THE DIRECTIONS OF AURORAL RAYS

II. METHODS OF DETERMINATION

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Summary

Two simple methods are described for finding accurately the direction of an auroral ray from a parallactic pair of photographs. Errors are discussed. The methods reopen an experimental field, namely, the study of the relationship of the direction of the geomagnetic field in the polar ionosphere to that at the ground.

I. Introduction

Hitherto no widespread use of parallactic photography appears to have been made for the determination of the slope of auroral rays. Indeed no accurate determinations of such slopes appear to exist. The need for such determination was stressed in an earlier paper (Cole 1963*a*, here called paper I) where it was shown that the study of the relationship of radiant regions of auroral coronas to magnetic disturbance was hitherto wrongly based. The methods discussed here reopen this field of experimental investigation. The colinearity of auroral rays with geomagnetic field lines has yet to be established with precision. Once established, this property could become a valuable tool to assist in mapping the polar geomagnetic field and in the study of polar geomagnetic disturbance.

It is the purpose of this paper to describe two simple methods for the determination of the slope of an auroral ray and to indicate the errors involved therein. It is found that the errors involved can be sufficiently small to warrant application of the methods in practice. The author knows of no other methods in the literature which fulfil this need to the precision indicated below. It is hoped that observers will make the necessary measurements to fill the obvious gap in our knowledge of auroral rays.

II. Method A

Commonly, parallactic pairs of auroral photographs have been taken with no other aid to the finding of directions than the star background (cf. Stormer 1955). Consider such a pair of photographs of an auroral ray.

A ray appears on a photograph as the segment of an almost straight line. Curvature of the geomagnetic field (discussed by Vegard and Krogness 1920) is not allowed for in the initial analysis; this is discussed below under "errors" (Section IV). The apparent slope of a ray seen by one observer (or his camera) can be found by interpolation amongst and calculation from the star background as follows.

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By interpolation from the stars the declination (δ) and right ascension (α) of any two points (subscripts 1, 2) on a ray projected onto the celestial sphere can be found. From these a normal ($\mathbf{n}$) (in the direction δ_n, α_n) to the plane containing the ray and an observer is found as the vector product of the two vectors in the directions of the two points. Thus

$$\mathbf{n} = \mathbf{a}_1 \times \mathbf{a}_2, \tag{1}$$

where $\mathbf{a}_1 \equiv (\cos\delta_1 \cos\alpha_1, \cos\delta_1 \sin\alpha_1, \sin\delta_1)$

and $\quad \mathbf{a}_2 \equiv (\cos\delta_2 \cos\alpha_1, \cos\delta_2 \sin\alpha_2, \sin\delta_2)$.

$\mathbf{n}$ is the normal to that great circle on the celestial sphere which contains the ray and the observer. Likewise, the second observer finds a normal $\mathbf{n}^*$ (in the direction δ_{n^*}, α_{n^*}) using any two points (3, 4) on the ray, thus

$$\mathbf{n}^* = \mathbf{a}_3^* \times \mathbf{a}_4^*.$$

Finally, the direction of the ray (δ_{R}, α_{R}) in the celestial coordinate system is given by the vector product

$$\mathbf{n} \times \mathbf{n}^* = (\mathbf{a}_1 \times \mathbf{a}_2) \times (\mathbf{a}_3^* \times \mathbf{a}_4^*). \tag{2}$$

Thus

$$\sin\delta_{\mathrm{R}} = \cos\delta_n \cos\delta_{n^*} \sin(\alpha_{n^*} - \alpha_n), \tag{3}$$

$$\tan\alpha_{\mathrm{R}} = \frac{\cos\delta_{n^*} \cos\alpha_{n^*} \sin\delta_n - \cos\delta_n \cos\alpha_n \sin\delta_{n^*}}{\cos\delta_n \sin\alpha_n \sin\delta_{n^*} - \cos\delta_{n^*} \sin\alpha_{n^*} \sin\delta_n}, \tag{4}$$

where

$$\sin\delta_n = \cos\delta_1 \cos\delta_2 \sin(\alpha_2 - \alpha_1), \tag{5}$$

$$\tan\alpha_n = \frac{\cos\delta_2 \cos\alpha_2 \sin\delta_1 - \cos\delta_1 \cos\alpha_1 \sin\delta_2}{\cos\delta_1 \sin\alpha_1 \sin\delta_2 - \cos\delta_2 \sin\alpha_2 \sin\delta_1}, \tag{6}$$

with similar expressions for $\sin\delta_{n^*}$ and $\tan\alpha_{n^*}$.

III. Method B

Jacka and Ballantyne (1955) developed special parallactic auroral cameras for use on Australian National Antarctic Research Expeditions. In these the image of an auroral form is produced on the film together with the image of an accurately ruled rectangular grid of lines. The direction of pointing of the camera, the orientation of the grid, and positioning of the grid with respect to the optic axis of the camera are known independently of the positions of photographed stars (which are not required in the method).

Consider the taking of a photograph of an auroral ray (pq) with such a camera whose object glass has centre C and whose optic axis is Cz (see Fig. 1). The image is located in the plane XOY so that PQ is the image of and is coplanar with pq. The focal length (f) of the camera equals CO. A point of special significance is the projection of the normal (to the plane Cpq) CN onto the plane XOY. This point is labelled N (coordinates X_n, Y_n) and will be called the pole of the plane Cpq.

It can be shown that the line ON is orthogonal to the image PQ of the plane Cpq as follows. The plane Cpq intersects the plane xCy in a line (Cl, not shown) which is orthogonal both to CN and Cz. This line Cl is parallel to the image PQ because the planes xCy and XOY are parallel. Therefore the plane Cnz (i.e. CNO) is orthogonal to PQ. Thus ON is orthogonal to PQ.

Clearly CN is orthogonal to CT, thus

$$ON = f^2/OT. \tag{7}$$

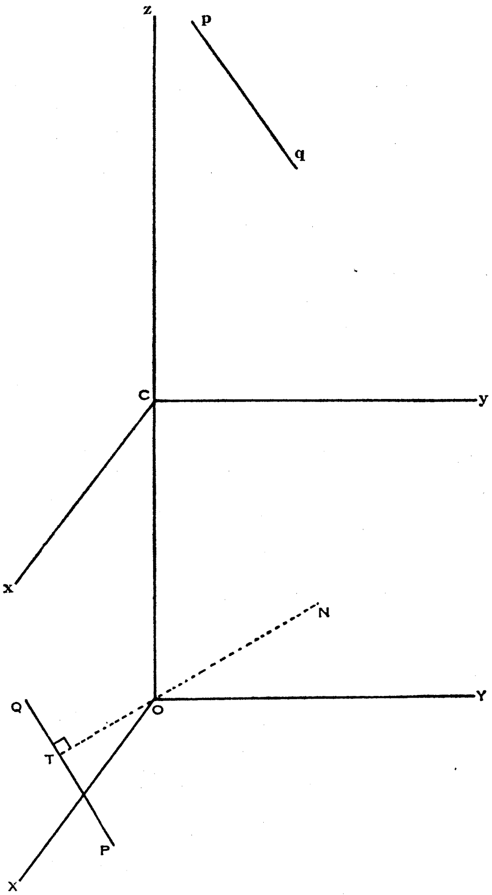

Fig. 1.—Method B: A camera C with optic axis OCz produces the image PQ of the ray pq in the focal plane XOY. The image (not shown) of a rectangular grid centred on O is also on the photograph. OT is orthogonal to PQ.

The point N can therefore be located on the photograph by constructing the perpendicular line to PQ through O (the centre of the rectangular grid of coordinates) and producing the line beyond O the distance given by (7), after having measured OT in the course of the construction, and knowing f for the camera.

Of course CN defines the normal to the plane Cpq. Knowing the direction of pointing (Cz) of the camera the direction of CN is then found by reference to the rectangular grid on the image.

In the system of coordinates (x, y, z), CN has direction cosines $(X_n, Y_n, -f)$. The equation of the plane Cpq is thus

$$X_n x + Y_n y - fz = 0. \tag{8}$$

Now a linear transformation connects the system x, y, z to a geographic system ξ, η, ν in which the ν axis is taken as the axis of rotation of the Earth, and the plane $\eta = 0$ passes through Greenwich. Thus

$$\begin{aligned} x &= l_1\,\xi + m_1\,\eta + n_1\,\nu + \beta_1, \\ y &= l_2\,\xi + m_2\,\eta + n_2\,\nu + \beta_2, \\ z &= l_3\,\xi + m_3\,\eta + n_3\,\nu + \beta_3. \end{aligned}$$

Thus the direction cosines of the direction CN in the geographic system are given by

$$\mathbf{n} \equiv (X_n l_1 + Y_n l_2 - f l_3;\; X_n m_1 + Y_n m_2 - f m_3;\; X_n n_1 + Y_n n_2 - f n_3). \tag{9}$$

Likewise from the second photograph of the ray

$$\mathbf{n}^* \equiv (X^*_{n^*} l^*_1 + Y^*_{n^*} l^*_2 - f^* l^*_3;\; X^*_{n^*} m^*_1 + Y^*_{n^*} m^*_2 - f^* m^*_3;\; X^*_{n^*} n^*_1 + Y^*_{n^*} n^*_2 - f^* n^*_3).$$

Finally, the direction of the ray in the geographic coordinate system is given by

$$\mathbf{n} \times \mathbf{n}^*.$$

IV. Errors

(a) *Exposure Time*

Exposure time introduces an error in positions found from stars of 15″ of arc for every 1 s of exposure time. The same error, of course, is involved in locating the beginning of the exposure period.

(b) *Motion of the Auroral Ray*

Auroral rays are known often to move normally to their length with speeds up to a few thousand metres per second. Consider a ray moving in longitude at a magnetic latitude of 70°. The slope of the ray will change by about 6″ of arc per second for every 100 m/s of its speed. Fast-moving rays with long exposures introduce errors which are too large. Slow-moving rays ($<$300 m/s) and short exposures ($<$3 s) introduce errors of $<$1′ of arc in the slope.

Commonly rays are almost stationary. Good quality photographs of rays (kindly provided by Dr. F. Jacka) have been examined and it is possible to delineate the slope of some ray images to an accuracy of better than 0·1°.

(c) *Method A*

It follows from (5) that the maximum error in estimating δ_n is

$$\epsilon\,(\delta_n) = |\tan \delta_n|\,(|\tan \delta_1| + |\tan \delta_2| + 2|\cot(\alpha_2 - \alpha_1)|)\epsilon, \tag{10}$$

where ϵ is the absolute error in arc measurement from the positions of stars. Thus $\epsilon(\delta_n)/\epsilon$ can be kept small by choosing $\delta_1 \approx \delta_2 \lesssim \frac{1}{4}\pi$ and $\alpha_2 \approx \frac{1}{2}\pi + \alpha_1$.

From (6) the extreme error in estimating α_n is

$$\epsilon\,(\alpha_n) = \sin\alpha_n\,\cos\alpha_n\left(\left|\frac{\delta N}{N}\right|+\left|\frac{\delta D}{D}\right|\right), \tag{11}$$

where δN is the error in estimating N, the numerator of (6), and D represents the denominator in (6). Because of their length these differentiations are not reproduced here. It is not simple to see from this expression what are the optimum conditions for minimum error. However, it is readily seen that if ϵ is 1′ of arc then $\epsilon(\alpha_n)$ or $\epsilon(\delta_n)$ can be as little as 5′ of arc in favourable circumstances. Since finding $(\delta_{\mathrm{R}}, \alpha_{\mathrm{R}})$ is just a replication of the process of finding (δ_n, α_n) the final errors $\epsilon(\alpha_{\mathrm{R}})$ and $\epsilon(\delta_{\mathrm{R}})$ may be as low as about 20–30′ in favourable situations. The calculation of errors seems to be as important as the calculation of $(\delta_{\mathrm{R}}, \alpha_{\mathrm{R}})$ and should be done in every case.

(d) *Method B*

This method appears to have errors of the same order as method A but somewhat larger, owing to the additional operations involved such as positioning the grid with respect to the optic axis of the camera.

(e) *Curvature of the Geomagnetic Field Lines*

Vegard and Krogness (1920) have investigated the change in magnetic zenith with altitude and find that at Bossekop a change of height of 200 km can cause a change of zenith distance of 1′ of arc and of azimuth 1° 31′ of arc. This error remains as an absolute error superimposed upon the error involved in finding the direction of a ray from a parallactic pair of photographs.

(f) *Overall and Desirable Accuracy*

Intuitively one would expect the greatest precision in determination of ray orientation when the two directions $\mathbf{n}$ and $\mathbf{n}^*$ are orthogonal and when the angle subtended by the ray at the observer is largest. Since the lowest point on a ray is likely to be about 100 km altitude, this implies a required base line of about 200–300 km.

During magnetic disturbance the horizontal component of the geomagnetic field can change by as much as 1000γ. This implies a change of magnetic zenith (at the ground) of about 2°. Whether or not the change of magnetic zenith in the ionosphere is of similar size is not known but theory would suggest that it may even be more (see paper I). Thus the method outlined here could provide the basis for the experimental verification of whether rays lie along the geomagnetic field and for the investigation of the relationship of auroral and magnetic zenith during magnetic disturbance. Evidence suggests (cf. Cole 1963*b*) that the motion of rays is faster during greater magnetic disturbance. Errors introduced by this motion are still smaller than the effect of change of magnetic zenith expected theoretically.

V. Discussion

In the experimental set-up the photographs should be taken so that they record rays which are close to the auroral zenith (see paper I) of the magnetic observatory.

For comparison of magnetic zenith with the slope of auroral rays the former must be transformed either to the celestial system for the instant of photography (method A) or to the geographic system (method B).

In paper I a method for finding the orientation of an auroral ray was outlined, based on the conventional determinations of height of its base and its top. The methods proposed above are much more direct.

There remains the problem of identification of a ray in two photographs. This may be difficult, but not insuperable, when they are profuse.

Finally a refinement (pedantic in the context) is made to paper I. Paper I discussed the situation generally encountered with auroral coronas in which the point S (at which rays meet or appear to meet below the surface of the Earth) is on the opposite side of the focal plane of the camera to the rays. When, however, S is on the same side of the focal plane as the rays, they naturally appear to converge weakly downwards. So that an observer looking at low elevation towards the polewards horizon may see rays converging downwards. However, this situation hardly concerns the accurate determination of the auroral zenith at a station near the auroral zone, since such distant rays are related to distant auroral zeniths. The limit to the applicability of the theory of paper I occurs when the optic axis of the camera is orthogonal to the rays. Thus the optic axis should lie within 90° of auroral (i.e. approximately magnetic) zenith. Strictly, this should have been stated as an assumption in the theory of paper I. This will usually be the case, for useful strong convergence will only be obtained if the optic axis points within 45° of the magnetic zenith.

VI. Acknowledgment

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VII. References

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