ON THE SWARM METHOD FOR DETERMINING THE RATIO OF ELECTRON DRIFT VELOCITY TO DIFFUSION COEFFICIENT

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Summary

The use of data from swarm experiments for electron energies approaching those corresponding to thermal equilibrium demands results of greater precision than has hitherto been available. In order to examine the possibility of producing such data, the swarm method for determining W/D has been extensively examined over a range of values of the parameter E/p where the agreement between the results of recent investigations is not good. A number of factors influencing the accuracy of measurements of this type are discussed. The results for hydrogen which are presented are considered to be subject to an error of less than 1%.

I. Introduction

The results of swarm methods for determining the ratio W/D of electron drift velocity to diffusion coefficient have found application in a number of recent papers in which collision phenomena between low energy electrons and gas molecules have been discussed (Gerjouy and Stein 1955 ; Huxley 1956, 1959 ; Shkarofsky, Bachynski, and Johnston 1961 ; Frost and Phelps 1961). For some of these applications, more especially those dealing with collision phenomena for electrons with mean energy of several electron-volts, the accuracy of existing data is sufficiently good. Not only are the results of a number of investigations substantially in agreement for this energy range but the degree of accuracy to which they have been obtained is adequate for most purposes. On the other hand, for those applications where the difference in energy between the electrons and gas molecules is important, small errors in the determination of W/D become significant as this difference approaches zero. Unfortunately it is in this energy range that the measurements become more difficult and the agreement between the results from various laboratories is not good.

In this paper an account is given of a systematic investigation of the swarm method for determining W/D using an apparatus which enables the dimensions of the diffusion chamber to be quickly and simply varied. A number of possible sources of error has been investigated both theoretically and experimentally, as a result of which it has been possible to determine the experimental procedures which lead to results of maximum accuracy. The results which have been obtained where these procedures were followed show good agreement over a wide range of values of the experimental parameters and it is considered that the values of W/D in hydrogen from $0{\cdot}1 < E/p < 5{\cdot}0$ V cm^{-1} mmHg^{-1} which are presented are subject to an error of less than 1%.

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II. Experimental Method and Apparatus

(a) *Theory of the Method*

The principle of the method was the same as that used by Crompton and Sutton (1952). Their apparatus was based on Huxley's modification of Townsend's original method of analysing the lateral diffusion of a stream of electrons (Huxley 1940—see Fig. 1). Electrons generated by a heated platinum filament enter the diffusion chamber through the hole of 1 mm diameter in the cathode after having drifted through a region below the hole in which the electric field is the same as the field in the diffusion chamber itself. Because of the relatively high gas pressures employed, ranging from 5 to 40 mmHg in the present experiments, this procedure ensures that the electrons have acquired a constant mean agitational energy before entering the diffusion space. The receiving

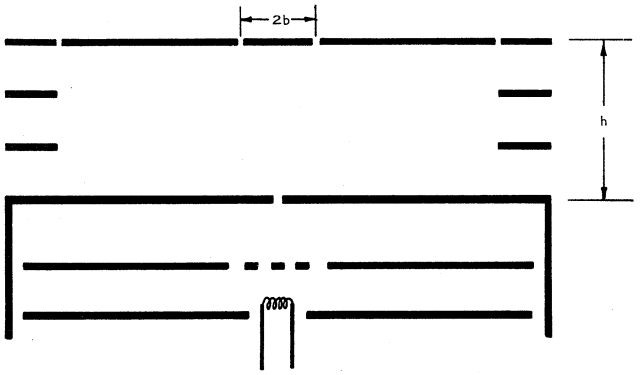

Fig. 1.—Schematic diagram of diffusion apparatus.

electrode is divided into insulated portions comprising a central disk and surrounding annuli to enable the distribution of electron concentration within the diffuse electron stream arriving at it to be analysed.

The differential equation describing the electron concentration within the diffusion space is

$$\nabla^2 n = W/D\,\partial n/\partial z = 2\lambda\,\partial n/\partial z, \tag{1}$$

where the z axis is parallel to the electric field. A general solution of equation (1) may be obtained in the form of a series whose $(k+1)$th term is

$$A_k e^{\lambda z}(\lambda r)^{-\frac{1}{2}} K_{k+\frac{1}{2}}(\lambda r) P_k^m(\cos\theta)\cos m\varphi.$$

Two simple solutions are the terms for which $m=0$ and $k=0$ and 1. These solutions are (Huxley 1940)

$$n = A e^{\lambda z} e^{-\lambda r}/r, \tag{2}$$

and

$$n = A e^{\lambda z}\,\frac{z}{r}\,\frac{\mathrm{d}(e^{-\lambda r}/r)}{\mathrm{d}r}, \tag{3}$$

where r is the distance measured from the origin (Fig. 2).

The boundary conditions at the upper and lower electrodes ($n=0$ everywhere at the upper electrode, $n=0$ except at the origin at the lower electrode) can in

effect be satisfied by the use of the dipole solution (3) if it is supposed that, in addition to the source at the origin, an image source of suitable strength is placed at the point (0, 0, $2h$) to make $n=0$ over the anode. The equation for n then becomes

$$n=Ae^{\lambda z}\left[\frac{z}{r}\frac{\mathrm{d}}{\mathrm{d}r}(e^{-\lambda r}/r)+\left(\frac{z-2h}{r_1}\right)\frac{\mathrm{d}}{\mathrm{d}r_1}(e^{-\lambda r_1}/r_1)\right], \tag{4}$$

where r_1 is the distance measured from the point (0, 0, $2h$).

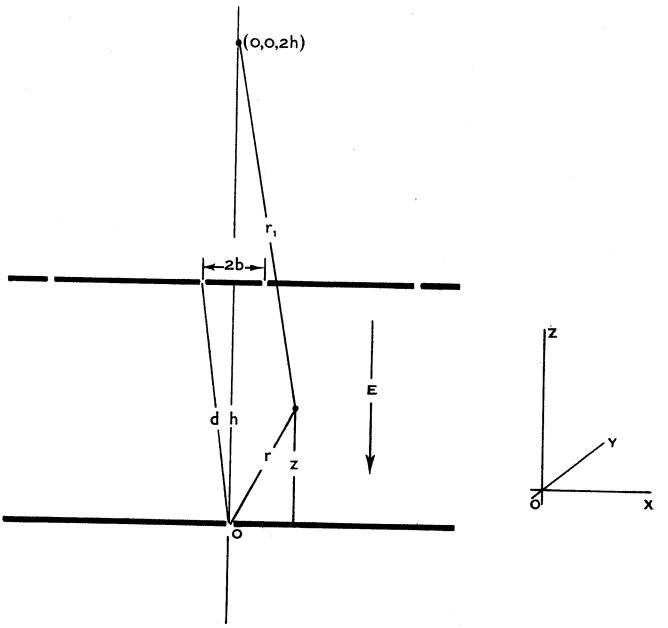

Fig. 2

If i_b is the current falling on the central disk of radius b and i is the total current arriving at the receiving electrode then it may be shown that the ratio $R=i_b/i$ is given by

$$R=1-(h/d-1/\lambda h+h/d^2\lambda)(h/d)e^{-\lambda(d-h)}, \tag{5}$$

where h is the distance between the cathode and anode and $d=\sqrt{(h^2+b^2)}$.

In practice, this solution, which accurately satisfies the prescribed boundary conditions, is known not to accord with the results of experiment (Huxley and Crompton 1955).* The expression for R which has been found to lead to consistent experimental results is

$$R=1-(h/d)e^{-\lambda(d-h)}. \tag{6}$$

In Section III (b) (i) it will be shown that, under certain experimental conditions, the same values of λ are obtained by analysing the experimental results using

* The equation given here was based on the diffusion flux to the anode from a single dipole source with $n=0$ over the anode. It differs from equation (5) only for small values of λh.

either equation (5) or equation (6). On the other hand, it is also possible to choose conditions which lead to significant differences between the results analysed with the two equations, and it is found that the use of equation (6) leads to results which are everywhere consistent whereas equation (5) does not. Equation (6) follows either from the use of equation (2) with the inclusion of an image term to make $n=0$ at $z=h$, or from the use of equation (3) without the inclusion of such a term. In the second case the flux to an element of the anode is taken to be proportional to $n\mathrm{d}S$, that is, the same as the flux to a geometrical plane at the anode (Townsend 1948). The empirically correct formula (6) for the current ratio results, therefore, from the use of solutions of the differential equation for n neither of which satisfies simultaneously the mathematically prescribed boundary conditions at the anode and cathode. It is apparent, therefore, that the actual boundary conditions are not those which might be expected, but it is impossible to decide from the results of experiments of the type described in the present paper which of the alternative solutions leading to equation (6) describes more accurately the physical boundary conditions in the apparatus. Nevertheless, the analysis of the results in Section III (*b*) (i) leaves no doubt as to the validity of this equation over a wide range of values of the parameters.

Measurement of the ratio R in an apparatus of known dimensions enables the quantity $\lambda=W/2D$ to be determined. A convenient parameter, which not only enables results taken under different experimental conditions to be compared but which also serves as an estimate of the ratio of mean electron energy to mean gas molecular energy, is the quantity k_1 defined through the relation

$$W/D=N_0eE/RTk_1, \tag{7}$$

where N_0 is Avogadro's number, e the electronic charge, E the electric field strength, R the gas constant, and T the absolute temperature.

It should be noted that the factor k_1 is related directly to the ratio W/D through equation (7) and that this ratio of the two macroscopic quantities W and D describing the diffusing electron stream is obtained directly from the measured current ratios by using equation (6). Consequently no assumption regarding the nature of the microscopic collision processes between electrons and gas molecules is required in deriving the values of k_1 from the experimentally measured quantities. In order to determine the true mean energy from the energy factor k_1 it is necessary to make certain assumptions about the behaviour of the individual electrons.

(*b*) *Apparatus*

The diffusion apparatus used in the present investigation was designed with a twofold objective. First, an apparatus of variable length was required for an investigation of electron attachment coefficients (Huxley, Crompton, and Bagot 1959). Secondly, since it is frequently necessary to take measurements for a wide range of values of the ratio W/D, depending on the nature and pressure of the gas and the electric field strength, an apparatus in which both the mode of division of the receiving electrode and the length can be widely varied is

essential if precise measurements are to be made. For example, the properties of the electronic motion in the inert gases make it preferable to use a short diffusion chamber with a central disk of large diameter to obtain approximately

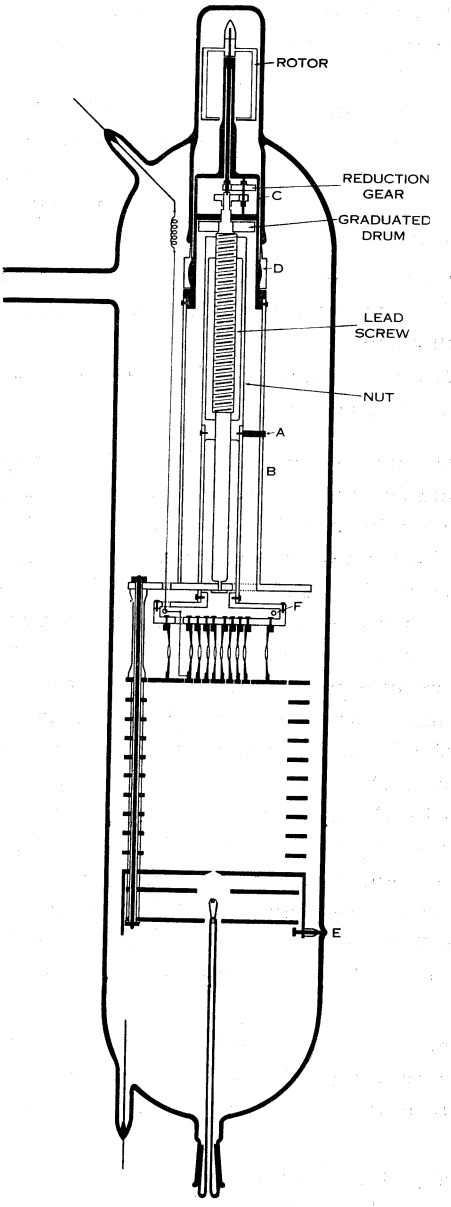

Fig. 3.—Cross section through the apparatus.

equal currents to the disk and outer annulus. At the other extreme a long chamber and small central disk are required to perform accurate measurements with electrons near thermal equilibrium regardless of the nature of the gas.

Figure 3 shows a cross section through the apparatus which was designed to the following specifications to satisfy the requirements outlined above :

(i) length of diffusion chamber to be variable from 1 to 10 cm with length setting accurate to 0·002 cm,
(ii) receiving electrode to be divided to enable the diameter of the central disk to be 1, 2, 3, or 4 cm,
(iii) variation in chamber length to be achieved without recourse to an external drive in order to eliminate sources of contamination such as shaft seals,
(iv) the use of non-magnetic materials throughout to eliminate stray magnetic fields,
(v) the avoidance of organic materials such as polytetrafluoroethylene (PTFE) in the construction and the use of any form of organic lubricant in the mechanism,
(vi) the apparatus to be constructed in such a way that outgassing at low temperatures is possible.

The drive for the mechanism used to vary the length consists of a copper rotor driven by a rotating magnetic field generated by the field coil of a small Magslip motor which can be placed over the section of the glass envelope encasing the rotor. The small torque which results is increased by means of the reduction drive of 25 : 1 in the gearbox and is then sufficient to drive the stainless steel lead screw which has a pitch of 1 mm. The nut which engages with the lead screw, and its extension, are made of a pure copper-tin alloy of composition such that its coefficient of expansion matches that of the stainless steel. Lateral motion of the receiving electrode attached to the extension of the nut is reduced to a minimum by the large separation which exists between the nut and the bearing which traverses the unthreaded section of the lead screw. Rotation of the nut is prevented by the pin A which runs in a guide slot cut in the stainless steel tube B. The bearings in the gearbox and the thrust bearings at the lower ends of the rotor shaft and lead screw are watchmakers' jewel bearings ; these and all other bearing surfaces are unlubricated. The micrometer wheel, graduated in hundredths of a millimetre, is used in conjunction with an index mark on the edge of a window cut in the casing C to position accurately the height of the electrode.

The extension attached to the nut carries a plate upon which are mounted the sections of the receiving electrode. Longitudinal slots in the extension coincide with the arms of a " spider " supporting the thrust bearing for the lead screw.

The central disk, of a nominal diameter of 1·0 cm, and the surrounding annuli, of nominal outside diameter 2, 3, 4, and 8·5 cm, which comprise the receiving electrode, require mounting in a manner which satisfies the following conditions :

(i) an insulation resistance of the order of 10^{14} Ω must exist between each section and between each section and the baseplate,

(ii) it must be possible to locate the sections accurately concentric with the axis of the apparatus,

(iii) the support must be sufficiently rigid to enable the face of the assembled electrode to be ground and polished to ensure a high order of co-planarity and surface finish.

These requirements were met by mounting each section on separate double-ended Housekeeper seals. In construction, the disk and each annulus was brazed to the appropriate seal and the seals then accurately positioned and bolted to the baseplate. The face of the whole assembly was then lapped and polished to a high surface finish. The dimensions of the sections of the electrode were measured in each case to the centres of the air gaps separating them. The two inner air gaps were 0·025 cm wide, whereas the outer ones were 0·05 cm wide.

The remainder of the construction of the diffusion chamber followed the design of similar apparatus described by Crompton and Sutton (1952). The faces of the ten guard electrodes are situated at 1 cm intervals from the face of the cathode so that integral values for the length of the diffusion space are obtained by setting the face of the receiving electrode co-planar with the face of any guard electrode. All the electrodes, including anode and cathode, were made of nichrome which was highly polished before being electroplated with a heavy deposit of gold.

The small clearance between the copper rotor and the glass envelope necessitates some form of adjustment at this point; furthermore, rigid mounting of the whole assembly would impose excessive strain on the mounting for any position other than the vertical one. To overcome these difficulties the assembly is carried on a ball-and-socket joint D which is brazed to a large diameter Housekeeper seal immediately above it. Small adjustment for centring the rotor is provided by three threaded studs E.

Leads from the guard electrodes are taken to tungsten-glass seals (of which only one is shown in Fig. 3) at the bottom of the glass envelope. Leads from the sections of the receiving electrode are brought to a circular glass ring F in which are sealed a number of right-angled tungsten rods. Connections are made to the tungsten glass seals at the top of the envelope by means of long phosphor-bronze springs which serve to keep the leads taut for all positions of the movable electrode.

Great care was taken to ensure cleanliness in the assembly of the apparatus with the result that pressures of the order of a micron can be maintained over a period of days without prior outgassing or the use of a liquid-air trap. During the course of the experiment an initial vacuum of better than 10^{-4} mmHg (the limit of the gauge used to measure the pressure) was obtained using liquid-air traps to reduce the level of condensable impurity to a minimum. Hydrogen was admitted to the apparatus through a palladium osmosis tube, the precautions described by Crompton and Elford (1962) being observed. Pressures were recorded on a precision capsule manometer (Crompton and Elford 1957) which was calibrated at the pressures used by a dead-weight pressure standard. The overall error in the measurement of pressure is of the order of 0·5% at the lowest pressures and somewhat less at higher pressures.

Voltages applied to the electrodes were derived from a Fluke type 301E power supply, which enabled the voltages to be set with an error of less than 0·1%.

In order to render negligible the effect of space-charge repulsion the total electron current entering the diffusion chamber was restricted to about 3×10^{-12} A. Checks were made at currents of 1 and 4×10^{-12} A but no change in the current ratios R could be detected. The ratios were measured with an error of the order of 0·1% by using a modification (to be described elsewhere) of the method employed by Crompton and Sutton. As in their experiments the currents to the sections of the receiving electrode were measured simultaneously while ensuring that each section remained at earth potential.

III. Results

(*a*) *Factors Determining Choice of Parameters*

Since the main aim of these experiments was to investigate the reliability of the method, measurements were made using as wide a range as possible of the parameters which could be varied, namely, those determined by the geometry of the diffusion chamber and the gas pressure. The following factors influenced the choice of the combination of the parameters used:

(1) The minimum value of electric field used was 3·0 V cm^{-1}. Below this value the influence of contact potential differences within the chamber is likely to become excessive in spite of the use of gold-plated surfaces to minimize such effects. The maximum voltage which could be applied in this set of experiments was 240 V, the limitation being electrical breakdown elsewhere in the apparatus.

(2) The pressure gauge covered the range 0–40 mmHg. At 5 mmHg the gauge cannot be read to much better than 0·5%; consequently pressures in the range 5–40 mmHg were used.

(3) The currents to the two portions of the receiving electrode were measured so that the smaller current was obtained as a fraction of the larger, the ratio being read from a 10-turn helical potentiometer with a resolution of better than one part in a thousand. Thus, for example, when the smaller current is 50% of the larger the ratio of the currents can be determined to 0·2%.

(4) Results were recorded using the maximum length of the diffusion chamber obtainable (h=10 cm), using h=2 cm, and using an intermediate value of h=5 cm. In hydrogen the limitations of pressure and electric field described above limit the number of values which can be recorded at h=1 cm: the wider range of measurements possible for h=2 cm determined the minimum value of h employed.

For each value of h and choice of the mode of division of the receiving electrode results were recorded at each pressure provided the distribution of current was such that the limitation of accuracy in the measurement of the

current ratios R enabled k_1 to be determined to 0·5% or better. The following combinations of h and electrode division were employed :

$$h=10 \text{ cm}; \quad b/h=0{\cdot}05,\ 0{\cdot}10,\ 0{\cdot}15$$
$$h=5 \text{ cm}; \quad b/h=0{\cdot}1,\ 0{\cdot}2$$
$$h=2 \text{ cm}; \quad b/h=0{\cdot}25.$$

For $h=10$ cm, ratios of the current received by the smallest annulus to that received by the remaining annular portion of the electrode were also measured while keeping the central disk earthed.

Table 1 records the results of the experiment. The entries in the table printed in italics are calculated from measurements for which the current ratios lie outside the accepted range defined above and are not, therefore, used in calculating average values. The mean value of k_1 for any given value of E/p is obtained by averaging all other results obtained using different gas pressures and geometries of the diffusion chamber; corresponding values of W/pD are also recorded. The r.m.s. deviation is always less than 0·5% and is of the order of 0·3% or less for all E/p greater than 0·2 V cm^{-1} mmHg^{-1}, while the largest discrepancy within the results for a given value of E/p is of the order of 1%.

In the following section an analysis is given of the errors to which the results obtained for a given set of experimental conditions may be subject. This analysis indicates the experimental conditions which must be used to obtain maximum accuracy.

(*b*) *Analysis of Sources of Error*

(i) *Solution of the Differential Equation.*—Equation (6) was used to calculate the results presented in Table 1. The validity of analysing the results of experiments of this type by the use of this equation has been established by Huxley and Crompton (1955) but the wider choice of the parameters d and h and hence of λ which is possible in the present experiments enables a more exacting check to be made.

A comparison of equation (6) with equation (5) shows that the largest discrepancy between them occurs for small values of λh and the ratio h/d; consequently, if the results are analysed using the alternative solutions, the largest discrepancies between the values of λ_P obtained by using equation (6) and the values λ_D obtained by using equation (5) will occur for small values of λ in a short apparatus if the diameter of the central electrode remains constant. This prediction is verified by Figure 4 which shows the magnitude of the discrepancy as a function of the current ratio R for $h=10$ and 2 cm with $b=0{\cdot}5$ cm in each case.

The smallest ratio R that can be measured with reasonable accuracy is $R=0{\cdot}1$. For $h=10$ cm this value of R corresponds to $Eh/k_1=4$ and Figure 4 shows that the discrepancy between the two solutions is then about 1·3%. On the other hand, for values of $Eh/k_1>20$ the discrepancy is less than 0·25%. Since the assumption as to the nature of the source then becomes unimportant, values of k_1 which are determined in an apparatus in which $h=10$ cm and $b/h=0{\cdot}05$ and where the experimental conditions are such that $Eh/k_1>20$ can be

TABLE 1

VALUES OF k_1 RECORDED IN HYDROGEN

h	← 10 →												← 5 →					← 2 →			Average Value	W/pD (cm^{-1} $mmHg^{-1}$)
b/h	0·05	0·1	central disk earthed	0·15	0·05	0·10	central disk earthed	0·15	0·05	0·10	central disk earthed	0·05	0·10	0·20	0·10	0·10	0·10	0·25	0·25	0·25		
p (mmHg)	5	5	5	5	10	10	10	10	20	20	20	40	5	5	10	20	40	5	10	20		
E/p (V cm^{-1} $mmHg^{-1}$)																						
0·10												1·81					1·82				1·81	2·18
0·15									2·25	2·26	2·26	2·24				2·27	2·24			2·27	2·26	2·63
0·20									2·67	2·69	2·68	2·67				2·69	2·67			2·69	2·68	2·95
0·30					*3·59*	3·58	3·57	3·56	3·56	3·57	3·55	3·55			3·58	3·57	3·56		3·58	3·56	3·57	3·33
0·40					*4·49*	4·47	4·45	4·45	4·45	4·45	4·44	4·45			4·47	4·47	4·46		4·45	4·45	4·45	3·56
0·50					*5·37*	5·35	5·35	5·35	5·34	5·34	5·33	5·34			5·36	5·36	5·36		5·33	5·34	5·35	3·70
0·60	*6·22*	6·23	6·22	6·21	*6·26*	6·23	6·21	6·22	6·21	6·20	6·20	6·22	*6·30*	6·23	6·24	6·24	6·24	6·23	6·21	6·22	6·22	3·82
0·70	*7·07*	7·06	7·05	7·06	*7·08*	7·07	7·04	7·05	7·05	7·05	7·06		*7·13*	7·09	7·09	7·09	7·10	7·07	7·07		7·07	3·92
0·80	*7·87*	7·84	7·86	7·86	*7·88*	7·85	7·85	7·87	7·84	7·85	7·86		*7·93*	7·89	7·88	7·89	7·88	7·87	7·84		7·86	4·03
0·90	*8·60*	8·59	8·60	8·60	*8·63*	8·60	8·60	*8·65*	8·59	8·60	8·59		8·66	8·64	8·64	8·66	8·64	8·62	8·59		8·62	4·13
1·00	*9·31*	9·29	9·29	9·31	9·33	9·32	9·29	*9·36*	9·30	9·29	9·29		9·36	9·31	9·34	9·36	9·35	9·34	9·29		9·32	4·25
1·20	*10·68*	10·60	10·60	10·62	10·65	10·64	10·61						10·71	10·62	10·65	10·65		10·67	10·61		10·64	4·46
1·50	*12·39*	12·33	12·37	12·40	12·40	12·38	12·36						12·45	12·41	12·40	12·43		12·40	12·36		12·39	4·79
1·80	*13·91*	13·88	13·93	13·98	13·96	13·93	13·92						14·01	13·93	13·96	14·00		13·97	13·94		13·95	5·11
2·00	*14·90*	14·84	14·89	14·95	14·95	14·91	14·87						14·97	14·91	14·94	14·98		14·93	14·86		14·9	5·31
2·40	*16·69*	16·65	16·67	16·74									16·78	16·67	16·75			16·72	16·72		16·7	5·69
3·00	*19·13*	19·09	19·11	19·23									19·23	19·22	19·24			19·21	19·20		19·2	6·19
4·00	*22·83*	22·74	22·79	22·89									22·96	22·89	22·99			22·88	22·93		22·9	6·92
5·00													26·08	26·24				26·26	26·12		26·2	7·56

used as standards of reference to determine the correctness of alternative formulae for R under other conditions.

The upper curve of Figure 4 shows that the choice of the incorrect expression for R can result in much more serious errors in the reduction of the data when $h=2$ cm. At the same time it is evident that the large discrepancies which result from the use of the different expressions enable the choice of the correct one to be made either by

(1) comparing the results derived from the alternative expressions with the " standard values " obtained for $h=10$ cm, or

(2) comparing the results obtained at the same values of E/p but at different pressures so that the results are taken over a range of values of Eh/k_1.

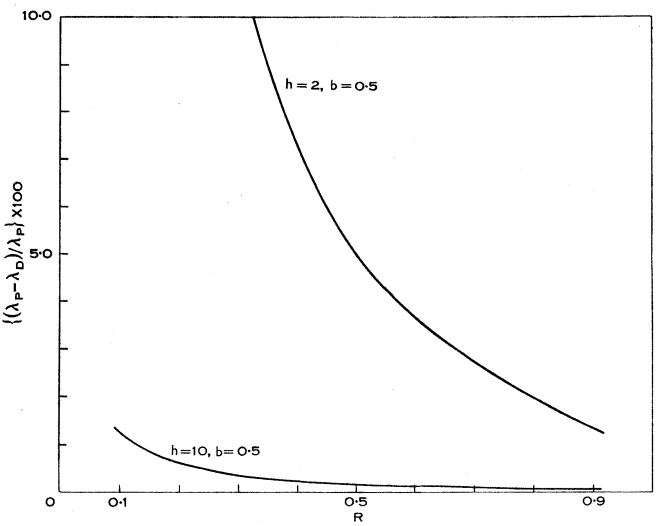

Fig. 4.—Curves showing the discrepancy (expressed as a percentage) between the values of λ derived by using the alternative equations (5) and (6).

In the present experiments the conditions imposed by the considerations discussed in Section III (a) were such that, for $h=2$ cm, the minimum value of Eh/k_1 used was just less than $1\cdot0$ (corresponding to $R=0\cdot47$), which occurred when $p=5$ mmHg and $E/p=0\cdot6$ V cm^{-1} mmHg^{-1}. The analysis of this result using the alternative expressions for R will lead to the largest discrepancies. On the other hand, a result is recorded at the same value of E/p for $h=10$ cm and $p=40$ mmHg with $Eh/k_1\approx40$ (corresponding to $R=0\cdot63$). Figure 4 shows that the discrepancy arising from the use of the alternative expressions for the evaluation of this result is of the order of $0\cdot1\%$ so that this result can be used as the standard for comparison.

Tables 2 and 3 contain an analysis of all the results obtained at $E/p=0\cdot6$ V cm^{-1} mmHg^{-1} for $b=0\cdot5$ cm and $h=2$ and 10 cm, based on equations (5) and (6) respectively. From these tables it is clear that agreement within

experimental error is obtained by the use of equation (6) but that the agreement resulting from the use of equation (5) is poor.

Table 4 contains all the results obtained for $h=2$ cm calculated using the alternative formulae. In each case the upper entry is the value calculated by using equation (5) the lower being calculated using equation (6). The table again demonstrates the superiority of equation (6).

TABLE 2

RESULTS ANALYSED USING EQUATION (5)

$E/p=0\cdot6$ V cm^{-1} mmHg^{-1}

p (mmHg)	5	10	20	40
$h=10$ cm ..	*6·29*	6·29	6·22	6·23
$h=2$ cm ..	6·62	6·39	6·31	—

(ii) *Axial Alignment of Source Hole.*—The provision of a movable electrode within the apparatus to vary the length of the diffusion chamber makes the accurate alignment of the centre of the receiving electrode on the axis of the apparatus more difficult than is the case when a rigid assembly of fixed length is used. Nevertheless, the provision of a large distance (11 cm) between the nut and the guide bearing (Fig. 3) and small tolerances between the bearing

TABLE 3

RESULTS ANALYSED USING EQUATION (6)

$E/p=0\cdot6$ V cm^{-1} mmHg^{-1}

p (mmHg)	5	10	20	40
$h=10$ cm ..	*6·22*	6·26	6·21	6·22
$h=2$ cm ..	6·23	6·21	6·22	—

surfaces reduce errors in alignment to a minimum. The concentricity of the movable electrode within each of the guard electrodes, which themselves are accurately aligned, is a useful test of alignment, and it is considered that the centre of the electrode remains at least to within 0·020 cm of the axis in all positions.

To determine the magnitude of the errors arising from misalignment a series of numerical integrations was performed to calculate the current ratios R for a number of combinations of b and h and a series of values of λ for increasing values of the distance of the centre of the disk from the axis. A typical curve showing the variation of R with this distance for a given geometry and a fixed value of λ is shown in Figure 5. Figure 6 shows the variation of the percentage error in the measured value of W/D (or k_1) as a function of the ratio R for a number of combinations of the parameters b and h for the case where the centre of the disk is off-axis by 0·020 cm. The errors from this cause are likely to be

TABLE 4

COMPARISON OF THE RESULTS FOR $h=2$ CM, USING THE ALTERNATIVE SOLUTIONS

E/p (V cm^{-1} mmHg^{-1})	$p=5$ (mmHg)	$p=10$	$p=20$	E/p (V cm^{-1} mmHg^{-1})	$p=5$ (mmHg)	$p=10$
0·15			*2·32*	1·0	*9·87*	*9·51*
			2·27		9·34	9·29
0·2			*2·74*	1·2	*11·20*	*10·85*
			2·69		10·67	10·61
0·3		*3·70*	*3·61*	1·5	*12·96*	*12·62*
		3·57	3·56		12·40	12·36
0·4		*4·58*	*4·52*	1·8	*14·57*	*14·24*
		4·45	4·45		13·97	13·94
0·5		*5·49*	*5·42*	2·0	*15·55*	*15·17*
		5·33	5·34		14·93	14·86
0·6	*6·62*	*6·39*	*6·31*	2·4	*17·35*	*17·03*
	6·23	6·21	6·22		16·72	16·72
0·7	*7·49*	*7·27*		3·0	*19·88*	*19·47*
	7·07	7·07			19·21	19·20
0·8	*8·34*	*8·04*		4·0	*23·58*	*23·26*
	7·87	7·84			22·88	22·93
0·9	*9·11*	*8·81*		5·0	*27·01*	*26·52*
	8·62	8·59			26·26	26·12

everywhere considerably less than indicated in Figure 6 and the results in Table 1 support this view.

(iii) *Finite Size of Source Hole.*—By considering the source as made up of a series of annular sources of increasing diameter the calculations discussed in the previous section can be extended to determine the magnitude of the error

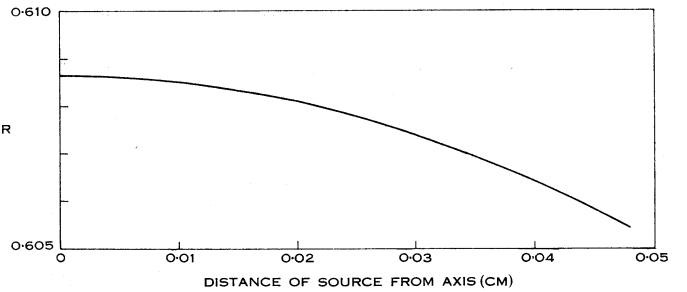

Fig. 5.—Showing a typical variation of the current ratio R with the distance from the axis of the point source ($h=10$, $b=0·5$, $\lambda=75$).

resulting from the use of a source of finite size. In the present experiments the source consists of a hole of 0·1 cm diameter in the cathode; contributions to the electron flux at the lower electrode arise, therefore, from annular sources of radii 0–0·05 cm. Since the exact distribution of the electron concentration over the hole is unknown, the distribution has been taken as uniform for the

purposes of these calculations. In practice the boundary condition $n=0$ which would be expected at the edge of the hole would lessen the contribution from the outermost annuli so that the calculations are likely to over-estimate the error from this cause.

Figure 7 shows, as a function of R, the maximum percentage error which can occur as a result of using a source hole of 0·1 cm diameter for a number of combinations of the parameters h and b. The curves show that, in the present experiments, the results recorded for $b=0·5$ at higher pressures could in some cases be just significantly higher than those recorded at lower pressures for the same value of E/p ; the absence of such a significant trend indicates that the assumption of a uniform distribution of n across the source hole has somewhat over-estimated the error.

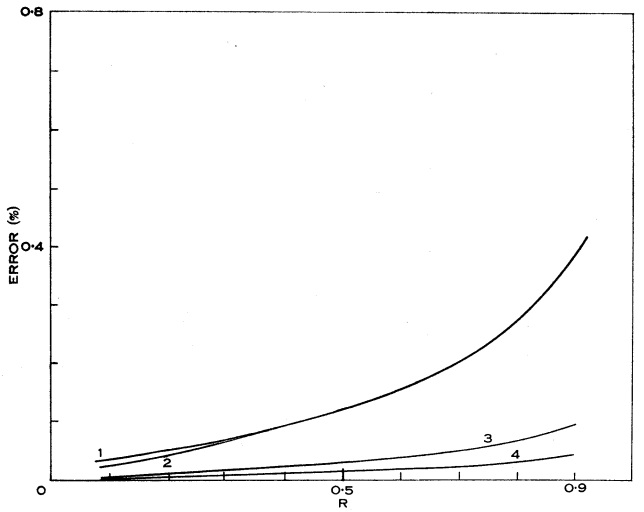

Fig. 6.—Curves showing the variation with R of the error introduced when the point source is 0·020 cm from the axis. Curve 1 : $h=2$, $b=0·5$. Curve 2 : $h=10$, $b=0·5$; $h=5$, $b=0·5$. Curve 3 : $h=10$, $b=1·0$. Curve 4 : $h=10$, $b=1·5$.

(iv) *Uniformity of Electric Field.*—A series of experiments has been carried out in another apparatus to investigate the influence on the experimental results of non-uniformity of the electric field within the diffusion space. Because of the wide separation of the anode and cathode relative to their diameter which occurs for the larger values of h, the field within the diffusion space is largely controlled by the guard electrodes, which no longer serve merely to reduce edge effects. These experiments have shown the need for a high order of precision in determining the spacing of the rings and a corresponding precision in the magnitude of the potentials applied to them. Distortion within the diffusion chamber can also arise from penetration of fields exterior to the guard electrodes, for example from an earthed shield surrounding the glass envelope. Distortion of this kind can be reduced by using an apparatus of large diameter with numerous guard electrodes of large radial depth. In the present experiments field dis-

tortion did not represent a significant source of experimental error except perhaps for the measurements made with $h=10$ cm at the lowest pressures and smallest values of E/p.

(v) *Surface Effects at the Anode.*—The influence of non-uniform surface potentials over the receiving electrode appears to be the factor most likely to limit the accuracy of experiments of this type at small values of the parameter E/p. Despite the use of gold-plated metal surfaces throughout the diffusion chamber to minimize effects of this kind, there is evidence to suggest that, under certain experimental conditions, the contact potential over the surface of the anode may have been neither uniform nor constant. For example, it was found that at a pressure of 5 mmHg when $h=10$, $b=1\cdot5$ cm, and $E/p=0\cdot6$ V cm^{-1} mmHg^{-1} the current ratio was $R=0\cdot661$ when measured at the commencement of a series

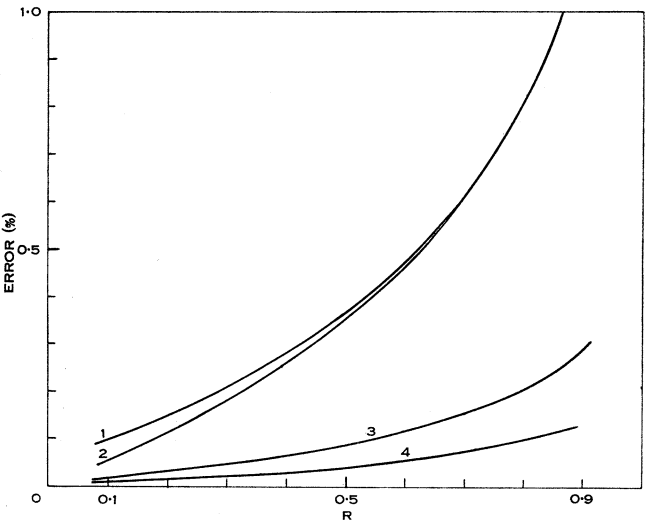

Fig. 7.—Curves showing the variation with R of the error introduced by using a source hole of 1 mm diameter (assuming n constant over the hole). Curve 1 : $h=2$, $b=0\cdot5$. Curve 2 : $h=10$, $b=0\cdot5$; $h=5$, $b=0\cdot5$. Curve 3 : $h=10$, $b=1\cdot0$. Curve 4 : $h=10$, $b=1\cdot5$.

of measurements at that pressure. When, on the other hand, the ratio was measured immediately after taking a value at the highest value of E/p of the series the ratio was found to be $R=0\cdot670$ and the ratio returned to its former value only after a period of the order of half an hour. The effect was quite reproducible and could not be attributed to changes in gas temperature nor to leakage currents in the electrometer circuits caused by dielectric soakage.

At the present time it is not known whether the variations in surface potential are caused by the bombardment of surface films on the electrode by the electron beam or whether the effect is an inherent property of a clean gold surface in a gas under these conditions. It is possible that the surfaces are contaminated by organic vapours from the mechanical pump used to exhaust the system, although liquid-air traps have been used to reduce contamination from this source.

Because of the presence of this effect some possibility exists that the results recorded at the lowest field strengths may be in error. This possibility was reduced by commencing every series of results for a given pressure at the lowest field strength. Moreover, since for a given value of E/p the effect diminishes as the pressure (and hence the electric field) is increased, errors from this source would lead to disagreement between the results recorded at different pressures. The results given in Table 1 indicate that the errors were not serious in these experiments when the procedure described above was followed.

(vi) *Temperature.*—The use of a filament as the source of electrons has the disadvantage that the gas in the apparatus may become heated during the course of the measurements, even although the heat dissipation from the filament is small. The effects of changes in the gas temperature are twofold. Firstly, the ratio E/N (N=molecular number density) will be proportional to the absolute temperature if the gas pressure is kept constant. Secondly, the energy ratio k_1 will be independent of temperature for values of E/p sufficiently small that the electrons are almost in thermal equilibrium with the gas molecules, and inversely proportional to temperature when E/p is large enough for the electron energy to be determined entirely by the electric field. Above some minimum value of E/p, therefore, the dependence of k_1 on E/p should be independent of temperature since, in hydrogen, k_1 is to a first order a linear function of E/p when $0{\cdot}1 \leqslant E/p \leqslant 5{\cdot}0$ V cm^{-1} mmHg^{-1}.

Experiments were performed in another apparatus equipped with a thermocouple and a cooling jacket surrounding the filament so that the gas temperature could be both controlled and measured accurately. Results were recorded at 15 and 20°C for $E/p = 0{\cdot}1$, $0{\cdot}2$, and $0{\cdot}4$ V cm^{-1} mmHg^{-1}. Although the current ratios were appreciably different, the values of k_1 obtained at the two temperatures differed by less than $0{\cdot}5\%$. It is, of course, necessary to know the gas temperature accurately to calculate the values of k_1 from the measured current ratios despite the fact that, in this case, k_1 is independent of temperature.

In order to keep errors due to unknown temperature changes to a minimum throughout the experiments described in this paper, the temperature of the laboratory was held constant at 20°C and the filament run only for short periods during the recording of each current ratio. Nevertheless, some of the random experimental error is attributable to changes of gas temperature since tests in the other apparatus showed that the gas temperature can exceed the ambient temperature by 2 or 3°C after prolonged use of the filament, when no cooling is provided.

(vii) *Presence of Negative Ions.*—In experiments using other apparatus there has been evidence of the presence of negative ions in the stream of electrons entering the diffusion chamber through the hole in the cathode (Crompton and Sutton 1952). In general, however, it has been found that the use of uncoated platinum filaments enables a stream of electrons free of negative ions to be generated.

The ratio of the drift speed to diffusion coefficient is normally very different for ions and electrons. Consequently a sensitive test for the presence of negative ions is afforded by comparing the values of k_1 measured using different modes of

division of the anode, particularly by comparing those measured with and without the inclusion of the central disk. A comparison of the results recorded in columns 4, 8, and 12 of Table 1 with other results recorded at the same pressures and with the same values of h shows that the results obtained in these experiments were not significantly affected by the presence of negative ions.

IV. Discussion

The range of values of E/p covered by the results in Table 1 has been investigated frequently (Crompton and Sutton 1952, and references therein; Cochran and Forester 1962) and a further investigation of it may therefore seem redundant. On the other hand, in some recent applications of the results of swarm experiments at low values of E/p (Huxley 1959; Frost and Phelps 1961) the quantity which has assumed importance is the quantity

$$(D/\mu - kT/e) = (kTe)(k_1 - 1),$$

where μ is the electron mobility. Since errors of a few percent in the determination of k_1 then become important as k_1 approaches unity, it is desirable to examine both the results of the swarm method and the method itself to see what degree of confidence in the results is justified for analyses of this kind.

Table 5

THE DISCREPANCY BETWEEN THE PRESENT RESULTS AND THOSE OF CROMPTON AND SUTTON AND OF COCHRAN AND FORESTER

E/p	Crompton & Sutton	Cochran & Forester
0·2	4·8%	25·0%
0·4	3·8	13·2
1·0	3·0	5·4
2·0	2·7	3·4
5·0	0·8	2·7

The discrepancies between the present results and two other recent determinations of k_1 in hydrogen by the swarm method are listed in Table 5.* The results of the present investigation agree with those of Crompton and Sutton to within the expected experimental error of the earlier work above $E/p = 1{\cdot}0$ V cm^{-1} mmHg^{-1}, but the errors are somewhat larger at the lower values of E/p. A number of factors may have contributed to errors of this magnitude in the earlier work. It will be seen that no results are recorded in Table 1 for $E/p = 0{\cdot}1$ V cm^{-1} mmHg^{-1} for $h = 2$ cm, corresponding to the length of the longer apparatus used by Crompton and Sutton. As in the earlier work, it was found that the current ratios become less reproducible for low field strengths,

* A comparison of the present results with those published by Townsend (1948) shows excellent agreement. There is some disagreement, amounting to 2 or 3%, between the latter results and those published by Townsend and Bailey (1921).

presumably because of the influence of non-uniform surface potentials as discussed in Section III (*b*) (v). Experience has shown that to obtain results of the accuracy claimed in the present experiments it is necessary to use field strengths of not less than about 3·0 V cm^{-1} so that measurements for $E/p=0\cdot1$ V cm^{-1} $mmHg^{-1}$ can be made reliably only at pressures in excess of 30 mmHg. The geometry of the earlier apparatus better suited for measurements at low values of E/p ($h=2\cdot0$ cm, $b/h=0\cdot25$) and the apparatus available for measuring the current ratios were such that the divergence of the electron stream would have been too small even at pressures considerably less than 30 mmHg for the ratios to be accurately determined. It was therefore necessary to use comparatively low pressures and to accept the consequent loss of accuracy resulting from the use of low field strengths. Furthermore, it was not possible to measure any of the experimental quantities with the accuracy possible in the present work.

The poor agreement between our results and those of Cochran and Forester at low values of E/p is surprising in view of the modern techniques which have been applied in each case. Unfortunately, insufficient experimental detail has been given in their paper to enable possible explanations of the discrepancies to be advanced. For example, no indication is given of the pressures which were used nor of the degree of self-consistency in the results taken at different pressures. Since, with one exception ($b=0\cdot3$ cm), the values of b and h for their apparatus lie within the range of values obtainable in our apparatus it is difficult to see how the discrepancies could arise from differences in geometry. The results in Table 1 for a diffusion chamber of length 2 cm agree well with those taken when the length was increased to 10 cm ; the length of the apparatus used by Cochran and Forester was 3 cm.

The analysis given in Section III (*b*) of the systematic errors inherent in the method shows that it is possible to design an experiment in which errors of this kind are reduced to less than 1% and the results presented in Table 1 have established that this order of accuracy is possible. An extension of the procedures discussed in this paper should enable results of comparable accuracy to be obtained at much lower values of E/p, and it is to be expected that, by designing the experiment to minimize the errors discussed in Section III (*b*), an improvement in accuracy may be achieved at the lowest values of E/p where maximum accuracy is required. It would therefore appear that the swarm method is capable of yielding data of an accuracy considerably greater than might be expected from a comparison of the results of earlier investigations.

V. Acknowledgments

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