ELECTRICAL SCANNING AND LOW PASS FILTERING IN RADIO ASTRONOMY*

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In previous studies of aerial smoothing in radio astronomy it has been supposed that the scanning is produced by mechanically moving the aerial. However, the introduction of a phase gradient across the aperture permits electrical scanning, and the theory of this is an interesting aspect of the general aerial smoothing problem.

Let the two-dimensional tangential aperture distribution of electric field be $E(x/\lambda,y/\lambda)$, polarized either in the y- or x-direction. Its Fourier transform is given by

$$P(l,m)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp\{-i2\pi[l(x/\lambda)+m(y/\lambda)]\}E(x/\lambda,y/\lambda)\mathrm{d}(x/\lambda)\mathrm{d}(y/\lambda).$$

The quantities l and m are direction cosines, and $P(l,m)$ is the *angular spectrum* (Booker and Clemmow 1950) of the plane waves, radiated and evanescent, set up in the (real and imaginary) directions (l,m) by the two-dimensional aperture distribution.

Let $P_0(l,m)$ be the angular spectrum associated with some particular aperture distribution $E_0(x/\lambda,y/\lambda)$ and now consider the effect of introducing a phase gradient across this distribution, i.e. consider

$$E(x/\lambda,y/\lambda)=\exp\{i2\pi[L(x/\lambda)+M(y/\lambda)]\}E_0(x/\lambda,y/\lambda).$$

By substitution in the defining formula for the angular spectrum we find

$$P(l,m)=P_0(l-L,m-M).$$

Thus, in the (l,m)-plane, the pattern $P_0(l,m)$ is shifted without change of form as the result of the introduction of an electrical phase gradient.

A real direction (l,m) corresponds to a zenith angle χ given by $\sin\chi=(l^2+m^2)^{\frac{1}{2}}$, and since $\sin\chi$ cannot exceed unity for real values of χ, it follows that real directions correspond to points on the (l,m) plane that are on or within the horizon circle $l^2+m^2=1$. These points can be regarded as orthogonal projections, onto the aperture plane, of the intersection of a celestial hemisphere of unit radius with a line leaving the antenna in a direction (l,m). This projection was described by Scheuer (1954). Points outside the horizon circle represent the imaginary zenith angles associated with evanescent fields. Confining attention to real directions we can say that the spreading (associated with aperture foreshortening) of an electrically shifted beam is compensated by the orthogonal

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projection, thus producing a simple shift of $P_0(l,m)$ without change of form. As the beam is shifted, we see very graphically how parts of it slide over the horizon circle into the evanescent zone.

Let the distribution of celestial brightness temperature be $T(l,m)$. Because of the remoteness of the celestial objects, $T(l,m)$ will be zero where $l^2+m^2>1$, and it is therefore fully represented by projecting the distribution over the celestial hemisphere onto the aperture plane. The observed antenna temperature $T_a(L,M)$, when the beam is shifted to (L,M), is then given by

$$T_a(L,M)=K\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(1-l^2 \text{ or } m^2)P_0(l-L,m-M)P_0^*(l-L,m-M)T(l,m)\mathrm{d}l\mathrm{d}m/n,$$

where K is a normalizing factor and the factor $(1-l^2$ or $m^2)$ depends on whether the tangential component of the aperture electric field is parallel to the y- or x-axis, and the direction cosine n is given by $l^2+m^2+n^2=1$.

The normalizing factor K is chosen, in accordance with custom, so as to make the observed antenna temperature T_a equal to T when a uniform sky brightness temperature T exists in all directions to which the aperture can radiate (i.e. where $l^2+m^2<1$). If the angular spectrum spills over the horizon circle, the renormalization called for by the conversion of radiated to evanescent power is taken care of by K. Hence finally

$$T_a(L,M)=\frac{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(1-l^2 \text{ or } m^2)P_0(l-L,m-M)P_0^*(l-L,m-M)T(l,m)\mathrm{d}l\mathrm{d}m/n}{\iint_{l^2+m^2<1}(1-l^2 \text{ or } m^2)P_0(l-L,m-M)P_0^*(l-L,m-M)\mathrm{d}l\mathrm{d}m/n}.$$

This result becomes quite simple for the high directivities and small beam shifts appropriate to radio astronomy. If these are such that the angular spectrum does not spread out towards or over the horizon circle, then the factor $(1-l^2$ or $m^2)$ is approximately unity over the significant range of integration, and the denominator is independent of L and M. We now obtain the result in the form of a standard convolution integral

$$T_a(L,M)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathbf{A}(L-l,M-m)T(l,m)\mathrm{d}l\mathrm{d}m,$$

where

$$\mathbf{A}(L-l,M-m)=\frac{P_0(l-L,m-M)P_0^*(l-L,m-M)}{\text{normalizing factor}}$$

is the quantity that we refer to as the response to a point source.

The projection suggests a way of extending the concepts of spatial frequency and sinusoidal components so as to apply to more than the small zone of the celestial sphere that is ordinarily considered under the high directivity approximation. Thus, a general sky distribution can be analysed into components which, when projected onto the aperture plane, are pure sinusoidal components. In other words, one Fourier analyses the projected sky distribution.

It is not obvious, however, that the concept of spectral sensitivity function, or filter characteristic, can also be extended. This concept depends on sinusoidal response to sinusoidal input, the necessary and sufficient condition for which is the existence of a convolution relation between the input (sky temperature) and output (antenna temperature). However, the full antenna smoothing integral only reduces to the familiar convolution integral when, by assuming high directivity, one is content to rule rectangular Cartesian coordinates on a zone of spherical sky (Bracewell and Roberts 1954). The special case of electrical beamswinging leads to a convolution integral on the aperture plane, but again, only for high directivity.

References

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