THE DIRECTIONS OF AURORAL RAYS

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Summary

The geometry of the radiation point of an auroral corona is examined. The radiation point of two rays is the antidirection of the point within the Earth at which the rays meet or appear to meet. It is therefore incorrect to identify the radiant point of a corona with local auroral zenith. Their difference in direction is commonly $0\cdot5°$ of zenith distance. The importance of rays as magnetic disturbance indicators in the height range 100–1000 km is stressed, particularly in view of possible deformations of the magnetosphere whose full effects may not be estimated from ground-based observations of the geomagnetic field.

I. Introduction

It has long been known that auroral rays lie approximately along the geomagnetic field. An auroral drapery is a curtain of light in which numerous rays exist. When seen from almost underneath, a drapery is called a corona and its rays appear to converge upwards. The apparent point of convergence is called the "radiation point" of the corona. Early investigators estimated the position of this point by the use of a theodolite. These early investigators found that the radiation point is usually significantly removed (by amounts of order 1°) from the magnetic zenith as determined by surface observations of the geomagnetic field.

Vegard and Krogness (1920) estimated the position of the radiation points of auroral coronas by means of *single* photographs of such coronas. They state "*For the determination of the radiation point we have adopted the following procedure:—*

If ray streamers are to be regarded as parallel straight lines they are all situated in planes which pass through the radiation point, and on the picture the rays will coincide with great circles through the radiation point. In the actual cases the radiation point falls inside or near the border of the picture, and the nets show that the great circles through such points will be practically straight lines on the picture.

Now as before we draw by means of the lantern an enlarged copy of the photograph and we find the points of intersection for the various ray-streamers. When we have a large number of rays we get a large number of points of intersection and these points will fall inside a fairly small area. The radiation point is determined from the mean position of these points of intersection."

It is apparent that other investigators, e.g. Stormer (1938) and Abbott (1958), have used essentially the same method as Vegard and Krogness. Some investigators have compared the position of radiation points so determined with the simultaneous magnetic zeniths as determined by ground-based observations of the geomagnetic

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field. Early results obtained by theodolite suggested that the radiation point was lower than the magnetic zenith (Vegard and Krogness 1920).

It is the purpose of this communication to investigate the concept of radiation point. It will be seen that the radiation point as defined above is a meaningless concept for use in determining the relationship of auroral rays to the magnetic zenith. It is suggested that a more appropriate concept is "local auroral zenith", which is here defined as the direction of a (hypothetical) ray (regarded as a segment of a straight line) passing through the observer. Vegard and Krogness (1920) and Abbott (1958) used the phrase "auroral zenith" superfluously merely as a substitute for radiation point or radiant point. In the present paper, however, auroral zenith connotes a different concept from radiation point, as will be seen below. It will be demonstrated that conclusions based upon the concept of radiation point are likely to be unreliable unless it is clearly understood what is meant by it. In particular, past investigations of the relationship of radiation point to magnetic disturbance will be seen to need reinterpretation.

II. Theory

(a) *Auroral Rays*

In the first place it is essential to realize that auroral rays are features lying approximately along the geomagnetic field and as such are not parallel but diverge absolutely outward from the Earth. Seen from the Earth's surface, however, they appear to converge upwards in the sky (see below). The convergence is more apparent when the drapery forms a corona.

(i) *Case I, Collinear Rays.*—Consider a model of two collinear auroral rays (r_1, r_2). See Figure 1. The optic axis of a camera is taken as the z axis and (x, y) axes are chosen orthogonal to z through the centre of the camera lens C. Let the ray r_1 join the points $(0, Y, 0)$ and $(0, 0, Z)$, and the ray r_2 join the points $(X, Y, 0)$ and $(X+\Delta X, O, Z)$ in the (x, y, z) system of coordinates. The focal plane of the camera is (x^1, O, y^1) and x^1 and y^1 are taken parallel to x and y respectively. The focal distance is f $(= CO)$.

The image of a point P is the intersection of the straight line PC with the plane $(x^1, 0, y^1)$. Thus one finds the following set of features and images (cf. Fig. 1).

Feature in (x, y, z) *System*	*Image in* $(x^1\ y^1)$ *System*
Point $(0, Y, 0)$	Point $(0, -\infty)$
Point ${}^1P_\infty$	Point ${}^1I_\infty$ or $(0, fY/Z)$
Ray r_1	Image i_1 (full line)
Point $(X, Y, 0)$	Direction $\tan^{-1}(Y/X)$
Point ${}^2P_\infty$	Point ${}^2I_\infty$ or $(-f\Delta X/Z, fY/Z)$
Ray r_2	Image i_2 (full line)

It is apparent that the images i_1 and i_2 may be produced beyond ${}^1I_\infty$ and ${}^2I_\infty$ to meet at a point R. The angle (ψ) between i_1 and i_2 is $\tan^{-1}(X/Y)$. It is clear that R is the (mathematical) image of the point S at which the (extended) rays actually meet beneath the Earth. For, in the projection implied, the projection of a straight line is a straight line and therefore the projection of the intersection of two straight lines is the intersection of their projections. Corresponding portions of r and i are drawn similarly in Figure 1.

It is apparent that the considerations of Vegard and Krogness (1920), Stormer (1938), and Abbott (1958) have been with a set of points R which a photograph of an auroral corona may yield. (Skew rays will yield more than one point R. See case II below.) The auroral zenith, however, is a point of the family I_{∞}. The photograph of a corona whose rays are distributed along a parallel of geomagnetic latitude

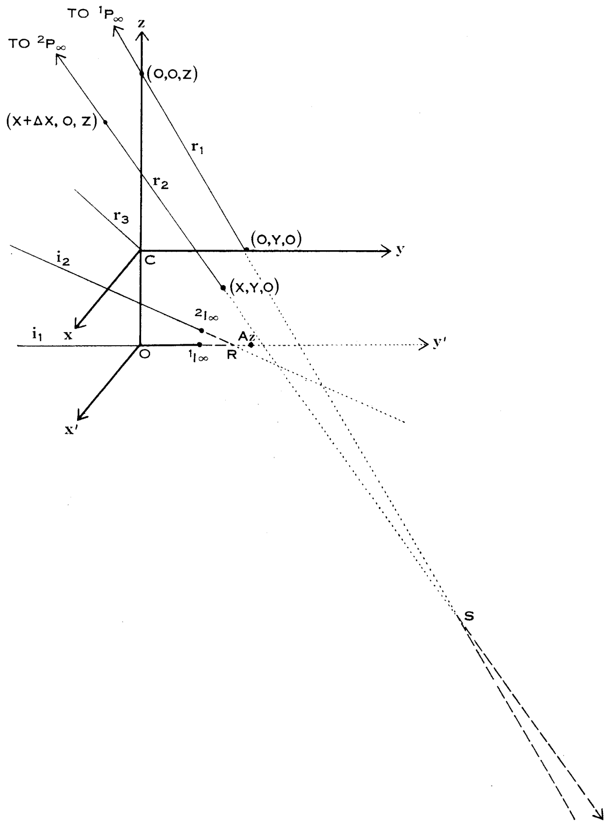

Fig. 1.—Schematic representation of two auroral rays (r_1, r_2) being photographed by a camera C to produce their images (i_1, i_2) in the focal plane (x^1, O, y^1).

would yield only one point R. This point R is in the antidirection of the point of intersection of these rays on the axis of the geomagnetic field (here assumed axially symmetric). This point R therefore has little to do with local auroral zenith at the Earth's surface and cannot be used as an indicator of its position. Rather, local auroral zenith must be derived by interpolation amongst or extrapolation from the set of points I_{∞}.

The question remains of how to determine the positions of the points I_{∞}. In general it is considered that points I_{∞} can only be obtained with precision with parallactic pairs of photographs. The tops and bottoms of auroral rays are often well-defined auroral features, so that with a pair of parallactic photographs the direction (and position) in space of auroral rays may be determined. This will yield by calculation a set of points I_∞. Each of the points I_∞ will have associated with it a point (E) on the Earth's surface to which one ray points. By interpolation amongst or extrapolation from the set of points E to the position of the observer, the point of the set I_∞ associated with the position of the observer (namely, local auroral zenith) may be found. Strictly speaking, of course, the rays are not straight, due to the curving of the geomagnetic field. Consequently the direction of the auroral zenith fixed in this way will depend on the height above the Earth of the segment of the ray at which its direction is determined. This effect would have to be taken into account in more accurate analysis, if the quality of photographs allowed it.

(ii) *Case II, Skew Rays.*—Consider now a model of two skew auroral rays, i.e. ones which if extended in straight lines would never meet. To discuss this case Figure 1 could be modified so that r_2 is the ray joining $(X+\Delta X, 0, Z)$ to $(X, Y+\Delta Y, 0)$. In this case the coordinates of ${}^2I_\infty$ become

$$(-f\Delta X/Z, f(Y+\Delta Y)/Z),$$

and $\tan\psi = X/(Y+\Delta Y)$.

In this case R is the image of the point at which the skew rays (seen from C) appear to converge under the ground. A corona in which the rays were skew would, however, yield a group of points R. The mean position of such points R is called the radiation point. It is obvious that the radiation point has little to do with "local auroral zenith" as defined above. The radiation point is merely the antidirection of the mean apparent point of convergence of the straight line extensions of auroral rays downwards.

(b) *Error involved in taking the Radiation Point as Local Auroral Zenith*

(i) *Position of the Radiant Point.*—Each point of the set I_∞ is the radiant point of a ray. The set of points I_∞ defines a "radiant region" for the whole corona. It is clear that a corona radiates upwards to a radiant region and not to a radiant point, and it radiates from a (mean) point below the horizontal plane whose antidirection is the radiant (or radiation) point R. There is no *a priori* reason why R should be the same as local auroral zenith.

Thus if two rays (r_1, r_2) at the same latitude have a longitude difference of $\Delta\phi$ then, on a dipole model field, the angular separation of ${}^1I_\infty$ and ${}^2I_\infty$ is

$$\beta_\phi = \Delta\phi \sin(I+\theta), \tag{1}$$

where I is the inclination and θ is the latitude. Thus if a corona extends 1000 km in longitude (which is common) near the auroral zones ($\theta \simeq 60$–$70°$), then $\beta_\phi \simeq 6°$. If two rays (r_1, r_4) are of the same longitude but of latitude separation $\Delta\theta$ then ${}^1I_\infty$ and ${}^4I_\infty$ are separated by

$$\beta_\theta = \Delta\theta[1+2\operatorname{cosec}^2\theta/(4+\cot^2\theta)], \tag{2}$$

assuming a dipole model field. Commonly rays are distributed over a latitude range of 5°, thus near the auroral zones $\beta_\theta \simeq 8°$.

Consider now a "potential" corona of size $\beta_\phi \simeq 6°$ and $\beta_\theta \simeq 8°$. Depending on what portion of the corona "lights up", the radiation point R derived may fall practically anywhere within a solid angle of about 50°. *A priori*, one could say that R may be up to 3 or 4° of zenith distance from local auroral zenith. It is clear that the error in taking R for A_z is intolerable.

(ii) *Error under Optimum Conditions*.—In this section optimum conditions for the estimation of A_z from R are discussed and the likely error found.

A "half-corona" is defined as a corona in which the rays in only one half of the sky are clearly developed. These are by far more common than "full-coronas" and form the majority of cases examined by Vegard and Krogness (1920) and Abbott (1958). Consider the special case of a half-corona situated polewards of the observer.

In Figure 1 suppose CZ is vertical and ZCY the meridian plane of a dipole magnetic field. An auroral ray (r_3) from C would intersect the plane (x^1, O, y^1) at a point A_z the image of the local auroral zenith. Suppose further that the rays r_1 and r_2 rise from a circle of geomagnetic latitude in the close vicinity of C. It is required to find the angle RCA_z ($=\delta$) (see Fig. 1). It is clear that the angle RCA_z equals the angle SCS^1 where S^1 (see Fig. 2) is the point on the geomagnetic axis at which rays on a circle of latitude through C meet. We first locate the point S^1 (see Fig. 2). Putting $DS^1 = L$ where D is the location of the dipole,

$$L = R \sin \alpha / \cos (\alpha - \theta), \tag{3}$$

where R = distance from D to base of ray, θ = geomagnetic latitude, α = angle between radius vector and dipole field (see Fig. 2). It is apparent that there are changes in L, (i) due to change in latitude of lines of force from r_3 to r_1, and (ii) due to changes in R, i.e. the height above the Earth at which the tangent plane at C (i.e. the plane of camera) intersects the ray r_1.

$$\mathrm{d}L = \frac{\partial L}{\partial \theta}\,\mathrm{d}\theta + \frac{\partial L}{\partial R}\,\mathrm{d}R. \tag{4}$$

Putting $SS^1 = \mathrm{d}L$ it can be shown that the angle subtended by $\mathrm{d}L$ at C is

$$\delta = \frac{\mathrm{d}L \cos^2 (\theta - \alpha)}{R \cos \theta}. \tag{5}$$

Thus, since the last term in (4) is much smaller than the others,

$$\delta \simeq \frac{(\mathrm{d}\alpha/\mathrm{d}\theta) \cos (\theta-\alpha) \cos \alpha + \{1 - (\mathrm{d}\alpha/\mathrm{d}\theta)\} \sin \alpha \sin (\theta - \alpha)}{\cos \theta} . \, \mathrm{d}\theta.$$

δ is the error in selecting the radiation point in place of the local auroral zenith, using a single fan of a corona. Putting $\theta = 67°$, $\alpha = 12°$ and $\mathrm{d}\alpha/\mathrm{d}\theta \simeq -0{\cdot}56$. Thus $\delta \simeq -0{\cdot}13\ \mathrm{d}\theta$.

Now in practice the point at which a ray of an auroral corona meets the horizontal plane of the observer is seldom less than 200 km and is more probably about 500 km from the observer. Putting $Y = 300$ km (see Fig. 1), $\mathrm{d}\theta \simeq 3°$ and $\delta \simeq -0{\cdot}4°$.

The above calculation of error (δ) is based on a simple geomagnetic field model more appropriate probably to quiet than to disturbed conditions. The calculation of δ was proferred to indicate the probable magnitude of error involved in taking R for A_z. During magnetic disturbance a more complex field, especially in the ionosphere, is likely to exist. This should not greatly affect the size of δ; however, the position of R or A_z with respect to magnetic zenith as observed at the ground may then assume a variety of relationships (see next section). It is clear that the confusion of R with A_z, as has been the case in the past, has lead to some weightless conclusions regarding the relationships of auroral zenith to magnetic zenith.

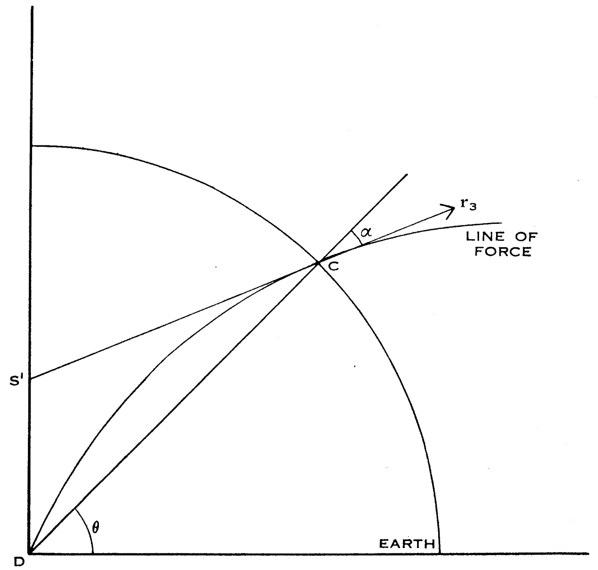

Fig. 2.—A dipole magnetic field line. The tangent at C meets the dipole axis at S^1.

III. Auroral Zeniths and Magnetic Disturbance

It is apparent from the above theory that studies of the relationship of radiation point to magnetic zenith are of limited value. However, studies of the relationship of local auroral zenith (as here defined) to local magnetic zenith are of the utmost importance. The writer knows of no such study.

It is suggested here that parallactic photography of auroral rays could help in the understanding of the dynamics of the outer magnetosphere. During magnetic disturbance the magnetic zenith (as determined from ground observations) fluctuates in direction. These fluctuations are due in part to electric current in the ground and in part to current in and above the ionosphere. With regard to auroral rays there appear two important questions to answer.

(1) With what accuracy may auroral rays be said to lie along the geomagnetic field?

(2) In what way does local auroral zenith fluctuate with respect to local magnetic zenith as determined at the ground?

It is desirable that the first question be resolved with an accuracy better than $0 \cdot 1^{\circ}$, for a large fluctuation in magnetic zenith (at the ground) is about 1° (corresponding to a disturbance in H of about 1000 γ). This experiment would require rocket magnetometer flights in conjunction with accurate parallactic photography of the corona. Preferably it would require magnetic measurements up to a height of 500 km (some auroral rays even extend to 1000 km!).

Whether or not this experiment can be done and yield useful results, the second investigation could proceed on the *reasonable assumption that at all times auroral rays are aligned along the geomagnetic field.* On this assumption auroral rays become valuable indicators of the direction of the geomagnetic field in a region of the atmosphere, 100–1000 km, not readily accessible to *in situ* magnetic measurements. The accurate parallactic photography of rays simultaneously with ground observations of magnetic disturbance would add another dimension to the experimental investigation of polar geomagnetic disturbance which is at present sadly lacking.

It is important to relate apparent auroral zeniths to magnetic disturbance. Variations in auroral zenith (i.e. magnetic zenith in the ionosphere) will be controlled by ionospheric and magnetospheric currents. These in turn are dependent on the interaction of interplanetary winds and ionospheric winds through the intermediary of the magnetosphere (Cole 1962*b*). Let us consider these currents in two groups.

(i) *Current Orthogonal to the Geomagnetic Field.*—Such currents flow intensely in the auroral zone at heights between 100 and 200 km (Cole 1962) sometimes in the form of a jet. Consider an isolated jet at a height of 150 km. Its magnetic effect at the ground below it will be twice that at 450 km altitude and in the opposite direction. Thus if such a jet is the sole source of disturbance the fluctuation in magnetic zenith (at the ground) should be twice that at 450 km altitude and of opposite sign in zenith distance.

(ii) *Current along the Geomagnetic Field.*—Recent theory of solar wind generated geomagnetic disturbance (Piddington 1959; Cole 1961) suggests that current often flows along the geomagnetic field lines from and to the interplanetary medium. Such current will cause a twist of the geomagnetic field, so that the auroral zenith will be displaced in azimuth. According to Piddington (1962) such current flowing along the geomagnetic field from the solar wind together with ionospheric Pedersen current with which it finds continuity has zero magnetic effect at the ground. He claims that ground level disturbance is attributable to the ionospheric Hall currents associated with this system. This claim may be experimentally tested by the study of the relationship of auroral zenith to magnetic zenith (at the ground) during disturbance.

(iii) *Deformation of the Magnetosphere.*—Implicitly associated with geomagnetic disturbance is the deformation of the magnetosphere. If auroral zeniths do not fluctuate in a manner predictable from sea-level magnetic disturbance, it means that there are disturbances in the magnetosphere whose effect is not registered at the ground. Since auroral regions are presumably connected to those outer parts of the magnetosphere whose interaction with interplanetary winds is the greatest, valuable insight into this interaction may follow from renewed study of auroral rays by parallactic photography along the lines suggested above. Such study would be made more valuable still by the simultaneous study of the motion of such rays (cf. Cole 1963).

IV. Discussion

Hitherto the study of auroral zeniths in relation to magnetic zeniths has been wrongly based. On this score alone the study should be recommenced. In these days of interest of the Earth environment in interplanetary space the study of auroral zeniths takes on a new importance—an importance it never had 30 and 40 years ago when interest *per se* in auroral zeniths was great. Compared to rocket investigations of the ionospheric geomagnetic field, parallactic photography of the aurora is inexpensive. However, any full program of such photography should be accompanied by rocket reference determinations of the magnetic field.

V. Acknowledgments

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