ON NUCLEON MAGNETIC MOMENTS

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Summary

The magnetic moments of the proton and neutron are computed for the pseudoscalar-pseudovector symmetric meson field considering the magnetic moments as the time average of the proton and the meson magnetic moments. Using the perturbation theory, the meson contribution is calculated from the self-energy of a nucleon in presence of a weak homogeneous magnetic field where the term due to meson magnetic energy was introduced from the expression for the force exerted on the emitted meson by the magnetic field. The self-energy is made finite with the idea of a fundamental length as introduced by Heisenberg and the cut-off so obtained fits exactly with that suggested by Chew in the extended-source theory. It is also found that mesons need to be present only about 25% of the time to explain the observed magnitudes of the anomalous nucleon magnetic moments.

I. Introduction

Using the covariant formalism and renormalization technique, several attempts (Bethe and De Hoffmann 1955) have been made to explain the well-known deviation of the nucleon magnetic moment from that obtained by Dirac. But the experimental agreement is far from satisfactory. The dispersion-theoretic approach (Frazer and Fulco 1960), however, has considerable success in solving the anomaly in the nucleon magnetic moment although the basic assumptions inherent in such a approach are yet to be verified rigorously. On the other hand the study of the extended source model in meson theory (Chew 1954) gives considerable insight into the low energy phenomena where the nucleon can be treated non-relativistically. And so far as the nucleon magnetic moment is concerned the experimental agreement is quite promising (Kundu 1958). In view of these facts it is interesting to investigate the problem in the non-relativistic approximation using the perturbation theory.

The present paper is an attempt to explain the nucleon magnetic moments with the assumption that the magnetic moment of the proton is due to the free proton for $(1-\epsilon)$ seconds plus the free neutron with the positively charged meson for ϵ seconds. Assuming charge conservation, the contributions are evaluated from the self-energy of a nucleon during its emission and reabsorption of a pseudoscalar charged meson in presence of a weak homogeneous magnetic field. From dimensional analysis the meson magnetic energy is defined as the force exerted on it per wave number. The usual perturbation theory adopted here results in an infinite self-energy and is made finite with the introduction of a fundamental length (Heisenberg 1938) that restricts the emission of mesons by a nucleon at rest with wave number greater than a certain value. The cut-off limit so obtained is found to be consistent with

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that suggested by Chew in the non-relativistic theory. The ratio of the anomalous neutron magnetic moment to that of proton is found here to be independent of the coupling constant and the cut-off limit and agrees well with that obtained experimentally. The meson also couples with neutron and proton for approximately one-quarter of the total time, a feature which is interesting and significant for the present problem.

II. Theoretical Procedure

As stated in the introduction, the magnetic moment of the proton μ_{P} corresponding to the process

$$\mathrm{P} \rightleftharpoons \mathrm{N}+\pi^{+}$$

is given by

$$\mu_{\mathrm{P}} = (1-\epsilon)\mu_0+\epsilon\mu_{\pi^+}+\epsilon\mu_0', \tag{1}$$

where $\epsilon\mu_{\pi^+}$ is related to the proton self-energy in the magnetic field, and μ_0, μ_0', μ_{π^+} denote respectively the magnetic moments of the free proton, neutron, and charged meson.

From the perturbation theory, the self-energy of a proton is well known to be

$$W_{\mathrm{P}} = (H_{01}'H_{10}')/(E_0-E_1), \tag{2}$$

where H_{01}' is the matrix element for the emission of a positively charged pseudoscalar meson by the proton, H_{10}' is the matrix element for the absorption of the same meson by the same proton and E_0,E_1 are the energies in the initial and intermediate states of the system.

Now, writing the pseudoscalar charged meson field interaction density in the form (March 1951)

$$\mathscr{L} = -\chi(g\tau_{\mathrm{NP}}\cdot\tfrac{1}{6}\omega_{\alpha\beta\gamma}\overset{*}{\psi}_{\alpha\beta\gamma}+f\tau_{\mathrm{NP}}\tfrac{1}{24}\omega_{\alpha\beta\gamma\delta}\overset{*}{G}_{\alpha\beta\gamma\delta})+\text{complex conjugate}, \tag{3}$$

with

$$\omega_{4lk} = \overset{*}{\phi}_{\mathrm{N}}\alpha_l\alpha_k\phi_{\mathrm{P}}, \qquad \omega_{123} = -\mathrm{i}\overset{*}{\phi}_{\mathrm{N}}\alpha_1\alpha_2\alpha_3\phi_{\mathrm{P}}, \qquad G_{1234} = \overset{*}{\phi}_{\mathrm{N}}\beta\alpha_1\alpha_2\alpha_3\phi_{\mathrm{P}},$$

$$\tau_{\mathrm{NP}} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \qquad \tau_{\mathrm{PN}} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix},$$

and g,f as the strength of the interaction, we find for a nucleon at rest

$$H' = -\mathrm{i}f\tau_{\mathrm{NP}}(\boldsymbol{\sigma}\,\mathrm{grad}\,\psi'^{*})_0+\mathrm{i}f^{*}\tau_{\mathrm{PN}}(\boldsymbol{\sigma}\,\mathrm{grad}\,\psi')_0, \tag{4}$$

where we put $\psi' = G_{\alpha\beta\gamma\delta}$ and the index "0" signifies that the gradient is to be taken at the point occupied by the nucleon.

The emission and absorption matrix element for a positively charged pseudoscalar meson will then be (on using (4))

$$H'_{n_k^+,n_k^++1} = -i\chi^{\frac{1}{2}}f^*\tau_{PN}\left(\frac{\hbar c}{2V}\right)^{\frac{1}{2}}$$

$$\times\sum_k\left(\frac{\hbar c}{\hbar c\sqrt{(k^2+\chi^2)}}\right)^{\frac{1}{2}}(n_k^++1)^{\frac{1}{2}}(\boldsymbol{\sigma}\,.\,\mathbf{k})\exp(i\mathbf{k}\,.\,\mathbf{r}_0), \tag{5}$$

and

$$H'_{n_k^+,n_k^+-1} = i\chi^{\frac{1}{2}}f\tau_{PN}\left(\frac{\hbar c}{2V}\right)^{\frac{1}{2}}$$

$$\times\sum_k\left(\frac{\hbar c}{\hbar c\sqrt{(k^2+\chi^2)}}\right)^{\frac{1}{2}}(n_k^+)^{\frac{1}{2}}(\boldsymbol{\sigma}\,.\,\mathbf{k})\exp(-i\mathbf{k}\,.\,\mathbf{r}_0), \tag{6}$$

where $k\hbar = p$, is the momentum of the emitted virtual meson, $\chi = m_\pi c/\hbar$ (m_π = meson mass), and V is the volume in which the functions are assumed to be periodic.

Since the meson is subjected to a weak homogeneous magnetic field, the matrix element for single meson emission and absorption processes reduces to

$$(H'_{01})_H = -i\chi^{\frac{1}{2}}f^*\tau_{PN}^{(1)}\left(\frac{\hbar c}{2V}\right)^{\frac{1}{2}}\sum_k\left(\frac{\hbar c}{\hbar c\sqrt{(k^2+\chi^2)}+E_{\pi+,H}}\right)^{\frac{1}{2}}(\boldsymbol{\sigma}^{(1)}\,.\,\mathbf{k})\exp(i\mathbf{k}\,.\,\mathbf{r}_0^{(1)}), \tag{7}$$

and

$$(H'_{10})_H = i\chi^{\frac{1}{2}}f\tau_{NP}^{(1)}\left(\frac{\hbar c}{2V}\right)^{\frac{1}{2}}\sum_k\left(\frac{\hbar c}{\hbar c\sqrt{(k^2+\chi^2)}+E_{\pi+,H}}\right)^{\frac{1}{2}}(\boldsymbol{\sigma}^{(1)}\,.\,\mathbf{k})\exp(-i\mathbf{k}\,.\,\mathbf{r}_0^{(1)}), \tag{8}$$

where $E_{\pi+,H}$ denotes the energy of a positive meson in the magnetic field H.

In presence of the magnetic field we find also that for ϵ seconds

$$E_0-E_1 = -(E_{\pi+}+E_{\pi+,H}), \tag{9}$$

since $E_P=E_N$ and for the above interval of time the proton magnetic energy $E_{P,H}=0$.

Now, on using the relations (7), (8), and (9) we get from (2), the self-energy of the proton in the direction of the magnetic field as

$$W_{P,H} = \tfrac{1}{3}\chi ff^*\tau_{NP}^{(1)}\tau_{PN}^{(1)}(\hbar c/2V)$$

$$\times\sum_k(\boldsymbol{\sigma}^{(1)}\,.\,\mathbf{k})(\boldsymbol{\sigma}^{(1)}\,.\,\mathbf{k})\frac{\hbar c\exp\{i\mathbf{k}\,.\,(\mathbf{r}_0^{(1)}-\mathbf{r}_0^{(1)})\}}{-[\hbar c\sqrt{(k^2+\chi^2)}+(\mu_{\pi+})H]^2}, \tag{10}$$

where we put $E_{\pi+,H} = (\mu_{\pi+})H$ and $E_\pi = \hbar c\sqrt{(k^2+\chi^2)}$.

On simplification (10) reduces to

$$W_{\mathrm{P},H} = -\chi ff^* \frac{\hbar^2 c^2}{6V} \sum_k \frac{k^2}{[\hbar c\sqrt{(k^2+\chi^2)}+(\mu_{\pi^+})H]^2}. \tag{11}$$

Replacing $\sum_k$ by the integral

$$\int d\Omega \int \frac{V}{(2\pi)^3} k^2 \, dk$$

and multiplying both the numerator and denominator by $[\hbar c\sqrt{(k^2+\chi^2)}-(\mu_{\pi^+})H]^2$, we obtain on neglecting terms containing higher powers in H,

$$W_{\mathrm{P},H} = -\frac{ff^*\chi}{12\pi^2}\int_0^\infty \frac{k^4\,dk}{k^2+\chi^2}+\frac{2}{3\pi}\left(\frac{ff^*\chi}{4\pi\hbar c}\right)(\mu_{\pi^+})H\int_0^\infty \frac{k^4\,dk}{(k^2+\chi^2)^{3/2}}. \tag{12}$$

The force acting on a meson of momentum p and charge e, emitted by a nucleon at rest, in presence of a magnetic field H is given by

$$\begin{aligned} F &= (e/m_\pi c) \,.\, p \times H \\ &= (e\hbar/m_\pi c)\tfrac{1}{4}\pi k H, \end{aligned}$$

where the angle between p and H is θ and $(\sin\theta)_{\mathrm{av.}} = \frac{1}{4}\pi$.

Now, from the dimensional consideration we define the meson magnetic energy as

$$E_{\pi,H} = F/k.$$

Thus for neutral meson

$$E_{\pi^0,H} = 0, \tag{13}$$

and for charged meson

$$E_{\pi^\pm,H} = \pm\tfrac{1}{4}\pi(e\hbar/m_\pi c)H. \tag{14}$$

Using (1) and (12), we find

$$\epsilon\mu_{\pi^+} = \frac{2}{3\pi}\left(\frac{ff^*\chi}{4\pi\hbar c}\right)\mu_{\pi^+}\int_0^\infty \frac{k^4\,dk}{(k^2+\chi^2)^{3/2}}, \tag{15}$$

where $\mu_{\pi^+} = \frac{1}{4}\pi(e\hbar/m_\pi c)$.

The integral in (15) leads to an infinite value of ϵ, i.e. of μ_{P}, and is made finite with the introduction of a fundamental length, according to which it is not possible to invent an experiment of any kind that permits a distinction between the positions of two particles at rest, the distance of which is below a certain limit d—the fundamental length. Thus we get an upper limit of the momentum p as h/d and that mesons with wave number k greater than $(2\pi/d)$ can not be emitted by a nucleon at rest. It is expected that d should not exceed the value of the nuclear range $1/\chi$ and which in turn gives $k_{\max.} \leqslant 2\pi\chi$.

The equation (15) thus transforms to

$$\epsilon = \frac{2}{3\pi}\left(\frac{ff^*\chi^3}{4\pi\hbar c}\right)\frac{1}{\chi^2}\int_0^{2\pi\chi}\frac{k^4\,\mathrm{d}k}{(k^2+\chi^2)^{3/2}}. \tag{16}$$

Now, on evaluating the neutron-proton interaction potential with the matrix element given by (7) and (8) it will be easily seen that $(ff^*\chi^3/4\pi\hbar c)$ corresponds to the coupling constant and taking its value to be $0\cdot058$ (Kundu 1958), we find from (16)

$$\epsilon = 0\cdot21, \tag{17}$$

and from (1), the proton magnetic moment

$$\mu_{\mathrm{P}} = 2\cdot991\mu_0,$$

where we put $M_{\mathrm{P}}/m_\pi = 6\cdot67$.

Proceeding exactly as in the case of proton with the scheme

$$\mathrm{N} \rightleftharpoons \mathrm{P}+\pi^-,$$

and noting that

$$E_0-E_1 = -(E_{\pi^-}+E_{\pi^-,H}+E_{\mathrm{P},H})$$

the self-energy of the neutron in the direction of the magnetic field H is found to be

$$W_{\mathrm{N},H} = -\tfrac{1}{3}\chi ff^*\frac{\hbar^2c^2}{4\pi^2}$$

$$\times\int_0^\infty\frac{k^4\,\mathrm{d}k}{[\hbar c\sqrt{(k^2+\chi^2)}+(\mu_{\pi^-})H][\hbar c\sqrt{(k^2+\chi^2)}+(\mu_{\pi^-})H(1-\frac{1}{4}\pi m_\pi/M_{\mathrm{P}})]}. \tag{18}$$

Multiplying both the numerator and denominator by

$$[\hbar^2c^2(k^2+\chi^2)-2\hbar c\sqrt{(k^2+\chi^2)}(\mu_{\pi^-})H(1-\tfrac{1}{8}\pi m_\pi/M_{\mathrm{P}})+(\mu_{\pi^-}^2)H^2(1-\tfrac{1}{4}\pi m_\pi/M_{\mathrm{P}})]$$

and retaining terms linear in H, we obtain

$$W_{\mathrm{N},H} = -\frac{ff^*\chi}{12\pi^2}\int_0^\infty\frac{k^4\,\mathrm{d}k}{k^2+\chi^2}$$

$$+\frac{ff^*\chi}{\hbar c}\frac{1}{6\pi^2}(\mu_{\pi^-})H\left(1-\frac{\pi}{8}\frac{m_\pi}{M_{\mathrm{P}}}\right)\int_0^\infty\frac{k^4\,\mathrm{d}k}{(k^2+\chi^2)^{3/2}}. \tag{19}$$

As before we write for the neutron magnetic moment

$$\mu_N = (1-\epsilon')\mu_0' + \epsilon'\mu_0 + \epsilon'\mu_{\pi^-}, \tag{20}$$

where $(\epsilon'\mu_0 + \epsilon'\mu_{\pi^-})$ will correspond to the coefficient of H in $W_{N,H}$.

Thus, we get from (19) and (20)

$$\epsilon'(\mu_0+\mu_{\pi^-}) = \frac{2}{3\pi}\left(\frac{ff^*\chi^3}{4\pi\hbar c}\right)(\mu_{\pi^-})\left(1-\frac{\pi}{8}\frac{m_\pi}{M_P}\right)\frac{1}{\chi^2}\int_0^\infty \frac{k^4\,dk}{(k^2+\chi^2)^{3/2}},$$

which finally gives

$$\epsilon' = 0\cdot 22,* \tag{21}$$

where the upper limit of the integral is taken as $2\pi\chi$.

The neutron magnetic moment will therefore be

$$\mu_N = -2\cdot 086\mu_0,$$

and the anomalous ratio of the neutron to the proton magnetic moment reduces to

$$-\mu_N/(\mu_P-1) = 1\cdot 05.$$

It is worth while to mention that the more exact expression for the energy should be given by

$$E = \alpha(F/k),$$

where α is a numerical factor.

Proceeding exactly as in the case with $\alpha = 1$, one can easily find that agreement with the experimental results is possible for $\alpha = 1\cdot 25$ and $(ff^*\chi^3/4\pi\hbar c) = 0\cdot 08$. This estimate of the value of the coupling constant is consistent with that obtained from the dispersion theoretic calculations and for $\alpha = 1\cdot 25$ the meson magnetic energy reduces to $\sim(e\hbar/m_\pi c)$.

III. Discussion

The present approach explains quite successfully the neutron and proton magnetic moment with the same value of the cut-off limit and coupling constant as used in the non-relativistic theory. The fact that meson needs to be present for 25% of the time is an interesting feature of the present procedure and may have some useful consequences to be exploited experimentally. The idea of removing the infinity with the introduction of a fundamental length, however, leads to a non-relativistic theory and the value of the coupling constant may impart certain inaccuracy. It is worth mentioning that in spite of many shortcomings of the perturbation theory it could well explain the nucleon magnetic moments. The effect of nucleon recoil may be considered with the idea of reduced mass.

* According to the principle of charge independence the time average ϵ for $P \rightarrow N+\pi^+$ should be equal to ϵ' for $N \rightarrow P+\pi^-$. However, this may not be true in the presence of a magnetic field. And it can be seen from the present approach that this violation of the charge independence has considerable effect on the difference in the neutron and proton anomalous magnetic moments.

IV. References

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