A CORRECTED FORMULA FOR THE SPECTRAL SENSITIVITY FUNCTION IN RADIO ASTRONOMY*

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According to Bracewell and Roberts (1954) the spectral sensitivity function $\bar{A}(s)$ is calculated from an aerial aperture distribution $E(x/\lambda)$ by

$$\bar{A}(s)=\frac{\int_{-\infty}^{\infty} E(x/\lambda-s)E^*(x/\lambda)\mathrm{d}(x/\lambda)}{\text{normalizing factor}}.$$

The derivation limits the result to one-dimensional apertures and to aerials of high directivity. From this they concluded that, since aperture distributions cut off, so also do spectral sensitivity functions.

This conclusion is not strictly correct because a real aerial cannot be represented by an aperture distribution that cuts off at a finite boundary. There is always a fringing field, of extent comparable with the wavelength, and this fringe makes the boundary fuzzy. Then, in turn, the spectral sensitivity function will have a tail.

It is convenient, however, and customary, to think in terms of arrays of infinitesimal elements or of apertures cutting off discontinuously at their edges.

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The Fourier components in the corresponding aperture distributions having crest-to-crest distances less than λ should be omitted before the radiation pattern is considered, because the fine-scale components are associated with evanescent modes that do not contribute to the distant radiated field. By filtering out the fine-scale components we round off the discontinuities of the aperture distribution and introduce a tail. The corresponding corrected spectral sensitivity function $\bar{A}_{\text{corr.}}(s)$ will be obtained merely by filtering $\bar{A}(s)$ and so is given by

$$\bar{A}_{\text{corr.}}(s) \propto \int_{-\infty}^{\infty} \bar{A}(u) \frac{\sin 2\pi(s-u)}{\pi(s-u)} \mathrm{d}u.$$

The full version of this formula is necessarily two-dimensional:

$$\bar{A}_{\text{corr.}}(s_x,s_y) \propto \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{J_1[2\pi(u^2+v^2)^{\frac{1}{2}}]}{(u^2+v^2)^{\frac{1}{2}}} \bar{A}(s_x-u,s_y-v)\mathrm{d}u\mathrm{d}v.$$

This new formula gives a superior representation in the pass band in the case of arrays of infinitesimal elements, for it properly smooths out the impulses in $\bar{A}(s)$ when $E(x/\lambda)$ is impulsive and, in particular, removes the normalizing difficulty when $\bar{A}(s)$ is impulsive at $s=0$. It is true that no difficulty has arisen over the spectral sensitivity function of conventionally designed dipole arrays, because they have been treated like their equivalent continuous apertures; but the new formula would be helpful in discussing an unusually spaced array or a slot array operated off frequency, for example.

The formula also gives numerical values for the sensitivity beyond the nominal cut-off, answering a problem posed by Lo (1961) and see also Bracewell (1962). The theory of restoration in the presence of errors (Bracewell 1958) will normally tell us to ignore contributions beyond the nominal cut-off, and so the cut-off is an effective one for radio astronomy.

References

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