A STUDY OF WHISTLING ATMOSPHERICS

III. OBSERVATIONS AT CLOSELY SPACED STATIONS

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Summary

Whistling atmospherics were recorded simultaneously at four stations with separations of about 150 km during the winter of 1960. Individual whistlers showed the same dispersions at all stations but significant differences in strength were sometimes observed. These features and the diffuseness of the whistlers are consistent with the idea of the energy emerging from the ionosphere through localized regions of size of the order of 100 km or less. A method has been developed of obtaining amplitude–frequency profiles. These were sometimes fairly flat but the less diffuse whistlers tended to show abrupt (up to 100 dB $(kc/s)^{-1}$) changes of amplitude of up to more than 20 dB for frequency ranges of about half a kilocycle per second. Such changes were, on occasions, dissimilar at the four stations.

I. Introduction

When simultaneous recordings of whistlers are made at stations separated by distances of the order of 1000–2000 km, large differences are found (e.g. Crouchley 1961) between the daily occurrence totals. A high proportion of whistlers is recorded at only one station and only occasionally do two stations record a particular whistler. In the present work four stations were used arranged so that adjacent stations were 100–170 km apart. Under such conditions a high proportion of individual whistlers was recorded at all four stations. The analysis of these whistlers yields strong evidence that they are received from localized regions of the ionosphere.

For this investigation, in addition to the usual study of frequency–time characteristics, a method has been developed (albeit somewhat laborious) for obtaining amplitude–frequency characteristics.

The recording sites used were located at Brisbane (geog. lat. 27° 32′ S., geog. long. 152° 55′ E., geomag. lat. 35° 8′ S., geomag. long. 226° 9′ E.), Southbrook, Bonalbo, and Dorrigo. The positions of these last three stations relative to Brisbane are indicated on the map (Fig. 1). All whistlers examined are believed to have been of the single-hop variety.

II. Experimental Details

(*a*) *Recording*

The recording equipment used was essentially the same at all sites and was similar to that used by Crouchley (1961) except for the following modifications: (i) a vertical aerial was used, (ii) a device for compensation for aerial characteristics was included, (iii) timing was accomplished by recording signals from a radio receiver tuned to standard time signals broadcast on 10 Mc/s.

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The aerial used was 10 m long and had a capacitance to ground of approximately 90 pF. To compensate for the frequency dependence of this capacitor, a resistance-capacitance combination was inserted between the preamplifier and the input to the recording amplifier. To adjust and calibrate the system, a low impedance generator was placed in series with the aerial. The gain of the system was then flat within ± 3 dB, from $1 \cdot 5$ to 15 kc/s.

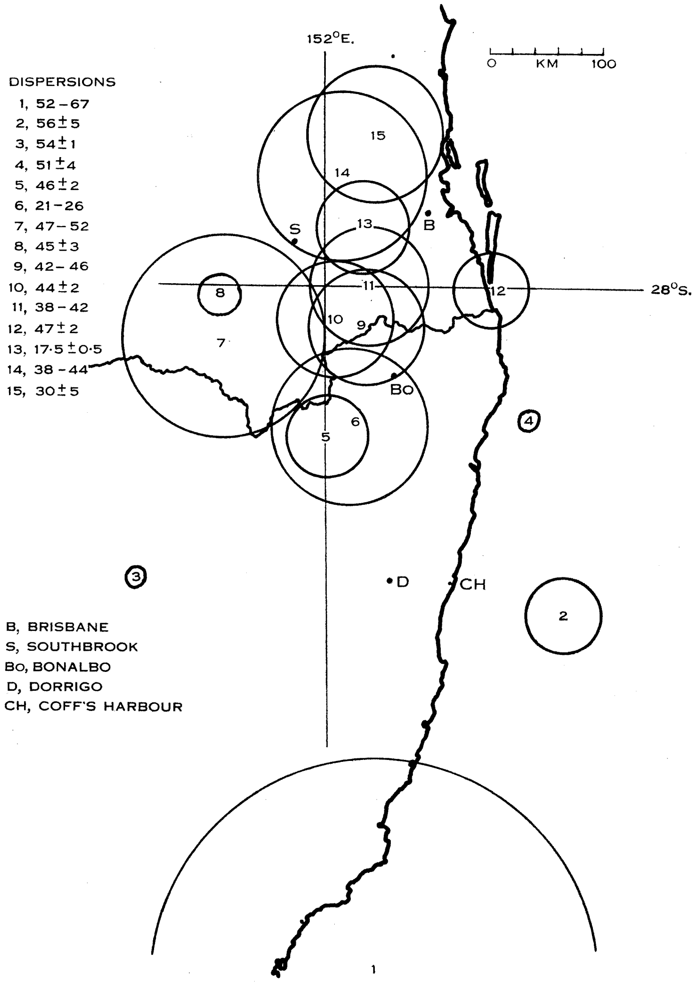

Fig. 1.—Map showing locations of stations, dispersions of whistlers, and approximate size and position of ionospheric sources.

A time-switch allowed recording between 0000 and 0045 h Aust. E.S.T., which was approximately the time of most frequent occurrence of whistlers in Brisbane during the I.G.Y. recording period (Crouchley 1961).

All recording sites were chosen for their lack of man-made electrical noise. Each was at the end of a long (in excess of 2 km) approximately straight 250 V power main, and distant at least 2 km from high-tension power mains. As a further precaution against such interference, the aerial and preamplifier were located approximately 150 m beyond the recording equipment in the direction of the line of extension of the power main. These precautions proved adequate to reduce man-made interference to a negligible level.

(b) *Preliminary Analysis*

Both tracks of the tapes could be monitored simultaneously by replay through two similar amplifiers of standard design. The time of occurrence (to the nearest second) of whistlers was noted from a stopwatch together with a subjective estimate of strength on a $\frac{1}{2}$, 1, . . ., 5 scale. By noting on the stopwatch the times of commencement of the tone broadcast every 5 min by JJY, corrections could be applied to the tabulated times of whistler occurrence. (These corrections were necessary because of slight differences in tape-deck speeds.)

(c) *Frequency-Time Relations*

Detailed analysis of individual whistlers was effected by use of a sound-spectrograph, the Kay Electric Sonagraph, which produced a frequency-time display over the range 0–8 kc/s, for approximately 2·4 s of record. Although the aural estimate of strength designated as strength $\frac{1}{2}$ was frequently made possible by the complete absence of man-made electrical interference, in practice whistlers so designated could not usually be recognized on a sonagram above the level of naturally occurring noise.

Frequency calibration of the sonagraph was accomplished by recording and reproducing a clipped 500 c/s tone, the frequency of which was checked against the mains frequency by the formation of a Lissajous' figure on a cathode-ray oscilloscope.

The dispersions of whistlers considered in this experiment were measured from sonagrams by two processes, (i) direct measurement of times and frequencies and a plot of t against $f^{-\frac{1}{2}}$, (ii) comparing the whistlers with curves drawn to the dispersion law with a series of values of dispersion.

(i) A graticule approximately the size of a sonagram was ruled on " Perspex " with horizontal lines coinciding with the harmonics of 500 c/s and with vertical markers of an arbitrary time scale. Sliding over this graticule was another smaller graticule covering the same frequency range, with the major intervals divided into tenth parts. These were used to measure to within 0·01 s the time intervals required for the plotting of t against $f^{-\frac{1}{2}}$ for accurate comparison of simultaneous whistlers. Times and frequencies were corrected for small differences in tape speeds by measuring the time interval, on the arbitrary scale, between selected outstanding sferics on the sonagrams. Correction factors for each of the stations at Brisbane, Bonalbo, and Dorrigo were deduced from these to bring the results to the same scale as Southbrook, which was taken as standard. The slope of the graph of t against $f^{-\frac{1}{2}}$ for all stations is then related to the dispersion by a single factor dependent on the arbitrary time scale, and the Southbrook tape speed. This factor was obtained from a knowledge of the duration of the time sample represented by the sonagram in both the arbitrary scale and the scale of seconds, and the replayed duration for the Southbrook tape of the 5-min intervals broadcast by JJY. The accuracy of comparison of dispersions computed by this method is $\pm 2\%$, which is also the accuracy of the stated dispersion values.

(ii) The dispersion of a whistler with ill-defined edges is so inexact that the above time-consuming method is preferably replaced by the second method mentioned above, that of fitting a graticule with curves drawn to the dispersion law. These curves were drawn on a thin transparent material and the dispersion estimated from the curves of closest fit. The accuracy of this result depended on the nature of the particular whistler, and on whether the initiating sferic could be identified, but cannot be better than ± 1 s$^{\frac{1}{2}}$.

(d) *Amplitude Measurements*

A comparison of relative amplitudes as a function of frequency was accomplished by recording each whistler on the magnetic drum of the Sonagraph, and then taking a series of sonagrams with the " reproduce " attenuation progressively increased in steps of 2 dB and with the automatic gain control circuit

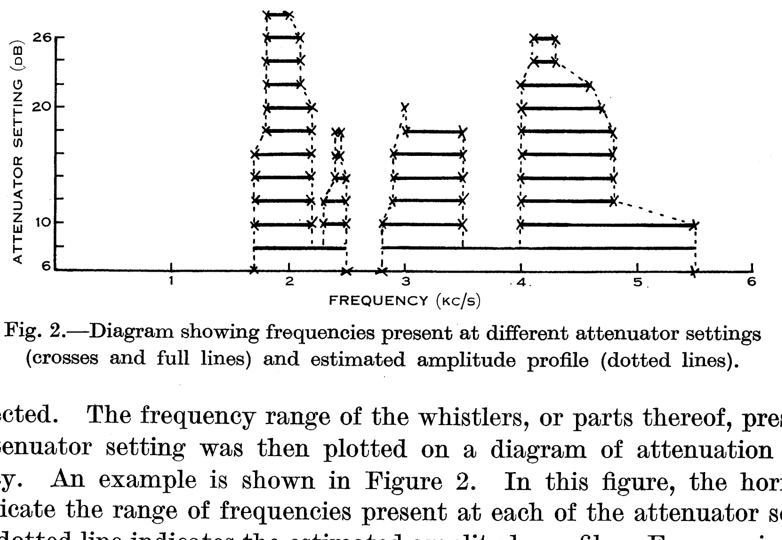

Fig. 2.—Diagram showing frequencies present at different attenuator settings (crosses and full lines) and estimated amplitude profile (dotted lines).

disconnected. The frequency range of the whistlers, or parts thereof, present at each attenuator setting was then plotted on a diagram of attenuation versus frequency. An example is shown in Figure 2. In this figure, the horizontal lines indicate the range of frequencies present at each of the attenuator settings and the dotted line indicates the estimated amplitude profile. Frequencies could be measured to ± 100 c/s and, as it was usually possible to decide that any particular frequency was present on one sonagram but not on the next, relative amplitudes should be accurate to better than ± 2 dB. Figures 4 and 5 of this paper show only the amplitude profiles, constructed as above, with crosses marking the limits of the frequency bands present at each attenuator setting.

Although the sensitivities of the recorders were initially adjusted to be similar, day-to-day variations of gain occurred. Measurements made at the respective aerials of the received amplitude of V.L.F. stations broadcasting on 16–19 kc/s indicated a possible variation of amplitude between stations of ± 4 dB at these frequencies. Such changes may be attributed to variations of soil conductivity with such factors as weather conditions. That no systematic error was involved is evident by the totals of numbers of whistlers recorded during the three months of operation being approximately the same for each station. However, it was considered unsafe to attach quantitative significance to the absolute amplitude found by the above method; only the relative amplitudes for different frequencies were accepted as significant.

(e) *Correspondence of Whistlers*

The stop-watch method of timing the occurrence of whistlers is often sufficient to identify simultaneous whistlers recorded at different stations. However, a much more precise identification can be readily made by adjusting the relative positions of the sonagrams until the background sferic patterns are in precise coincidence. In all actual cases the chance of a spurious coincidence was extremely small.

III. Occurrence

(a) *General*

Two stations began recording near the end of May 1960 and all four stations were operative by early June. On 82 days between then and the closure of the stations in mid September, three or four stations were operating simultaneously. On 10 of these days 40 or more whistlers were received of strength 2 or more at any one station; on 11 days between 10 and 40 strong whistlers were recorded; and on 61 days only a few weak whistlers were recorded.

(b) *Comparison between Stations*

The occurrences of whistlers at the various stations were compared by the following two methods.

(i) Correlation coefficients (r) were calculated between the daily totals of the number recorded (irrespective of strength) using, for each pair of stations, values for approximately 60 days. The values of r so calculated were: Southbrook-Brisbane $0 \cdot 96$ (124 ± 4 km spacing), Brisbane-Bonalbo $0 \cdot 92$ (154 ± 4 km), Bonalbo-Dorrigo $0 \cdot 97$ (167 ± 4 km), Southbrook-Bonaldo $0 \cdot 91$ (156 ± 4 km), Southbrook-Dorrigo $0 \cdot 89$ (304 ± 4 km), Brisbane-Dorrigo $0 \cdot 88$ (315 ± 4 km). A plot of these coefficients against distance shows that they lie, almost within the limits of their standard errors, on a straight line passing through the point $r = 1 \cdot 00$ for zero spacing between the stations. On this plot r falls to a value of $0 \cdot 90$ in a distance of approximately 250 km and thus 80% (r^2) (Bennett and Franklin 1954) of the variation in occurrence of whistlers at one station may be accounted for in terms of this variation at another 250 km away.

This plot also suggests that the stations were similar in properties and there is no evidence at these distances of the north-south correlation being different to the east-west correlation.

(ii) For each pair of stations the second method was restricted to a consideration of whistlers which showed a strength of at least 1 at one or other station, and whistlers were considered to be simultaneous if the reported times were within two seconds (± 1 s per station). The number N_1 of such whistlers which appeared at both stations was counted and also the numbers N_2 appearing at station A only and N_3 appearing at station B only were found. The number,

$N = N_1 + N_2 + N_3$, thus gives the total number of whistlers of strength at least 1 appearing at either or both stations. The percentage $100(N_2+N_3)/N$, i.e. the percentage of these whistlers appearing at only one station, varied from about 10% up to, on a few occasions, 40%. Thus usually 90% of the whistlers of strength at least 1 in a region could be heard at two stations 300 km apart, but occasionally this value fell to 60%.

Both methods therefore suggest that some whistlers fall below the level of detectability, i.e. that significant changes in strength occur, in a distance of 300 km. This point is further discussed in the following section.

(*c*) *Location*

On certain nights the distributions of strengths at the several stations were consistent with the supposition that whistlers reached them by propagation in the Earth-ionosphere waveguide from a common, fixed region of the ionosphere of radius less than 100 km. Such an area may be termed the "ionospheric source" of the whistlers. As an example Table 1 lists the distribution of strengths for whistlers received on July 18.

Table 1

DISTRIBUTION OF WHISTLER STRENGTHS ON JULY 18

Station ..	Southbrook			Brisbane			Bonalbo			Dorrigo		
Strength ..	$\frac{1}{2}$	1	2 or more	$\frac{1}{2}$	1	2 or more	$\frac{1}{2}$	1	2 or more	$\frac{1}{2}$	1	2 or more
No. of occurrences ..	83	113	61	6	86	99	118	23	3	85	19	0

From this table and a similar tabulation for the other days concerned, the impression is gained that a strong whistler may often noticeably decrease in amplitude over a distance of 150 km. Thus, while the detailed way in which strength varies with distance close to (i.e. within a few wavelengths of) the ionospheric source is not known, it seems reasonable to assume that the source of whistlers is nearest to the station which records the largest number of strong whistlers. On this basis Table 1 suggests that the ionospheric source of whistlers was closer to Brisbane than to the other stations and would therefore appear to have been roughly north to north-west of Brisbane and probably not more than 100 km range from that station.

The dispersions of the whistlers recorded on days for which ionospheric source location by this method was possible were measured by fitting dispersion curves, and the deduced values were plotted on a map in the area of the source. With the exception of two days of unusually low dispersion, there was a reasonably consistent increase in dispersion with latitude (Fig. 1).

An estimate of the area of the region from which whistlers came may be obtained from consideration of the diffuseness of the whistlers (Crouchley and Finn 1961). Taking a figure for change of dispersion with latitude of $2 \cdot 2$ s$^{\frac{1}{2}}$ per degree (from graph of Allcock (1959)), the diameter of the region producing a whistler of diffuseness 1 is approximately 100 km. In Figure 1 circles on the map show the probable locations of the sources and their approximate sizes.

IV. Characteristics of Simultaneous Whistlers

(a) *Dispersion*

A small minority of the whistlers sonagrammed consisted of a single sharply defined trace. However, the great majority consisted of multiple traces which may or may not have been sharply defined, or of single bands lacking in structural component lines and of poor definition (Plate 1).

Some suitably sharp simultaneous whistlers were chosen for dispersion comparison. The dispersions were determined to within 2% by an Eckersley-Storey type plot (e.g. Fig. 3). In all cases the dispersions of simultaneous whistlers were the same at the four stations to within 2% (Table 2). By extra-

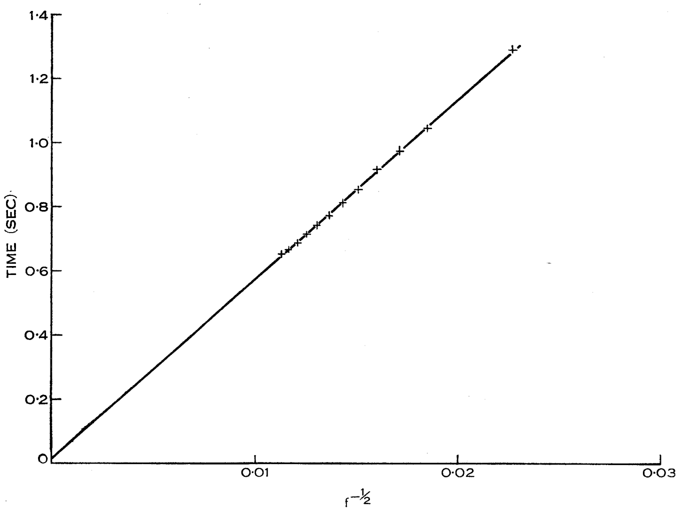

Fig. 3.—Eckersley-Storey plot of a whistler for determination of dispersion. f is frequency in cycles/second.

polating the graph backwards to cut the frequency axis, the time of the initiating sferic was found. Such sferics often showed a dispersion " hook ", as is typical of an impulsive disturbance propagated in the Earth-ionosphere waveguide (e.g. see Outsu 1960). Plate 1, Figure 2 shows a four-component whistler received on August 29, 1960. Traces 1 and 3 have dispersions of $56 \cdot 4 \pm 1$ $s^{\frac{1}{2}}$ and $59 \cdot 1 \pm 1$ $s^{\frac{1}{2}}$ respectively and a common time origin. Similarly traces 2 and 4 have dispersions of $57 \cdot 3 \pm 1$ $s^{\frac{1}{2}}$ and $60 \cdot 0 \pm 1$ $s^{\frac{1}{2}}$ respectively and a time origin $0 \cdot 04$ s (by extrapolation) later than the origin for traces 1 and 3. The strong sferics eparated by $0 \cdot 04$ s at the left of the plate do in fact coincide with the time origins required by the extrapolation. (Times of origin may be determined to $\pm 0 \cdot 02$ s.)

(b) *Amplitude-Frequency Comparisons*

Diagrams showing the variation of amplitude with frequency, over the range 80 c/s to 8 kc/s, were prepared by the method described in Section II (*d*) for

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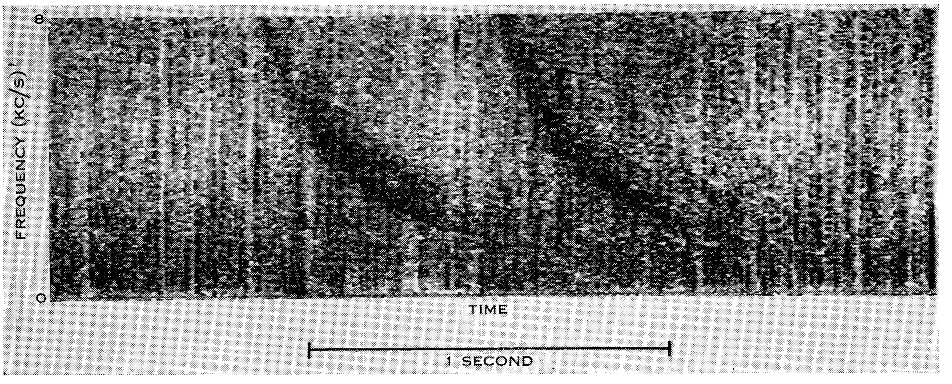

Fig. 1.—Sonagram of type (i) whistler.

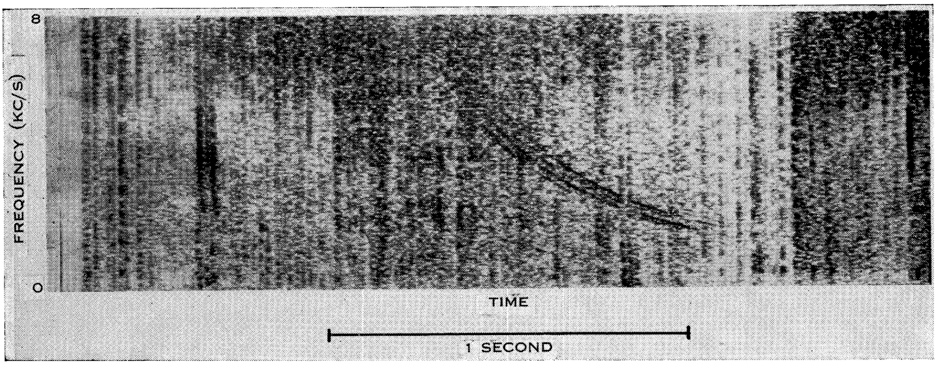

Fig. 2.—Sonagram of type (ii) whistler.

TABLE 2

DISPERSION COMPARISONS

Date	Time	Southbrook	Brisbane	Bonalbo	Dorrigo
21. vii.60	0000–12	42·9	41·3	41·7	42·0
	0000–13	41·7	42·3	43·2	42·0
24. vii.60	0015–33	44·9	45·0	44·8	45·0
17.viii.60	0014–35				
	Leading edge	48·0	48·0	48·3	49·0
	Trailing edge	52·1	54·0	53·1	53·7
29.viii.60	0021–18				
	Trace 1	56·9	—	—	55·9
	Trace 2	57·1	56·9	57·6	57·5
	Trace 3	59·1	—	—	—
	Trace 4	60·0	—	60·4	59·4
13. ix.60	0026–17	52·0	—	—	52·3
	0028–44	52·9	53·5	53·2	53·2
14. ix.60	0012–18	65·5	65·8	64·3	64·9
	0022–35	67·1	67·7	67·1	66·5

all of the whistlers discussed in the previous subsection. The 12 traces examined in this fashion for each of the four stations could be classified into two broad types, (i) those for which the amplitude was nearly constant over most of this frequency range, and (ii) those exhibiting large fluctuations of amplitude within this frequency range.

(i) The five whistlers falling into the first category were relatively diffuse (Plate 1, Fig. 1) and showed a lower-frequency limit between 1 and 2 kc/s, with variations between stations of the order of a quarter of a kilocycle per second. The upper-frequency limit, which was estimated from an ordinary sonagram with the tape replayed at half-speed, was on some occasions in excess of 16 kc/s (the upper recording limit) and sometimes set by noise at about 8–10 kc/s with a variation between stations of from 0 to 4 kc/s. The shapes of the amplitude graphs from the four stations were similar and in reasonable quantitative agreement (within the limits of experimental error), but the absolute values differed from station to station by a few decibels and were occasionally different by up to about 8 dB. Such whistlers showed gradual amplitude changes with average slopes over their " flat " portions of between $+2$ and -1 dB $(\mathrm{kc/s})^{-1}$. The low-frequency cut-off had an " average " slope of between 6 and 30 dB $(\mathrm{kc/s})^{-1}$ for four of these whistlers, with a variation of up to 20 dB $(\mathrm{kc/s})^{-1}$ between stations, while for the other this slope was about 100 dB $(\mathrm{kc/s})^{-1}$. Figure 4 shows an example of the amplitude frequency plots for this type of whistler.

(ii) The other whistlers, falling into the second category, were very sharp (Plate 1, Fig. 2), either single or multiple traces, and showed amplitude variations which were on occasions in excess of 20 dB (Fig. 5) but more commonly about

12 dB. The rate of change of amplitude at the sudden changes was commonly in excess of 100 dB (kc/s)$^{-1}$ but in such cases it was only possible to estimate this value from a consideration of the errors of measurement (Section II (*d*)) and not directly. However, on some occasions the changes were less abrupt with a slope of about half this value. An average of three maxima was observed in the frequency range 0–8 kc/s, but this number never exceeded four. The average widths of maxima for a particular whistler were between one-half and one kilocycle per second while the minima were typically one-third to one kilocycle per second

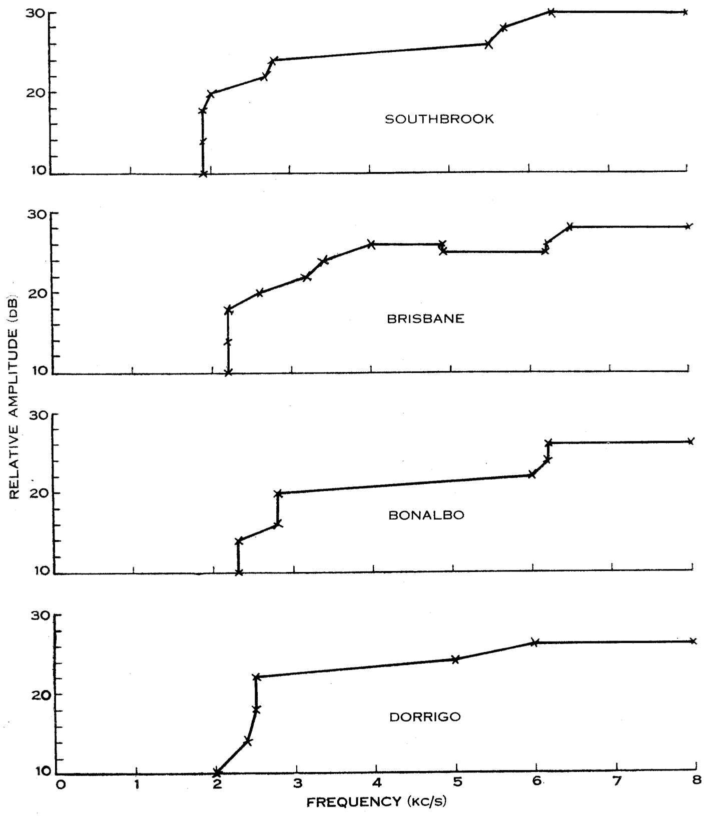

Fig. 4.—Example of type (i) whistler amplitude-frequency profile.

wide. On a few occasions the patterns from the four stations showed a rough agreement but on others there were marked differences in the number, centre frequencies, widths, and strengths of the maxima. The minimum frequencies observed were of the same order as those of type (i) whistlers, but the highest frequencies observed were usually between 4 and 8 kc/s, although an occasional trace showed at higher frequencies.

From a comparison of ordinary sonagrams (as distinct from sets taken for amplitude measurements) it would seem that the first type is more common than the second type. Whistlers recorded in the one hour usually appear to be fairly similar in characteristics but it would be necessary to determine many more amplitude profiles to reach a definite conclusion.

It is estimated that the whistlers examined had electric field strengths of the order of 50–100 μV/m.

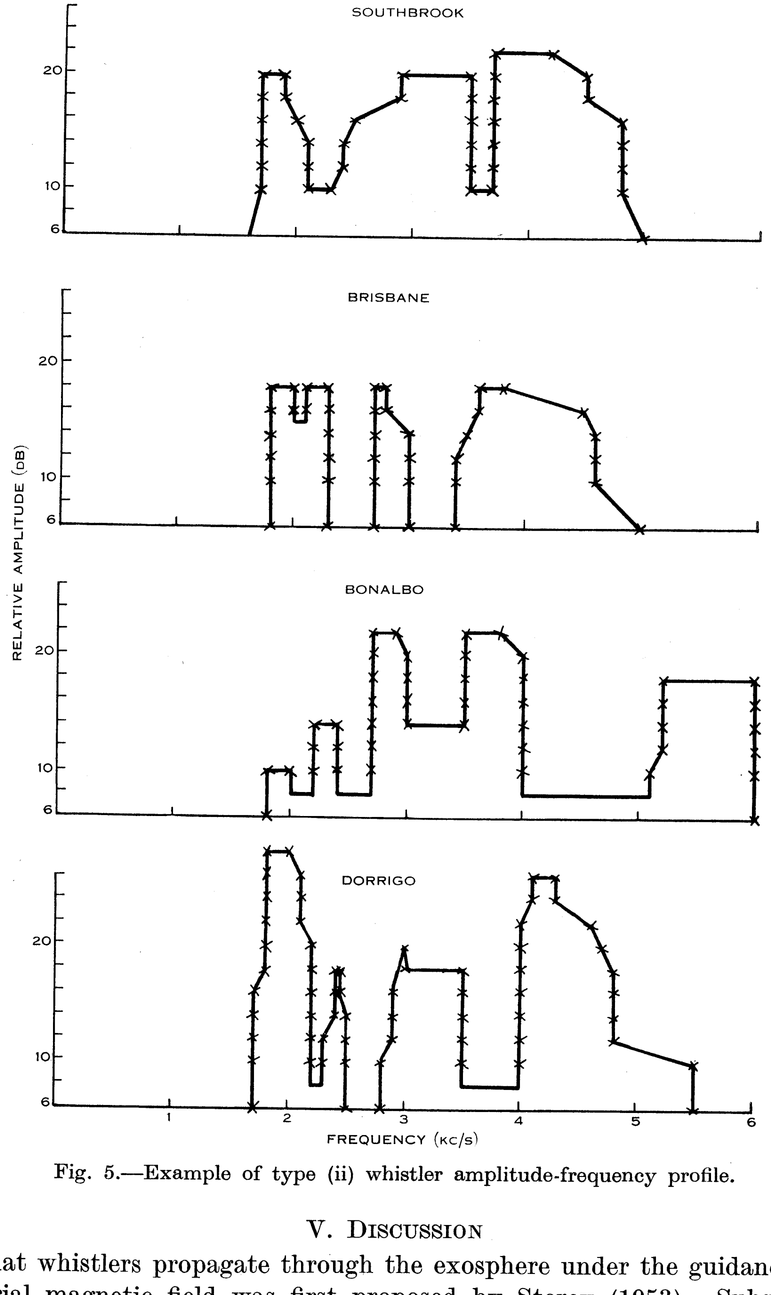

Fig. 5.—Example of type (ii) whistler amplitude-frequency profile.

V. Discussion

That whistlers propagate through the exosphere under the guidance of the terrestrial magnetic field was first proposed by Storey (1953). Subsequently various workers (e.g. Smith, Helliwell, and Yabroff 1960) have suggested that columns of enhanced ionization, the so-called " ducts ", are responsible for a more restricted guiding of energy from the initiating lightning discharge. In the former case a departure of up to 19° from the field lines may theoretically occur and hence a large region of the ionosphere in the hemisphere opposite to the lightning flash might be expected to be illuminated. Thus, unless energy can only penetrate the ionosphere in special, relatively small places, a large area of the ground should be illuminated. In the latter case energy would be expected to emerge only from the end of the duct. An estimate of the illuminated region has been made by Crouchley and Finn (loc. cit.).

The experimental results presented in the present paper suggest that the energy received by a whistler receiver comes from a localized region of the order of 100–200 km size. Thus in Section III (*c*) it is reported that the strength of whistlers on occasions decreases markedly in 150 km, a result which would be difficult to understand if a region of the order of 1000 km diameter were illuminated. Furthermore, within the limits of experimental error, the dispersion of a whistler does not change from station to station. Again, such would not be expected to be the case with a large area of illumination, for then one would reasonably expect the dispersion to depend on the latitude of the observing station. Thus Allcock (loc. cit.) presents a graph of dispersion against latitude indicating an increase of approximately 2 $s^{\frac{1}{2}}$ per degree. On this basis one would expect dispersions at Dorrigo to be about 6 $s^{\frac{1}{2}}$ greater than at Brisbane or Southbrook. The experimental results, however, show differences of less than one-quarter of this and the lower latitude stations are not consistently lower in dispersion. If, as is suggested by the numerical values of Figure 1, the actual increase in dispersion with latitude for whistlers arriving at different latitudes was, during these experiments, somewhat greater than the value due to Allcock, then this result becomes still more significant. Further, the estimates of size of the " ionospheric source " are consistent with this interpretation of the relative strength and dispersion data.

The interpretation of the amplitude-frequency data is, however, much more difficult. Since different stations sometimes see different amplitude patterns it seems likely that the effects are produced at the end of the path rather than along the path. On the view presented above, the stations were observing radiation from a source of size comparable to a free-space wavelength, radiating into the Earth-ionosphere waveguide. The nearest approach to a theoretical discussion of such a problem is that of Wait (1957), who deals with the case of a vertical aerial radiator. Under such conditions a series of maxima and minima are predicted in the radiation pattern near to the source, with a transition to an exponential decay at several wavelengths distance from the radiator. These maxima and minima may be interpreted as interference between rays which arrive directly and those which undergo successive reflections at the Earth and the ionosphere. Similarly, Poeverlein (1961) has pointed out the possibility of the production of a standing wave pattern between the Earth and the ionosphere with amplitude peaks separated by a frequency of the order of 1 kc/s and with large variations in intensity from maxima to minima. Also, if the lower boundary of the ionosphere is rippled with a scale of tens of kilometres as suggested by

Bowman (1960) then interference effects may be produced in a manner similar to a diffraction grating. Further, the waves are emerging from a dispersive region of high refractive index and thus a small variation in the angle of incidence on the lower face of the ionosphere will cause a large change in the angle of refraction. Thus, if some process along the path tended to make the angle between the wave-normal and the vertical in the ionosphere change in an oscillatory fashion with frequency, then a station, depending on its position, might expect to receive certain frequencies and not others. The observed changes in amplitude are in some cases very abrupt and such sudden changes do not usually occur in interference phenomena. They are more reminiscent of the sudden change in intensity observed when the frequency of a variable-frequency radio wave exceeds the critical frequency of a reflecting medium. It is possible that the type (i) whistler is formed by the superposition of a large number of type (ii) whistlers so that the individual structure is obscured.

VI. Conclusions

The energy received by a whistler recorder emerges from the ionosphere in localized regions which may, around geomagnetic latitude 35°, have a size from a few tens of kilometres up to 200 km. Marked variations in the amplitude of the received signal, both with frequency and position, may occur close to these regions, particularly for the smaller ones, but the dispersion is the same for all stations. A significant change in the subjective strength of a whistler may occur in a distance of the order of 200 km.

VII. Acknowledgments

The recording of whistling atmospherics at closely spaced stations was undertaken at the suggestion of Professor H. C. Webster. The authors are grateful to him for this suggestion and for his unfailing interest in the project. Such recordings would not have been possible but for the cooperation and assistance of Mr. W. L. Brown (Southbrook), Mr. W. Little (Bonalbo), Mr. A. McRae and Mr. H. Ashford (Dorrigo), and Mr. M. Thorpe (Brisbane), who permitted the installation of equipment on their respective properties and/or operated it. Miss B. Ross and Mr. R. Green also monitored many of the tapes which were recorded. The financial assistance of the Radio Research Board of C.S.I.R.O. is gratefully acknowledged.

VIII. References

Allcock, G. McK. (1959).—*J. Atmos. Terr. Phys.* **14**: 185.

Bennett, C. A., and Franklin, N. L. (1954).—"Statistical Analysis in Chemistry and the Chemical Industry." p. 37. (Wiley: New York.)

Bowman, G. G. (1960).—*Planet. Space Sci.* **2**: 133, 150.

Crouchley, J. (1961).—*Aust. J. Phys.* **14**: 22.

Crouchley, J., and Finn, R. J. (1961).—*Aust. J. Phys.* **14**: 40.

Outsu, J. (1960).—*Proc. Res. Inst. Atmos. Nagoya Univ.* **7**: 58.

Poeverlein, H. (1961).—*J. Res. Nat. Bur. Sci. Sect.* D **65D**: 465.

Smith, R. L., Helliwell, R. A., and Yabroff, I. W. (1960).—*J. Geophys. Res.* **65**: 815.

Storey, L. R. O. (1953).—*Phil. Trans.* A **246**: 113.

Wait, J. R. (1957).—*Proc. Inst. Radio Engrs., N.Y.* **45**: 760.