NEW SPECTROGRAPHIC LENS SYSTEM

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Summary

The performance of a three-prism stellar spectrograph of the Mount Stromlo Observatory is discussed. A new spectrographic objective (relative aperture $f/3 \cdot 5$) has been designed for the largest practicable extension of the spectral image field perpendicular to the optical axis—considering stellar spectra in the wavelength region 3900–4900 Å to be recorded in good definition. The complete system specifications are indicated and the design of the spectrographic objective is described.

I. Introduction

The new spectrographic lens system of the Cassegrain stellar spectrograph of the Mount Stromlo Observatory was required to replace the existing optics, relative aperture of the camera objective $f/3 \cdot 5$, focal length 8 in., and to record a considerably extended spectral range (3900–4900 Å), preferably perpendicular to the optical axis. This spectral region corresponds to about 1·2 in. actual length of the spectrum in the image field of the spectrograph. In addition, it was also desired to photograph the region Hβ to Hα by adjusting the tilt of the plate, the position of the collimator remaining unchanged. The resolving power of the spectrographic objective was required to be 50–100 lines/mm and the existing dispersing device was to be used.

It is well known that this problem cannot be solved easily, because the ideal camera objective of a stellar spectrograph should consist of only a few thin lenses (preferably uncemented) of glasses of high transmission and therefore cannot cover a wavelength region of the required extent. In particular, conventional designs, which generally serve entirely different purposes, could not be used. For example, with the existing camera objective, Dallmeyer Pentac $f/2 \cdot 9$, focal length 8 in., it was possible to achieve satisfactory definition only for the region 3900–4150 Å; the remaining part of the range 3800–4500 Å, of fair image quality, showed the coma effect distinctly even with the aperture limited to $f/4 \cdot 6$. This effect seriously interfered with the accuracy of line measurements, particularly in the photometric study of line profiles (the appearance of line curvature, due to the nature of the prisms, was avoided by a curved collimator slit). Furthermore, optical image formation to the limit of resolution of 10 μ on the photographic plate was quite impossible.

In comparison with conventional photographic systems the spectrographic objectives are corrected for extreme image sharpness (corresponding to the photographic emulsion to be used) in particular with respect to:

(*a*) spherical aberration,
(*b*) asymmetrical aberration (coma),
(*c*) flatness of the meridional image field,

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because the image formation of spectral lines is related to the meridional image field (Gullstrand 1890, 1906). Astigmatism is of secondary significance but must not impair the image sharpness. Other aberrations can be neglected. The condition (*c*) represents the chief difficulty in our case, because the use of curved plates is excluded and therefore achromatization of all wavelengths is required.

The new lens system was designed and constructed at the Defence Standards Laboratories, Maribyrnong. It is substantially the subject of this paper to indicate the spectrograph performance and to describe the design of the lens system.

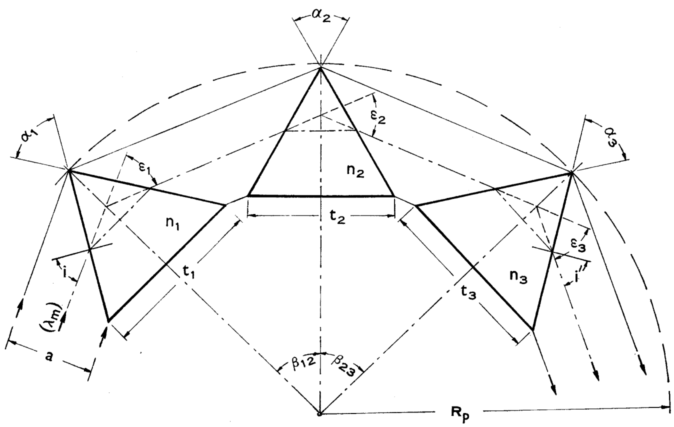

Fig. 1.—Dispersing device (refractive index $n_1=n_3$).

II. Spectrograph Performance

The optical image formation of the collimator slit in the focal plane of the camera objective (assumed here to be perpendicular to the optical axis) requires precise definition of the optical axis and the principal ray paths in the dispersing device (Plaskett 1909). This device is a three-prism system designed to give specified resolving power (R) and adequate slit width for effective light transmission. The prism system is specified by the refracting angles (Fig. 1)

$$\alpha_1=64°\ 6',\quad \alpha_2=60°\ 8',\quad \alpha_3=64°\ 6', \tag{1}$$

and by the refractive indices (Table 1) determined by the Hartmann (1898) formula

$$n=n_0+c/(\lambda-\lambda_0)\quad (\lambda \text{ in } \mu). \tag{2}$$

If the vertices of the prisms lie on a circle of radius $R_p=7\cdot 61$ in., then the angular separations are

$$\beta_{12}=\beta_{23}=56°. \tag{3}$$

Minimum deviation occurs for the wavelength $\lambda_m=4190\cdot 5$ Å, i.e. the wavelength on the optical axis adjusted to equal incidence angles ($i=i'$).

The prism edges of the first prism define the rectangular clear aperture (diaphragm) of width

$$a = t_1(1 - n_1^2 \sin^2 \tfrac{1}{2}\alpha_1)^{\frac{1}{2}}/(2 \sin \tfrac{1}{2}\alpha_1), \tag{4}$$

where t_1 denotes the maximum glass path parallel to the optical axis. The principal ray paths are determined with respect to the optical axis, and the

TABLE 1

PRISM REFRACTIVE INDICES

	n_0	c	λ_0
n_1	1·58635	0·01314	0·20190
n_2	1·60898	0·01491	0·20220

distance of the entrance pupil of the camera objective is a function of the wavelength. The total minimum deviation is

$$\varepsilon = \varepsilon_1 + \varepsilon_2 + \varepsilon_3,$$

and the change in deviation corresponding to the change in wavelength

$$\frac{d\varepsilon}{d\lambda_m} = \frac{2t_1}{a}\frac{dn_1}{d\lambda_m} + \frac{t_2}{a}\frac{dn_2}{d\lambda_m}, \tag{5}$$

where $dn/d\lambda$ is a negative quantity. Multiplying the angular dispersion (5) by the camera focal length (f_2) gives the linear dispersion

$$\frac{d\varepsilon}{d\lambda_m} f_2 = \frac{ds}{d\lambda_m} \tag{6}$$

(ds = virtual separation of the spectral lines λ_m and $\lambda_m + d\lambda_m$).

To compute the linear dispersion from Table 1 and the given dimensions $a = 5 \cdot 5$ cm, $f_2 = 19 \cdot 5$ cm, we use (1) and (4), and get $t_1 = 12 \cdot 0$ cm, $t_2 = 10 \cdot 2$ cm, $t_3 = 12 \cdot 0$ cm, $dn_1/d\lambda_m = -2786 \cdot 6$ cm^{-1}, $dn_2/d\lambda_m = -3170 \cdot 7$ cm^{-1}, $1/(ds/d\lambda_m) = 28 \cdot 4$ Å/mm.

The resolving power of the prism system is related to the smallest resolvable wavelength difference by

$$R_0 = \lambda/d\lambda$$

for an infinitely narrow slit according to Rayleigh's theory. However, in astronomical practice the faintness of light sources requires that the slit should be opened as widely as permissible, i.e. until the slit image corresponds to the resolving power of the emulsion selected for the problem. By consideration of (6) the ratio of the practical to the theoretical resolving power may be written

$$\frac{R_E}{R_0} = \frac{ds}{[ds]} \qquad \left(\frac{ds}{[ds]} = \frac{d\lambda}{[d\lambda]}\right),$$

where the limit $[ds]$ is set by the photographic grain size E, so that

$$R_E = \frac{\lambda_m}{E\, d\lambda_m/ds}, \quad \frac{R_E}{R_0} = \frac{\lambda_m f_2}{aE}, \tag{7}$$

and in the ideal case

$$[ds]=\frac{f_2}{a}\lambda_m \qquad (R_E=R_0).$$

For the conditions quoted we assume $E=20\ \mu$ for stellar spectroscopy and get from (7)

$$R_E/R_0=0\cdot 074,$$

i.e. the attainable resolving power is smaller than 1/10th of the theoretical resolving power, and spectral lines of the wavelength difference

$$\lambda_m/R_E=0\cdot 57\ \text{Å}$$

are just resolvable. The corresponding slit width is

$$\frac{f_1}{f_2}E=\frac{R_0}{R_E}\ \frac{\lambda_m f_1}{a} \tag{8}$$
$$=0\cdot 08\ \text{mm},$$

when the collimator focal length $f_1=78\cdot 3$ cm.

III. The Collimator Objective

The new collimator objective is required to have the aperture ratio $f/14$, focal length $78\cdot 3$ cm, and the adjustable slit length of 20 mm (i.e. a field angle maximum of about 45 min of arc). Hence the primary monochromatic correction to be considered involved spherical aberration and coma (aplanatic correction).

Spectrographic image formation of the collimator slit is defined by the image formation of the spectrum of the dispersing device on a flat photographic plate and residual chromatic aberration of the collimator objective is to be considered. Secondly, maximum sharpness of the line images requires excellent correction of the spherical aberration over the whole image field (Hartmann 1900). Therefore, for the photography of extended wavelength regions it is most desirable that the collimator objective should have excellent chromatic correction and preferably should be an aplanatic apochromat.

This requirement could easily be satisfied because of the comparatively small aperture ratio. The system specifications are :

Radii of Curvature (in.)	Axial Separations (in.)	Refractive Indices at λ_m
$r_1=15\cdot 960$		
	$0\cdot 25$	$n_I=1\cdot 49776$
$r_2=10\cdot 930$		
	$0\cdot 10$	$n_{II}=1\cdot 52972$
$r_3=\ 6\cdot 325$		
	$0\cdot 10$	$n=1$
$r_4=\ 6\cdot 360$		
	$0\cdot 25$	$n_I=1\cdot 49776$
$r_5=53\cdot 940$		

Different n-values may be computed from (2) and Table 2. These optical glasses are particularly selected for durability of the cemented surfaces (see Section IV). Figure 2 represents a meridional section of the collimator objective.

TABLE 2

COLLIMATOR OBJECTIVE REFRACTIVE INDICES

	n_0	c	λ_0
n_{I}	1·47229	0·00668	0·15680
n_{II}	1·49642	0·00857	0·16185

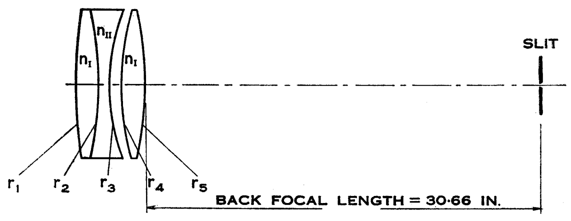

Fig. 2.—Collimator objective.

IV. DESIGN OF THE CAMERA OBJECTIVE

The camera objective is required to photograph, with good definition, the wavelength regions perpendicularly extended as far as possible at both sides of the optical axis. It has been indicated that these regions cannot be axially symmetric and depend on the variable distance of the entrance pupil. The constants of the objective, related to the optical axis at minimum deviation and the full spectrograph aperture, are:

relative aperture $f/3\cdot5$, focal length $19\cdot5$ cm.

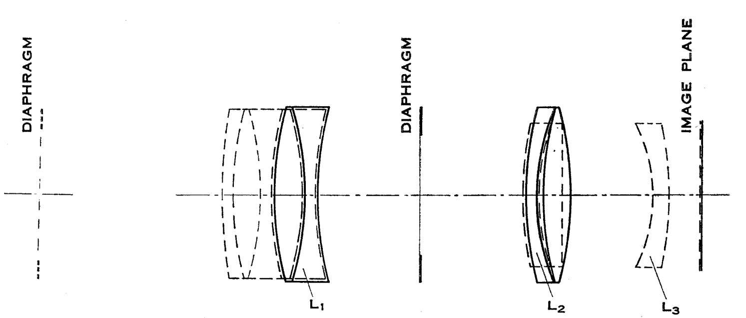

Fig. 3.—Petzval objective, relative aperture $f/3\cdot5$, designed 1840 (continuous lines); initial modification (broken lines).

The clear aperture of the front component of the objective is required to be 7·6 cm in diameter to avoid vignetting, and to be at 6 cm axial distance from the prism system.

To solve this problem the initial developments were based on the Petzval objective, which consists of two components L_1 and L_2 (Fig. 3) and is a perfect aplanat but not an anastigmat. Although the astigmatism is comparatively small, the field curvature is most disturbing while the attainable chromatic correction is only about the same as that of conventional photographic objectives. Thus the requirements were

(*a*) correction of the Petzval sum, and

(*b*) largely extended achromatization of the modified Petzval objective, together with the extreme image sharpness of the original system.

Considering Figure 3, it is well known that requirement (*a*) is easily satisfied by an additional Smyth lens L_3. However, the fulfilment of requirement (*b*) is difficult to achieve, particularly towards shorter wavelengths, and hard to reconcile with the requirement that the ideal spectrograph objective should avoid image deterioration caused by cemented surfaces and light absorption of thick lenses. With cemented surfaces the image deterioration is due to back reflection (causing light losses and secondary images) and the limited durability of such surfaces (Eberhard 1903). Since thick lenses can be achromatized most simply by cementing optical glasses of particular relative dispersion, an attempt

Table 3

CAMERA OBJECTIVE REFRACTIVE INDICES

	n_0	c	λ_0
n_{III}	1·59941	0·01005	0·16565

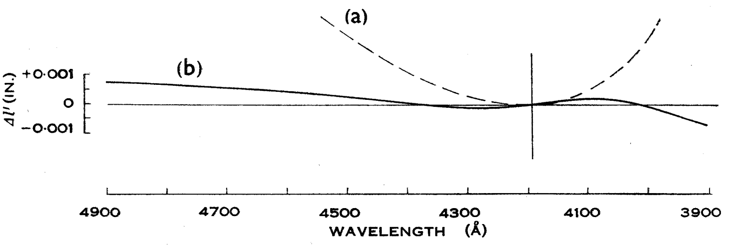

Fig. 4.—Longitudinal chromatic aberration ($\Delta l'$). (*a*) Petzval objective; (*b*) modified objective.

was made to eliminate the above-mentioned secondary effects. For this purpose equal thermal coefficients of expansion were chosen and also minimum surface powers, such that

$$|(n'-n)/r| < 1/(10f_2)$$

(r = radius of curvature and n, n' denote the respective refractive indices at each side of a cemented surface), and the average value of n,n' is about in agreement with the refractive index of the cement. However, for easier optical alignment of a cemented component with several cemented surfaces, it was then decided to split the modified five-element component (representing the original component L_1) into two three-element components.

Spectrographic objectives with this modification require components L_2 and L_3 chiefly for the correction of coma, astigmatism, and meridional field flatness. It is advantageous to choose glass for the positive element of component L_2 with a higher refractive index than the glasses in Table 2 but with equal relative dispersion, e.g. as in Table 3, and the component L_2 is then not cemented if the thermal properties of the glasses differ.

To make full use of the largely extended axial achromatization (Fig. 4) the Smyth lens L_3 has been figured. With reference to Tables 2 and 3, the system specifications are:

Radii of Curvature (in.)	Axial Separations (in.)	Refractive Indices at λ_m
$r_1 = 8 \cdot 391$		
	$0 \cdot 50$	$n_I = 1 \cdot 49776$
$r_2 = 4 \cdot 254$		
	$0 \cdot 12$	$n_{II} = 1 \cdot 52972$
$r_3 = 4 \cdot 000$		
	$0 \cdot 34$	$n_I = 1 \cdot 49776$
$r_4 = 37 \cdot 240$		
	$0 \cdot 01$	$n = 1$
$r_5 = 4 \cdot 857$		
	$0 \cdot 49$	$n_I = 1 \cdot 49776$
$r_6 = 4 \cdot 671$		
	$0 \cdot 10$	$n_{II} = 1 \cdot 52972$
$r_7 = 3 \cdot 260$		
	$0 \cdot 36$	$n_I = 1 \cdot 49776$
$r_8 = 57 \cdot 260$		
	$3 \cdot 89$	$n = 1$
$r_9 = 7 \cdot 990$		
	$0 \cdot 10$	$n_I = 1 \cdot 49776$
$r_{10} = 6 \cdot 430$		
	$0 \cdot 30$	$n_{III} = 1 \cdot 63907$
$r_{11} = 80 \cdot 000$		
	$2 \cdot 50$	$n = 1$
$r_{12} = 1 \cdot 872$		
	$0 \cdot 15$	$n_{II} = 1 \cdot 52972$
$r_{13} = 3 \cdot 820$		

Figure 5 represents a meridional section of the camera objective and r_{12} designates the central radius of curvature of the aspherical surface to be defined in polar coordinates:

Let φ be the angle (in degrees) between the radius of curvature (r_m) and the optical axis relative to all surface points in the meridional section. If the surface constant is

$$A = 32 \cdot 673(10^{-3})\varphi,$$

then the meridional surface section is determined by

$$r_m = 1 \cdot 872 - (0 \cdot 49A^4 - 5 \cdot 89A^6)(10^{-3}).$$

The maximum figuring depth of the effective surface is about 20 wavelengths.

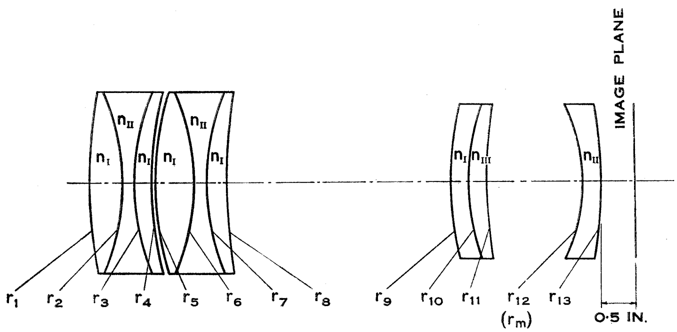

Fig. 5.—Camera objective (r_{12}=central radius of curvature, r_{m}=variable radius of curvature).

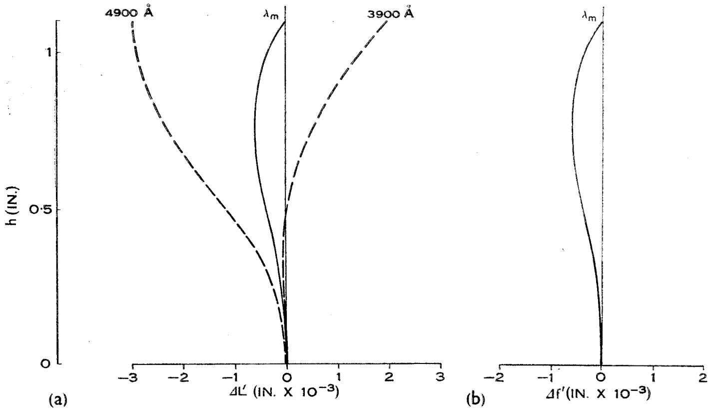

Fig. 6.—Aplanatic correction (h=semi-aperture). (a) Longitudinal spherical aberration ($\Delta L'$) and its variation with wavelength; (b) deviation from the sine condition ($\Delta f'$).

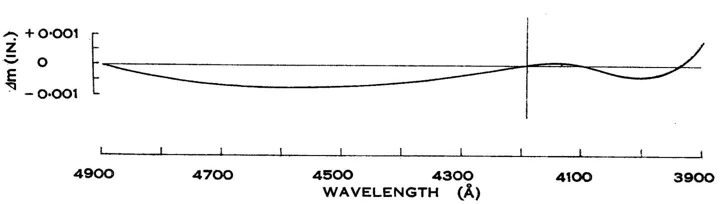

Fig. 7.—Meridional image field (Δm=linear deviation of the meridional image points measured parallel to the optical axis).

The performance of this objective in practice is marked by

(1) the highly aplanatic correction and its limited variation with wavelength (Fig. 6),

(2) the flatness of the meridional image field, which is almost perpendicular to the optical axis and well within tolerance ($\pm 0 \cdot 001$ in.) over the total wavelength region 3900–4900 Å (Fig. 7).

Since the intersection angles of the principal rays with the optical axis are less than 5°, highly uniform sharpness over the whole image field is indicated. The Petzval sum is $+0 \cdot 03$ in^{-1}, its change with wavelength is negligible, and the astigmatic differences at the edges of the image field ($+0 \cdot 01$ and $+0 \cdot 03$ in.) are insignificant. The total wavelength region corresponds to a spectrum of about $1 \cdot 2$ in. length on the photographic plate and can be extended to other regions by adjusting the tilt of the plate. Light transmission is further increased by blooming the glass-air surfaces for maximum transmission at 4200 Å.

In a practical resolving power test, carried out by H. Gollnow (Mount Stromlo Observatory), the system proved capable of resolving spectral lines which are $0 \cdot 3$ Å apart, corresponding to a resolving limit of 10 μ.

V. Conclusion

The limit of performance of a spectrograph with a given three-prism dispersing device and fixed values of the optical system constants is discussed. A new spectrographic objective (relative aperture $f/3 \cdot 5$) has been designed for the largest practicable extension of the spectral image field perpendicular to the optical axis. The corresponding spectral region 3900–4900 Å can be photographed with good definition. Other wavelength regions may be also recorded on tilted plates.

VI. Acknowledgment

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