelihood equations and relating them to the inverse of the variance–covariance matrix (Moran 1957). The resulting equation has the form :

$$\begin{bmatrix} \operatorname{var}(\hat{\gamma}) & \operatorname{cov}\left(\hat{\gamma}, \widehat{\frac{1}{\beta}}\right) \\ \operatorname{cov}\left(\hat{\ }, \widehat{\frac{1}{\beta}}\right) & \operatorname{var}\left(\widehat{\frac{1}{\beta}}\right) \end{bmatrix} = \begin{bmatrix} -E\left(\dfrac{\partial^2 L}{\partial\gamma^2}\right) & -E\left(\dfrac{\partial^2 L}{\partial\gamma\cdot\partial\left(\frac{1}{\beta}\right)}\right) \\ -E\left(\dfrac{\partial^2 L}{\partial\gamma\cdot\partial\left(\frac{1}{\beta}\right)}\right) & -E\left(\dfrac{\partial^2 L}{\partial\left(\frac{1}{\beta}\right)^2}\right) \end{bmatrix}^{-1},$$

and the right-hand side reduces to :

$$\begin{bmatrix} -N\psi'(\gamma)+\dfrac{n}{\gamma^2} & N\beta \\ N\beta & -N\gamma\beta^2 \end{bmatrix}^{-1},$$

where $\psi'(\gamma)$ is the tri-Γ-function and is $(d/d\gamma)[\psi(\gamma)]$.

This equation was applied to both sets of Rockhampton data. For the whole month :

$$N=1643\ ;\quad n=1190\ ;\quad \gamma=0\cdot100\ ;\quad \beta=2\cdot19\ ;$$
$$\psi'(0\cdot100)=101\cdot4333 \text{ (Davis 1933, Vol. 2, p. 29)},$$

and var $(\widehat{1/\beta})$ was calculated as $0\cdot0019$.

The standard error of this estimate is $0\cdot044$.

Then for the 3 days January 21, 22, and 23 :

$$N=159\ ;\quad n=107\ ;\quad \gamma=0\cdot097\ ;\quad \beta=5\cdot10\ ;$$
$$\psi'(0\cdot097)=108\cdot4833.$$

The variance of this estimate of $(1/\beta)$ was found to be $0\cdot0034$ and the standard error of the estimate is $0\cdot058$.

An approximate test of significance is based on the assumption that the ratio

$$\frac{\text{observed difference between the two } (1/\beta) \text{ values}}{[\text{sum of the variances of the two estimates}]^{\frac{1}{2}}}$$

is a standard normal variate. This gives

$$\frac{0\cdot457-0\cdot196}{[0\cdot0019+0\cdot0034]^{\frac{1}{2}}}\simeq\frac{0\cdot261}{0\cdot073}.$$

This is greater than $3\cdot5$ and so is highly significant.

Obviously the observed difference between the values would have been greater if the calculation had been carried out for two distinct samples of 28 and 3 days instead of including the latter in the larger group as was done here.

Relying then on the following two assumptions : (i) The independence of major rainfalls at Sydney and Rockhampton ; (ii) the adequacy of the fit of the truncated Γ-distributions ; it appears that there is good evidence to support the hypothesis that the period January 21–23 contains preferred dates for the occurrence of heavy rainfalls at Rockhampton.

V. Graphical Analysis of December and February Data

A brief investigation by graphical methods was extended to data for December and February for Sydney, Melbourne, and Adelaide.

Mean value curves for the three stations for December are shown in Figure 10, and as before, there seems little regularity between the curves.

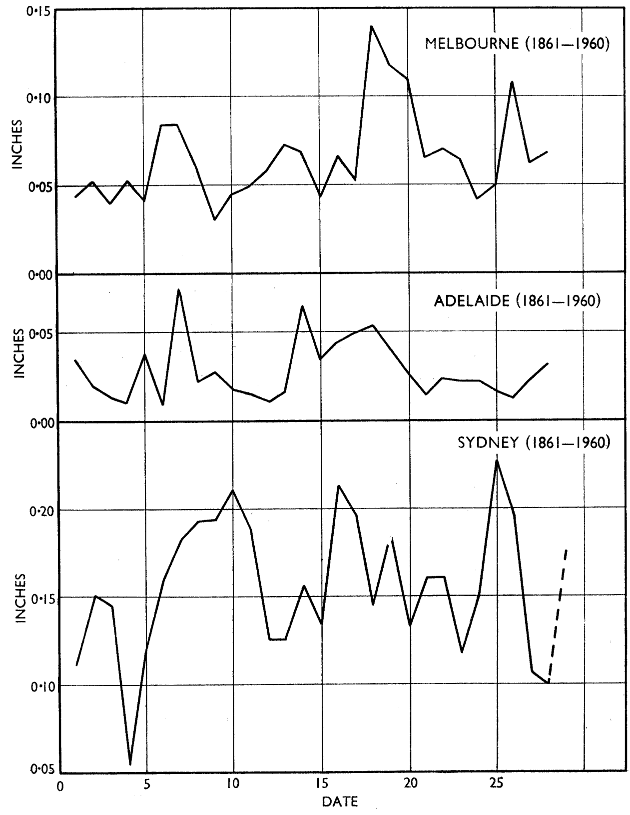

Fig. 12.—Mean daily rainfall for February at Melbourne, Adelaide, and Sydney.

As in the case of the January records, the Sydney series was divided into four consecutive subseries of 25 years and the resulting curves appear in Figure 11. The major peaks in the " total " series occur on the following dates :

1 9 13 18 31

while in the four subseries, major peaks occur as follow :

(1861–85)	..					15				25	29	
(1886–1910)	..	1			13			18				
(1911–35)	..			10	13							30
(1936–60)	..		2				17		19			

The corresponding curves for the February data appear in Figures 12 and 13. The first of these shows some measure of regularity between the graphs of the Melbourne and Adelaide rainfalls in that there are peaks which are common to both on February 7, 14, and 18, but the major peaks in Sydney occur on February 10, 16, and 25.

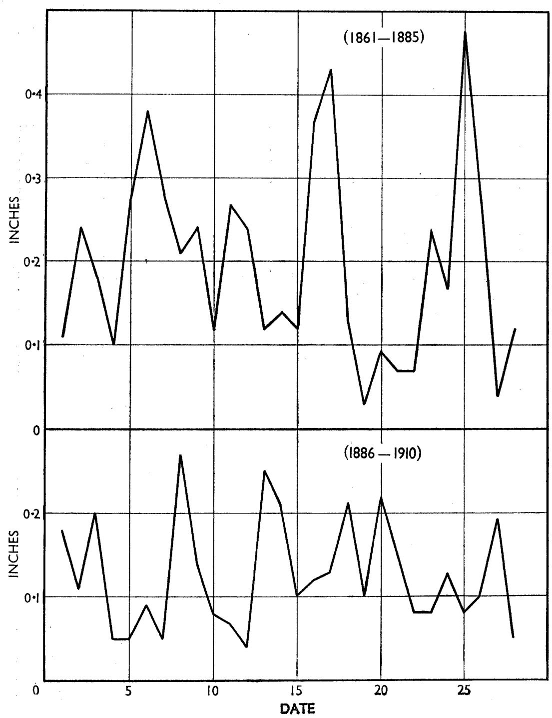

Fig. 13 (*a*).—Mean daily rainfall for February at Sydney, two 25-year curves.

From Figure 13 containing the four 25-year subseries of Sydney data, major peaks occur on the following dates:

1861–85	..	6					17				25
1886–1910	..		8			13			20		
1911–35	..				11	16					25
1936–60	..			10				19		22	

It is apparent that neither of these sets of Sydney curves possesses the regularity of the corresponding graphs for January, and the data for these 2 months appear to be randomly distributed in time.

VI. Conclusions

There seems fairly reliable evidence to support the hypothesis that there are preferred dates for the occurrence of heavy rainfalls in January at a number of Australian stations. The evidence cannot be classed as conclusive because of some apparent gaps in the pattern, since, for example, the effects seem quite pronounced at Sydney, Port Macquarie, and Rockhampton, less definite at Port Hedland and hardly perceptible at Brisbane.

These effects have been demonstrated by methods of non-parametric analysis in which peaks in mean daily rainfalls at individual stations persisted when the complete series of observations had been divided into a number of subperiods. The same peaks also persisted in the residual curves of Sydney January data, when up to and including the 10 highest falls on each date were omitted.

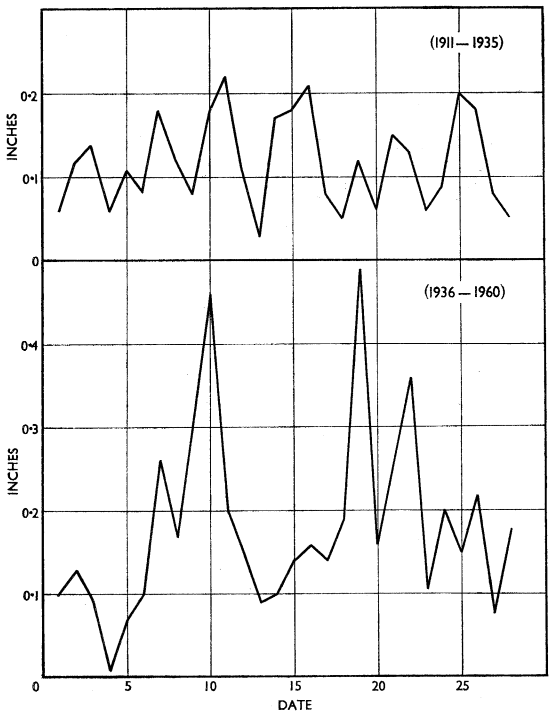

Fig. 13 (*b*).—Mean daily rainfall for February at Sydney, two 25-year curves.

Supporting evidence for the hypothesis came from the fact that the same peaks were found beyond the boundaries of a particular rainfall regime, that is as indicated above, they occurred at both Sydney and Rockhampton, and the striking feature of this section of the work was, that while these two stations showed peaks in average rainfall on the same calendar dates, the great majority of the heavy falls did not occur at each place in the same year.

This fact was used as the basis for a parametric test of Rockhampton data, when by applying a truncated Γ-distribution, a highly significant difference was found between all-January data and the data for 3 days, the days being selected at Rockhampton because they were peak dates in Sydney.

Thus the distribution-free methods and the parametric tests both tend to confirm the suggestion that there are non-random processes in the production of daily rainfalls in January, though the obvious question which was raised above and which has not been answered is—" If the effects are real why are they not apparent at all stations ? "

The survey of December and February series was very limited, but in both cases there was no evidence that the processes were other than random. This poses another basic question—" If the effects in January are real, why should they not extend to the adjacent months ? "

The answers to these questions can come only from examination of more data and an attempt will be made to include in a later investigation records from stations in all rainfall districts of Australia and covering all months.

VII. Acknowledgments

Thanks are due to Professor P. A. P. Moran for his advice on the statistical treatments in the paper and to Mr. J. V. Maher of the Bureau of Meteorology, Melbourne, for advice and assistance in calculating the theoretical frequencies of the Γ-distributions.

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APPENDIX

SYDNEY DAILY RAINFALL, JANUARY 1861–1960

(Hundredths of Inches)

Year \ Date	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
1861	0	0	2	0	6	3	13	1	0	0	0	0	1	0	13	37	2	9	13	0	89	4	0	29	0	0	0	0	0	36	103
62	4	1	0	0	0	0	1	6	0	0	0	16	0	0	0	0	11	48	109	3	3	93	31	22	23	0	0	0	0	0	0
63	0	6	0	0	0	5	0	0	0	0	0	1	19	95	0	4	0	0	0	0	5	375	25	0	5	55	16	0	33	0	0
64	21	0	0	0	·4	21	13	6	0	0	1	18	2	0	0	17	0	0	0	0	7	0	0	0	0	0	0	0	0	0	0
65	0	0	0	0	0	38	0	0	56	27	0	0	97	57	23	7	0	0	0	0	5	0	0	3	0	0	0	4	4	128	1
66	0	0	0	2	0	0	0	0	0	19	0	8	4	0	0	0	0	0	103	0	5	4	1	1	0	0	13	23	191	36	0
67	0	0	0	0	0	0	0	0	0	4	0	0	0	0	1	0	3	0	0	16	97	0	0	1	0	0	0	0	0	14	37
68	0	0	0	0	0	1	0	1	50	47	0	0	2	0	0	0	0	127	11	3	9	52	21	0	0	0	1	79	28	16	1
69	0	18	0	0	0	0	0	0	0	0	6	0	0	0	0	4	2	23	15	0	1	21	0	0	0	0	0	0	0	1	11
70	0	0	0	33	2	0	0	0	0	2	0	0	0	0	149	1	1	0	0	27	2	0	0	0	0	0	31	24	0	0	3
71	25	0	0	0	0	0	1	21	0	98	0	0	21	40	0	0	0	0	0	0	0	11	67	184	47	0	0	48	0	0	0
72	52	0	8	0	0	0	4	0	0	22	2	2	280	17	0	0	0	0	38	1	0	0	27	11	1	0	39	0	0	0	0
73	0	0	0	4	0	0	0	140	2	65	45	251	5	16	0	0	0	1	0	0	0	18	0	0	1	1	0	0	0	0	1
74	0	0	0	0	0	3	4	0	12	14	41	5	0	26	7	0	0	12	0	46	1	0	6	3	35	161	0	1	8	2	0
75	0	0	0	0	0	0	0	25	29	1	0	0	0	0	0	0	21	0	0	0	0	0	0	0	18	1	3	3	0	13	0
76	0	0	2	0	47	5	0	0	0	3	0	2	0	5	3	30	11	0	4	1	0	0	0	0	0	0	28	1	1	0	0
77	0	26	0	1	0	0	0	1	28	1	0	84	0	0	0	0	0	0	0	1	0	0	0	0	0	0	9	5	0	0	0
78	5	0	10	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	2	0	89	1	0	0	0	0	0	0	0	0
79	0	0	0	0	0	0	15	0	0	0	0	8	7	0	0	0	0	0	3	45	156	1	79	0	0	0	0	0	0	0	0
80	0	0	0	3	0	0	0	0	20	0	0	0	0	0	0	0	0	12	10	0	0	0	0	0	53	1	1	0	0	0	12
81	0	0	0	0	7	0	0	0	0	0	19	19	9	45	132	8	0	1	0	1	2	4	0	10	5	5	0	0	18	0	1
82	0	0	3	0	0	0	0	0	0	0	3	0	18	0	0	0	15	0	0	0	8	0	0	0	0	5	0	0	0	0	10
83	1	0	0	23	43	20	1	0	0	0	19	170	0	47	0	0	0	129	143	0	0	6	22	0	29	0	12	224	161	0	0
84	0	0	0	0	0	.0	0	0	0	0	0	0	14	0	0	0	11	34	2	0	2	0	0	0	0	0	23	0	0	0	0
85	0	11	22	0	0	0	0	0	0	20	0	156	1	0	0	0	0	5	10	12	55	16	0	0	8	5	58	9	3	0	0
86	0	1	0	2	0	0	0	0	0	2	0	8	0	0	0	2	1	0	1	0	0	5	1	0	9	22	27	195	0	0	0
87	0	0	0	3	0	9	0	0	0	0	0	0	0	0	0	14	19	0	1	70	194	17	7	6	8	137	134	11	0	0	1
88	0	0	0	8	0	0	0	0	0	0	0	0	0	24	0	0	3	0	2	0	0	0	0	0	0	0	0	0	0	0	3
89	0	0	0	56	0	0	2	0	0	0	0	0	0	0	6	0	8	23	2	0	16	1	0	0	2	0	15	17	7	54	1
90	0	0	0	30	1	99	0	0	0	0	0	5	1	29	28	29	35	4	1	0	0	0	0	0	5	0	0	0	0	249	86
91	0	0	0	0	0	21	3	0	0	13	68	6	0	6	0	93	121	82	0	0	0	1	2	0	48	1	1	0	19	23	15
92	27	0	0	0	0	0	0	0	1	7	181	167	105	0	0	0	71	2	0	0	0	0	0	0	0	0	43	33	0	0	1
93	117	75	5	0	35	0	0	1	0	0	0	0	0	21	4	0	0	0	47	18	0	0	105	8	1	0	0	0	1	7	11

APPENDIX (*Continued*)

Year \ Date	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
1894	4	0	0	0	0	5	1	11	6	7	0	1	0	20	0	3	0	2	0	0	1	0	0	6	12	7	35	14	0	23	4
95	13	19	19	26	2	0	0	16	25	10	4	0	0	0	5	55	10	9	19	18	32	80	213	21	0	1	145	24	38	1	3
96	0	0	0	0	0	0	0	6	0	0	0	0	0	0	27	14	5	0	2	23	3	3	0	0	11	69	1	0	0	3	0
97	0	0	0	0	39	0	0	0	0	93	2	0	1	2	0	0	1	5	0	0	0	0	3	0	0	0	20	2	45	0	0
98	0	1	0	0	0	0	0	0	0	3	1	0	66	11	0	85	1	1	0	0	1	58	0	4	205	97	0	5	0	0	0
99	0	0	0	0	0	0	0	0	0	0	19	49	0	0	0	0	0	0	132	0	0	1	0	0	0	0	0	0	0	4	0
1900	3	0	0	0	23	41	14	7	13	0	25	0	0	0	0	0	0	0	0	1	3	0	0	0	33	2	4	0	0	0	0
01	0	168	0	0	0	0	0	0	1	0	0	68	0	0	1	5	0	0	0	0	0	306	17	0	0	0	0	69	12	0	0
02	5	1	0	0	38	70	22	0	0	9	0	0	11	1	0	0	0	0	3	1	1	0	0	0	0	0	15	1	0	0	0
03	0	0	0	0	62	0	0	0	0	0	0	0	17	8	3	0	0	0	1	0	0	8	1	0	31	7	64	26	0	0	0
04	0	2	98	0	1	0	0	4	0	0	0	0	0	13	32	0	0	0	0	9	14	1	0	0	0	2	14	0	0	0	2
05	0	0	6	12	29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	8	0	0	0	0	11	0	44	64
06	39	3	0	0	0	0	0	9	25	0	0	15	0	65	5	0	0	1	0	0	0	1	0	0	0	56	0	0	0	0	0
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10	0	0	0	0	0	0	0	0	8	2	5	129	19	49	88	1	0	0	0	0	11	61	21	0	0	0	0	34	82	14	6
11	0	1	8	0	0	9	8	0	2	0	0	207	708	3	23	38	19	104	22	1	0	13	19	34	26	86	176	0	3	0	14
12	11	0	0	0	0	105	14	0	0	0	0	0	0	1	0	1	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	1	38	18	0	0	0	0	0	0	0	0	0	0	5	1	5	0	1	0	0	0	0	0	0	0	0	0	0	0
14	7	0	2	1	0	0	1	0	0	0	0	15	7	0	0	0	0	0	0	23	0	0	0	0	0	0	0	9	0	0	0
15	1	11	0	0	0	0	0	0	55	10	0	0	17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	14	0	10
16	2	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	1	0	18	20	1	0	0	0	0	0	0	103	0	0
17	0	0	41	2	0	2	0	0	0	6	1	7	0	0	1	1	0	1	0	0	0	2	73	5	0	3	0	15	21	19	109
18	39	0	0	9	0	3	11	9	1	2	123	653	162	1	11	1	1	0	0	1	0	3	0	0	0	0	0	0	49	113	128
19	0	0	10	16	47	0	0	10	0	0	0	1	0	7	0	11	0	0	0	45	0	0	0	0	0	0	0	0	0	7	0
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23	0	0	0	0	0	29	0	0	0	0	2	4	1	0	0	6	0	0	0	0	0	0	0	1	0	140	0	0	0	0	0
24	0	10	6	0	0	11	3	0	0	0	0	0	0	0	0	0	0	0	0	0	134	70	130	74	1	40	1	0	0	0	0
25	0	0	0	0	13	19	0	0	1	5	0	0	0	0	1	174	56	27	0	0	41	0	70	5	0	3	0	0	16	15	26
26	3	15	0	0	5	4	1	27	17	0	0	0	9	1	0	0	0	0	0	265	3	1	0	0	0	0	0	0	2	0	0
27	0	0	0	0	1	1	21	0	103	8	0	0	0	0	0	0	2	121	0	1	0	115	7	0	0	2	4	19	6	0	0

APPENDIX (*Continued*)

Year \ Date	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
1928	0	0	0	0	6	64	0	20	42	9	0	0	0	12	27	36	0	0	0	18	0	0	2	1	0	0	0	1	18	0	2
29	0	0	0	0	0	0	4	12	0	0	0	0	0	0	3	0	0	0	0	4	0	0	0	0	0	4	5	0	0	0	0
30	41	0	0	0	1	0	0	0	0	0	2	59	1	0	0	1	15	0	0	0	0	0	0	302	4	18	10	3	0	1	0
31	0	12	24	0	3	0	0	5	6	0	0	0	0	7	9	6	1	6	20	0	7	0	0	18	23	7	0	47	1	0	0
32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	24	1	0	0	0	0	0	0	0
33	0	3	2	9	12	0	0	0	1	0	0	0	0	0	1	18	0	0	0	0	21	71	372	20	1	0	57	32	0	25	1
34	0	2	5	1	0	27	137	0	3	0	10	0	0	1	0	0	0	0	0	0	0	17	0	0	0	0	1	3	0	0	28
35	0	0	0	24	6	132	0	0	0	27	0	0	0	65	16	0	21	8	0	0	0	0	0	5	0	0	2	0	0	0	0
36	1	2	0	0	94	32	1	29	2	0	7	3	77	81	1	0	33	82	1	0	0	0	0	0	0	0	0	0	0	2	0
37	16	32	0	14	0	0	0	0	83	2	0	20	3	0	0	0	3	0	0	4	0	0	0	0	2	8	0	0	3	0	22
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	31	5	27	56	55	10	54	4	2	0	39	74	0	0	579
39	18	0	0	0	41	0	0	0	0	0	0	3	0	0	0	2	53	75	0	0	30	0	3	0	0	9	77	5	2	5	0
40	14	0	0	0	0	0	24	0	0	0	0	0	0	0	0	6	4	3	4	0	0	0	0	0	0	0	0	0	0	1	0
41	3	4	58	38	2	47	14	5	4	22	1	0	0	0	0	0	6	4	9	0	0	0	0	0	0	137	23	0	0	0	0
42	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	4	0	0	0	2	0	0	0	0	39	0	0	0	0	7
43	69	21	0	101	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	3	49	0	0	0	1	0	0	0	0
44	35	85	1	0	0	0	0	0	0	0	74	0	0	0	0	22	0	0	0	0	0	0	0	3	13	7	0	2	0	0	0
45	0	0	0	10	0	0	0	0	0	0	0	72	28	13	8	0	49	0	2	0	1	0	1	0	0	0	1	2	0	1	53
46	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	28	12	21	0	2	0	0	0	0	0	0	0	0
47	0	15	1	13	6	0	34	0	0	11	0	0	0	0	0	0	26	0	0	0	0	0	0	0	14	0	0	0	0	45	2
48	0	210	26	25	0	71	0	0	2	0	0	20	74	102	185	18	22	3	0	0	0	0	0	17	68	0	0	0	0	32	27
49	89	0	0	0	1	0	1	0	78	575	192	46	2	0	0	63	1	13	0	0	0	0	0	2	16	1	0	0	0	0	0
50	24	88	8	0	0	0	0	19	0	0	0	0	0	0	0	1	0	0	448	19	0	0	0	1	0	1	0	0	0	0	0
51	13	0	0	0	0	0	0	43	0	0	2	0	0	88	0	8	37	224	167	130	131	0	12	107	32	31	14	7	1	6	1
52	0	0	0	0	0	5	130	44	0	0	0	0	0	0	0	10	0	0	0	0	0	0	0	0	0	0	1	4	0	0	0
53	2	0	0	4	0	56	17	0	0	0	0	4	0	0	0	63	21	0	0	0	0	0	82	4	5	10	10	0	0	0	0
54	91	33	115	25	0	0	9	2	0	0	7	55	10	2	0	1	12	0	0	1	0	0	0	0	0	0	0	0	74	18	12
55	0	0	0	0	0	0	0	2	0	0	0	0	0	1	11	3	0	0	0	0	2	32	598	131	29	19	0	0	55	15	0
56	0	0	1	0	0	1	0	0	0	0	5	28	0	0	0	5	0	0	0	2	97	0	0	0	0	112	86	52	3	0	1
57	0	1	0	35	16	0	7	0	2	2	0	0	0	0	0	0	2	4	0	21	0	10	107	0	0	0	4	0	0	0	0
58	54	0	0	4	0	0	18	4	0	0	10	4	0	0	0	0	83	11	0	37	6	9	48	0	0	0	0	0	107	15	99
59	21	67	2	26	48	0	0	8	4	18	14	1	1	0	1	0	0	0	0	0	8	80	18	15	0	0	204	108	0	0	0
60	0	0	1	40	50	0	0	0	0	0	0	0	0	1	0	0	0	0	41	1	0	14	59	0	0	0	0	0	0	18	2
Total	882	984	685	975	865	1021	571	544	745	1211	903	2399	1905	1095	856	929	867	1431	1445	1081	1443	1696	2409	1080	830	1342	1603	1413	1146	1129	1511