ON EXTERNAL RADIO EMISSION FROM THE EARTH'S OUTER ATMOSPHERE

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Summary

The possibility is considered that cyclotron radiation, generated by fast bunched electrons in the Earth's outer atmosphere, may be observable outside the geomagnetic field. It is shown that the intensity and other properties of such external radiation may be deduced from the known characteristics of v.l.f. hooks. It appears likely that the radiation will occur as short bursts in the frequency range from 200 to 1000 kc/s. The total radiated power may be from 1 to 1000 W, depending on the electron energy.

Introduction

It has been known for many years that radio emissions, originating in the Earth's outer atmosphere, may be observed at the ground in the frequency range from 1 to 20 kc/s. They are generally of two main types, short-duration bursts lasting about a second, called hooks, and more continuous emissions like solar noise storms with a duration of minutes or hours and called hiss.

Recent developments in the theory of these radiations have shown that many of their properties may be accounted for if they originate as electromagnetic radiation from fast electrons trapped in the geomagnetic field (Dowden 1962*a*, 1963). The evidence suggests that from time to time weakly-radiating electron streams arise in the outer atmosphere. The streams appear to be energetically unstable and may be caused to emit strong coherent radiation when triggered by some suitable event such as a whistler (Hansen 1963; Helliwell 1963). The region of coherent radiation is frequently small (<500 km) and the radiating electrons may have narrow energy and pitch distributions with $\delta E/E$, $\delta\phi/\phi \sim 1\%$.

The requirement that the radiation process be coherent, that is, one in which electrons radiate in phase, may be seen from a consideration of the observed intensity. This is of the order of 10^{-13} W m^{-2} (c/s)$^{-1}$ at the ground and is estimated to be 10^{-10} W m^{-2} (c/s)$^{-1}$ above the ionosphere (Dowden 1962*b*). For a wave frequency of 5 kc/s this latter value corresponds to an electron temperature of 10^{21} °K if the process is incoherent, or an electron energy of 10^{17} eV, that is, far higher than is likely for electrons in the Earth's outer atmosphere.

The actual emission process seems likely to be cyclotron radiation, the wave frequency being Doppler shifted downwards behind the electrons to a value which permits propagation along the geomagnetic field lines to the observer.

Now the electrons may also in certain circumstances emit cyclotron radiation in a plasma in a forward direction (Ellis 1962, 1963*a*). This will occur if the Doppler shift is sufficient to cause the wave frequency to be greater than the extraordinary

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mode cut-off frequency,

$$f_X = (f_0^2 + \tfrac{1}{4} f_H^2)^{\frac{1}{2}} + \tfrac{1}{2} f_H.$$

In the outer atmosphere this forward radiation would be unable to travel downwards below the level for which the wave frequency $f = f_X$, but would be able to escape outwards, unlike the backward radiation which may be responsible for hooks and hiss.

It is conceivable, therefore, that whenever the very low frequency emissions are observed, radiation is also emitted by the same electrons in an outward direction but at a higher frequency. Here we examine this possibility and the likely properties of the radiation.

Theory

The conditions in which coherent generation and amplification of electromagnetic radiation can occur in a plasma have been investigated by a number of authors (Twiss 1958; Bekefi, Hirshfield, and Brown 1961; Ginsburg and Zelezniakov 1962). It has been shown that the radiation absorption coefficient may assume negative values for a particular frequency if there exists a radiative process where this frequency is emitted only by electrons of particular energy, providing that the actual energy distribution of the fast electrons has a positive slope for this value of energy.

Wild, Smerd, and Weiss (1963) give for the absorption coefficient of an anisotropic fast electron plasma, of density M in direction θ,

$$K(\theta) = -\frac{c^2 M}{m^2 f^2} \int_0^\infty \int_0^{2\pi} \left(\frac{\partial}{\partial E} + \frac{\Delta\phi}{hf}\frac{\partial}{\partial\phi}\right) F(E,\phi)\, W_f(E,\theta-\phi)\, g(E,\phi)\, \mathrm{d}E\, \mathrm{d}\phi, \tag{1}$$

where $F(E,\phi)$ is the distribution of the electrons with respect to energy E and pitch ϕ, $W_f(E,\theta-\phi)$ is the electron emissivity function for a frequency f, and m is the refractive index of the fast plasma. $g(E,\phi)$ is the statistical weight, and $\Delta\phi$ is the change in direction of the electron caused by emission or absorption of a photon. If the absorption coefficient is negative then a wave travelling through the (high velocity) plasma will be amplified. The amplified wave may originate either externally (external excitation) or internally as radiation from individual electrons (internal excitation).

For Doppler-shifted cyclotron radiation, the electron emissivity function $W_f(E,\theta-\phi)$ is a delta function of the energy and pitch, whose values are specified by the Doppler equation,

$$f_s = \frac{s(1-v^2/c^2)^{\frac{1}{2}} f_H}{1-n(v/c)\cos\theta\cos\phi}, \tag{2}$$

where f_s is the wave frequency for harmonic number s. Negative absorption may occur if the electron stream has a sufficiently narrow energy and pitch distribution such that

$$\left(\frac{\partial}{\partial E} + \frac{\Delta\phi}{hf}\frac{\partial}{\partial\phi}\right) F(E,\phi) > 0$$

near the values of E and ϕ which satisfy (2).

In the present circumstances this is not a particularly stringent condition. Bekefi, Hirshfield, and Brown (1961), for example, have shown that even a slight modification to a Maxwellian energy distribution can lead to coherent cyclotron radiation. In addition, to obtain theoretically the observed spectral shape of v.l.f. hooks one has already to specify a highly restricted range of possible energies and pitch angles (Dowden 1962*a*).

In this paper we will not consider in any more detail the electron properties needed for coherent amplification and radiation but will assume that in fact these may be satisfied by real electron streams. Further, since a complete theory is not yet available, we will assume that, providing amplification is possible, the properties of the radiation will be determined essentially by the electron emissivity function, which in turn contains the effect of propagation in the cold ambient plasma.

We need first to consider the electromagnetic radiation emitted by an electron travelling with speed v through a plasma of density N and with superimposed magnetic field H. The power emitted per unit solid angle per electron at frequency f_s is given by Eidman (1958, 1959)

$$W_s(\theta) = \frac{\xi^2 e^2 f_s^2 n\pi}{c^3} \frac{[v_1 J_s'(A)+(\alpha c f_{\mathrm{H}}\gamma s/f_s n \sin\theta+\delta v_2)J_s(A)]^2}{|1-\beta_2 \cos\theta(n+f_s\,\mathrm{d}n/\mathrm{d}f_s)|}, \tag{3}$$

where

$$\begin{aligned}
A &= \frac{v_1}{c}\frac{f_s}{f_{\mathrm{H}}}\frac{n}{\gamma}\sin\theta,\\
\gamma &= (1-v^2/c^2)^{\frac{1}{2}},\\
\theta &= \text{direction of wave normal with respect to H},\\
f_{\mathrm{H}} &= eH/2\pi mc,\\
X &= Ne^2/mf^2\pi = f_0^2/f_s^2,\\
Y &= f_{\mathrm{H}}/f_s,\\
\delta &= -K\sin\theta+L\cos\theta,\\
\alpha &= K\cos\theta+L\sin\theta,\\
K &= 2Y(1-X)\cos\theta[Y^2\sin^2\theta\mp(Y^4\sin^4\theta+4Y^2(1-X)^2\cos^2\theta)^{\frac{1}{2}}]^{-1},\\
L &= [XY\sin\theta+KXY^2\cos\theta\sin\theta][1-Y^2-X(1-Y^2\cos^2\theta)]^{-1},\\
\xi &= 2n^2[\{1-X/(1-Y^2)\}(1-\alpha^2)+(1-X)\delta^2-2XY\alpha/(1-Y^2)]^{-1},\\
f_0 &= (Ne^2/\pi m)^{\frac{1}{2}},\\
s &= 0, 1, 2, 3, \ldots .
\end{aligned}$$

v_1, v_2 are components of velocity perpendicular and parallel to the field, $\beta_{1,2} = v_{1,2}/c$. $W_s(\theta)$ represents the power integrated over the whole frequency range for each harmonic s.

A condition for radiation is that the wave frequency f_s shall satisfy equation (2), which must be solved for allowable values of f_s and θ simultaneously with the refractive index equation,

$$n^2 = 1-\frac{X(1-X)}{1-X-\frac{1}{2}Y^2\sin^2\theta\pm\{\frac{1}{4}Y^4\sin^4\theta+(1-X)^2\cos^2\theta\, Y^2\}^{\frac{1}{2}}}. \tag{4}$$

The values of f_s, θ, and n so obtained may then be used in equation (3) to compute $W_s(\theta)$. The behaviour of $W_s(\theta)$ differs markedly depending on whether the wave normal direction θ is between 0 and $\frac{1}{2}\pi$ (forward radiation) or between $\frac{1}{2}\pi$ and π (backward radiation), and we consider these two cases separately.

(a) *Backward Radiation*

In the situation of interest here, i.e. radiation in the Earth's exosphere by electrons of energies of about 100 keV (Dowden 1962*b*) the backward Doppler shift is sufficient to reduce the radiated frequency γf_{H} to a value much less than the local plasma cyclotron frequency f_{H}. Consequently, the wave will propagate in the quasi-longitudinal extraordinary mode and we may use the corresponding approximation for the refractive index

$$n^2 \sim X/(Y\cos\theta - 1), \qquad X \gg 1. \tag{5}$$

Solving equations (2) and (5) we obtain for the direction of the radiation, taking $s = 1$ (fundamental mode),

$$\cos\theta = \frac{(1-\gamma Y)^2}{2\beta_2^2 X}\left[Y \pm \left\{Y^2 - \frac{4\beta_2^2 X}{(1-\gamma Y)^2}\right\}^{\frac{1}{2}}\right]. \tag{6}$$

Figure 1 shows the relation between the observed frequency f_s and θ in typical circumstances. It is seen that as θ is decreased from π (backward direction) the Doppler frequency gradually increases until a maximum value f_{m} is reached.

Figure 1 also shows the corresponding variation in the computed radiated power W_s ($s = 1$) obtained from equation (3). It is noteworthy that, unlike cyclotron radiation in a vacuum, the maximum in W_s does not occur in the direction of the magnetic field but in the direction for which the Doppler frequency is a maximum. This direction is obtained easily from equation (6) (two roots equal)

$$\cos\theta = (1-\gamma Y)^2 Y/2\beta_2^2 X.$$

The corresponding frequency is obtained similarly,

$$f_{\mathrm{m}} = \frac{\gamma f_{\mathrm{H}}}{2\beta_2 f_0/f_{\mathrm{H}}+1}. \tag{7}$$

(b) *Forward Radiation*

If we consider radiation for which the Doppler frequency is greater than the local plasma cyclotron frequency (i.e. with $\frac{1}{2}\pi > \theta > 0$) no simple approximation for the refractive index is valid. It is, therefore, convenient in this case to solve the Doppler refractive index equations (2) and (4) by graphical means. In the Earth's exosphere the plasma frequency is generally greater than the plasma cyclotron frequency. The extraordinary mode cut-off frequency f_{X} is therefore usually significantly greater than f_{H} and solutions of equations (2) and (4) cannot be obtained for the fundamental mode ($s = 1$) with electron energies of about 100 keV. That is, the electron speed is not sufficient to Doppler-shift the radiated frequency $s\gamma f_{\mathrm{H}}$ to a value greater than f_x and the radiation is hence impossible in this mode. However, solutions will always be obtainable by considering the higher order harmonics (i.e. with $s > 1$).

This situation is shown in Figure 2. The variation in the radiated power $W_s(\theta)$ for the third and fourth harmonics for the conditions of Figure 2 is shown in Figure 3. It is seen from Figure 2 that for the third harmonic, as the angle θ is increased a maximum value of $\theta = \theta_m$ is reached beyond which there are no solutions of equations (2) and (4). It is found that for this value of θ the denominator in equation (3)

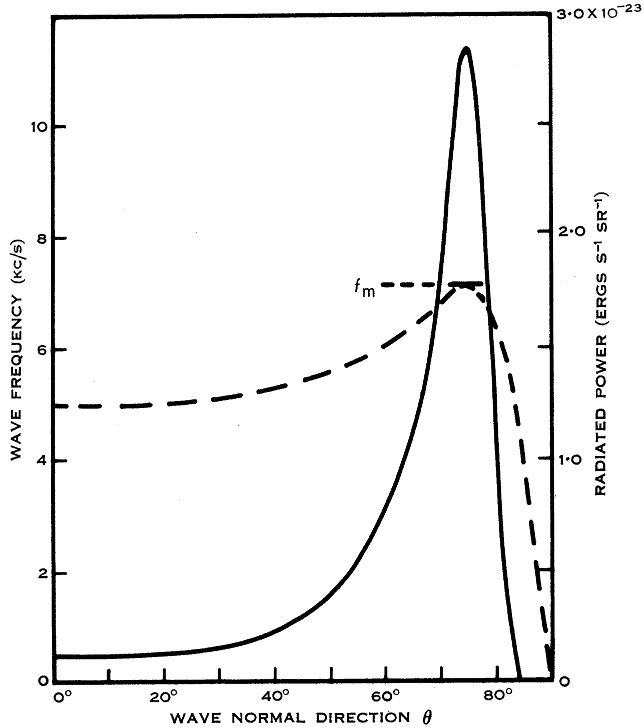

Fig. 1.—Variation of backward Doppler frequency with wave normal direction θ (dashed curve) and radiated power with θ (solid curve). Electron energy 110 keV; field line latitude 60°; emission latitude 30°; scale frequency 500 kc/s.

becomes zero, producing an infinity in the radiated power $W_s(\theta)$. This behaviour of $W_s(\theta)$ is typical whatever the order of the harmonic, providing there is a limiting value of $\theta = \theta_m$. The infinity will in general be suppressed as a result of refraction of the wave subsequent to emission. Since the radiation takes place in a medium of refractive index n, the angular power distribution is modified during the passage of the wave out of the medium according to

$$W(\alpha)\sin\alpha\,d\alpha = W(\theta)\sin\theta\,d\theta,$$

or

$$W_s(\alpha) = W_s(\theta)\frac{\sin\theta}{\sin\alpha}\frac{d\theta}{d\alpha}, \tag{8}$$

where $\alpha =$ direction of wave normal after refraction. We assume that the plasma density is proportional to the magnetic field intensity and hence planes of constant refractive index are normal to H. Now it is found that, for $\theta = \theta_m$, $d\alpha/d\theta$ is zero.

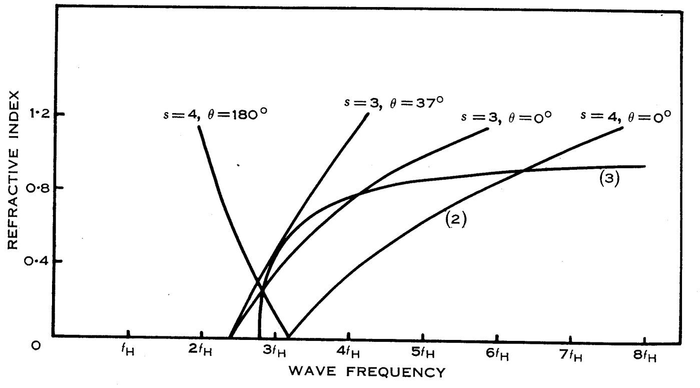

Fig. 2.—Graphical solution of the Doppler equation (2) and the refractive index equation (3) for forward radiation. ($\lambda = 40°$, $\beta = 0 \cdot 6$, $\phi = 29°$, $f_H = 100$ kc/s, $f_0 = 223$ kc/s.)

Conversely, however, refraction increases the radiated power in the direction for which the final direction α is a maximum ($\alpha = \alpha_m$). In Figure 4 the relation between

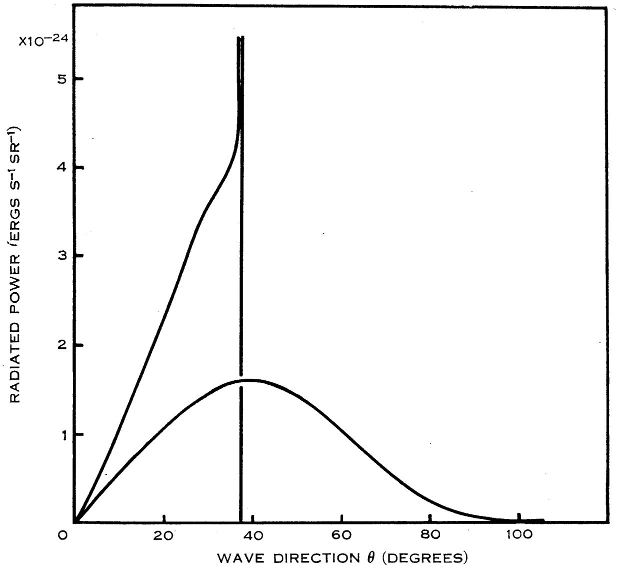

Fig. 3.—Variation of radiated power with θ for forward radiation. Parameters as in Figure 2.

θ and α for the third and fourth harmonics is illustrated. It is seen that $d\alpha/d\theta = 0$ for $\alpha = \alpha_m$ ($s = 3$), but that $d\theta/d\alpha = 0$ for $\theta = \theta_m$ ($s = 3$). For the fourth harmonic $s = 4$ there is no maximum value of θ but the enhancement in $W(\alpha)$, ($\alpha = \alpha_m$) still exists since $d\alpha/d\theta$, ($\alpha = \alpha_m$) $= 0$.

The angular variation in the radiated power after refraction obtained from equation (8) and Figure 3 is shown in Figure 5. We note the zero in $W(\alpha)$ for $\alpha = 0$ which

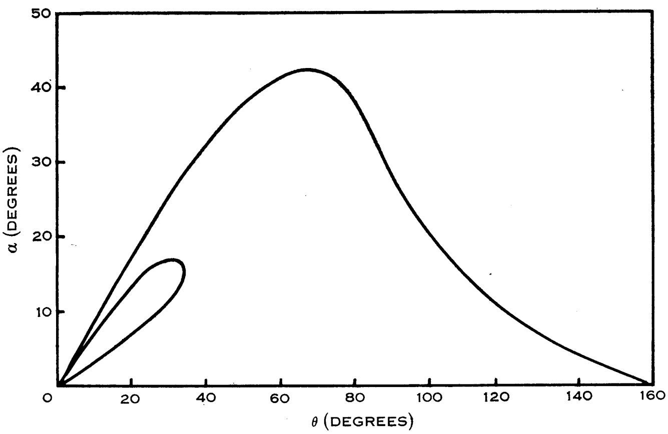

Fig. 4.—Relation between initial direction of wave normal θ and final direction α after refraction by the local plasma. Parameters as in Figure 2.

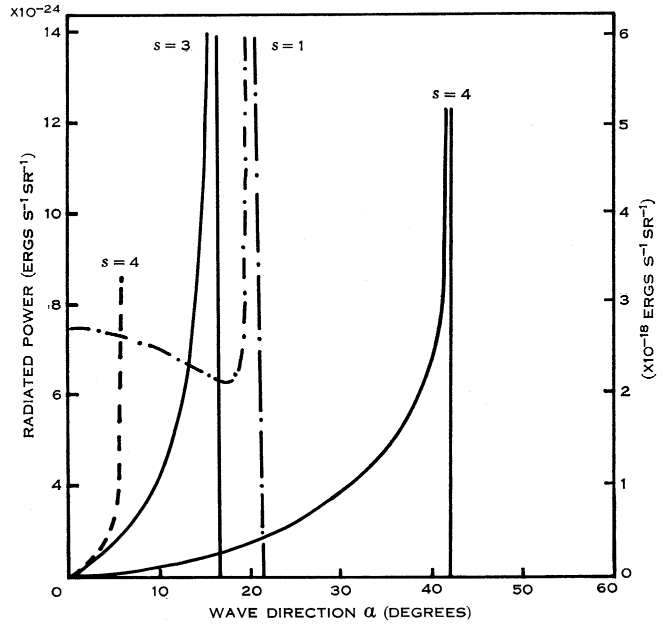

Fig. 5.—Angular distribution of power after refraction.
______ $E = 110$ keV, $\lambda = 40°$;
_ _ _ _ $E = 110$ keV, $\lambda = 30°$;
_._._. $E = 250$ keV, $\lambda = 52°$.

is characteristic of radiation in the higher harmonics, $s > 1$. For radiation in the fundamental mode ($s = 1$), where this is possible, $W(\alpha)$ has a maximum for $\alpha = 0$. This is illustrated in Figure 5, where the electron energy and plasma density are chosen to permit radiation for $s = 1$.

Discussion

(a) *Intensity and Frequency*

In applying the theory of cyclotron radiation in a plasma to electrons in the terrestrial exosphere we shall consider only electron energies and trajectories which are appropriate to the production of observable v.l.f. hooks. Since the intensity of hooks has been measured it is then possible to estimate the number of emitting electrons using equation (3) and hence to calculate the intensity of the concurrent forward radiation.

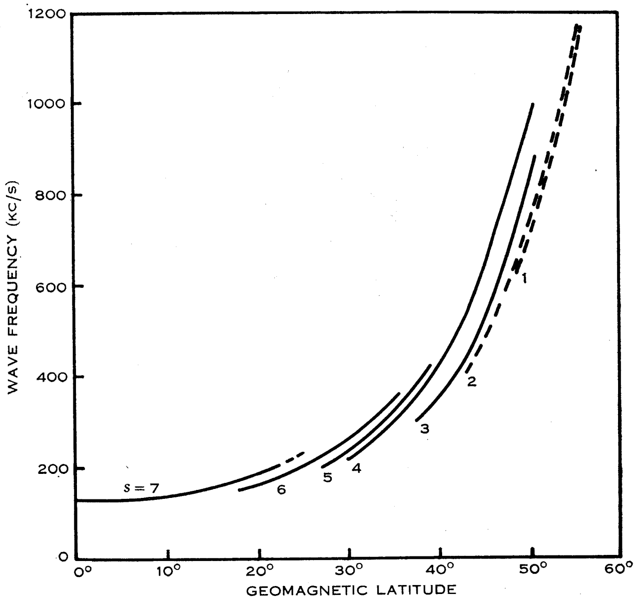

Fig. 6.—Variation of wave frequency with geomagnetic latitude produced by electron bunch travelling along the 60° field line. Solid curves, electron energy 110 keV; dashed curves, electron energy 250 keV.

We assume a bunch of electrons of energy 110 keV travelling along the geomagnetic field line ending at latitude $\lambda = 60°$, with a pitch angle of 10° in the equatorial plane. The scale frequency of the plasma $F = f_0^2/f_H$ is assumed to be 500 kc/s.

Figure 6 shows the variation in the frequency of the forward radiation which would be emitted by the bunch as it travels down the 60° field line. The frequency at each point is that for which the radiated power $W(\alpha)$ after refraction is a maximum. We note that the frequencies of each of the different harmonics at any instant is almost the same but that the radiation direction α_m is different for each. That is, the electrons emit harmonic radiation mainly along the surface of cones of angular radius α_m where $\alpha_m = \alpha_m(s)$ but where the frequency f_s is not a strong function of s for $\alpha = \alpha_m$. This behaviour contrasts with radiation in a vacuum where the harmonic frequencies are integrally related. Figure 7 shows the variation of α_m with wave frequency for the different harmonics. With the assumed model and considering only the fourth and lower harmonics the frequency range of the radiation is 220–1000 kc/s.

Since we have assumed that the plasma density $N \propto H$, the plasma frequency $f_0 \propto H^{\frac{1}{2}}$ and, as the distance along the field line from the equatorial plane is increased, the ratio f_0/f_{H} decreases and $f_{\mathrm{x}} \to f_{\mathrm{H}}$. The harmonic number for which solutions of equations (2) and (4) can be obtained consequently decreases as the electrons travel downwards. With the model chosen earlier, that is, 110 keV electrons and $F = 500$ kc/s, at no point before reflection of the electrons does the harmonic number become less than 3 and the electrons can nowhere radiate ahead in the fundamental or second harmonic modes.

The flux density of the v.l.f. radiation is observed to be of the order of 10^{-13} W m^{-2} (c/s)$^{-1}$ at 5 kc/s (Dowden 1962*c*) and the bandwidth of hooks is about 100 c/s.

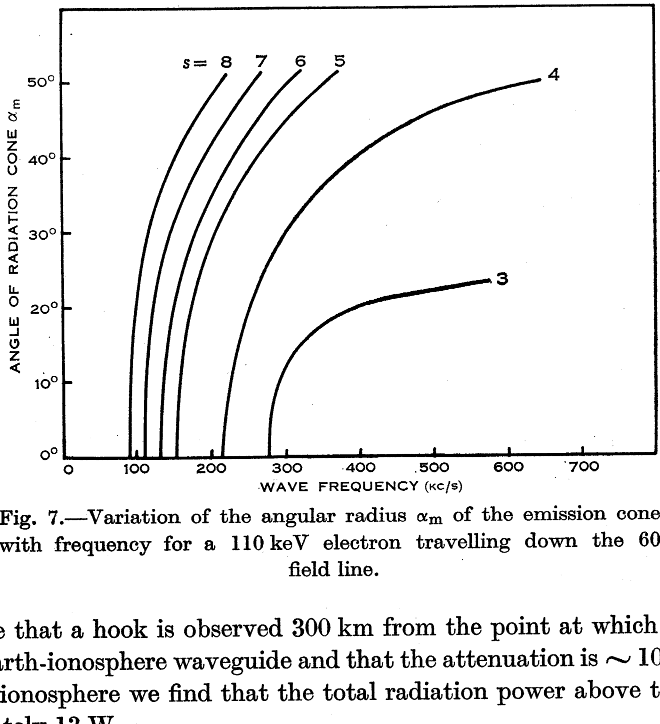

Fig. 7.—Variation of the angular radius α_{m} of the emission cones with frequency for a 110 keV electron travelling down the 60° field line.

If we assume that a hook is observed 300 km from the point at which the radiation enters the Earth-ionosphere waveguide and that the attenuation is $\sim$ 10 dB in passing through the ionosphere we find that the total radiation power above the ionosphere is approximately 13 W.

Now for propagation in the v.l.f. mode, the wave packets may be guided down the field lines almost independently of the initial direction of the wave normal, as is the case for whistlers (Smith 1960). Hence we have, assuming coherent emission and with the total power P consequently proportional to the square of the number M of radiating electrons,

$$\iint M^2 W(\theta)\sin\theta \, \mathrm{d}\theta \, \mathrm{d}\phi = P.$$

Using the example illustrated in Figures 1 and 5 we find that when the latitude of the electron bunch is 30° the number of electrons $M \sim 2\times10^{14}$.

We may compute the power radiated P_s from 2×10^{14} electrons using Figure 5. We have

$$P_s = \iint M^2 W_s(\alpha)\sin\alpha \, \mathrm{d}\alpha \, \mathrm{d}\phi,$$

or $P_4 \simeq 0 \cdot 03$ W at 220 kc/s at latitude 30° (fourth harmonic). When the electron bunch reaches $\lambda = 40°$, radiation in the third harmonic is possible and we obtain $P_3 \sim 0 \cdot 34$ W at 340 kc/s. The power decreases rapidly with harmonic number for the higher harmonics. The flux density at a distance from the bunch may be obtained as follows. Since the wave frequency changes rapidly with time as the electron travels down the field line, the minimum bandwidth will be given by

$$\begin{aligned} \delta f &= \left(\frac{\mathrm{d}f}{\mathrm{d}t}\right)^{\frac{1}{2}} \\ &= 1 \cdot 5 \text{ kc/s at } \lambda = 40°. \end{aligned}$$

Taking the major part of the radiation to be confined to a cone with an angular thickness $\delta\alpha$ of about 1° we have for the flux density S at a distance of 10^5 km from the source

$$\begin{aligned} S &= P(2\pi \sin \alpha \, \mathrm{d}\alpha \delta f \, . \, R^2)^{-1} \\ &\sim 2 \times 10^{-19} \text{ W m}^{-2} \text{ (c/s)}^{-1} \text{ at 340 kc/s} \qquad (s = 3). \end{aligned}$$

There is no information on the background flux density of the galactic radiation at less than 1 Mc/s, but it may well be of this order. It seems likely that the external cyclotron radiation would be of observable intensity at smaller distances.

If we assume that the electron energy is higher, solutions to the radiation equations can be obtained in the fundamental mode. If the electrons travel along the $\lambda = 60°$ field line as before but their energy is 250 keV and the plasma scale frequency is taken to be 180 kc/s, then radiation is emitted with $s = 1$ at latitudes between 50° and 55° (Fig. 6). The angular power distribution is shown in Figure 5.

The power in this mode is very much higher than that calculated above for $s = 3, 4$. If we take the same number of electrons as before, radiating coherently, we get a total power of 6×10^2 W compared with $0 \cdot 34$ W for $s = 3$. Since this exceeds the kinetic energy of the electrons, it is obvious that either the radiation process does not proceed very far or that the radiation is not entirely coherent.

(b) *Other Properties*

Figure 7 shows that the emission cone angle α_{m} is generally less than 50° for $s = 4$ and less than 25° for $s = 3$ with the assumed model. The radiation will be observable most strongly from positions which lie on the emission cones and hence from a relatively limited region of space. The ray trajectories for the third harmonic with the electron bunch at a latitude of 40° are shown in Figure 8. The ray proceeds ahead of the electron bunch until reflected near the extraordinary mode reflection level where $f = f_{\mathrm{X}}$. Providing the observer is within about 25° of the field line direction he would observe a short pulse at a single frequency as the changing radiation cone swept through his position. However, a stationary observer would be unlikely to be observing on the correct frequency for which a possible cone would intersect him. Only if wide-band observations were made or if the relative position of the observer varied and the electron bunches occurred in different longitudes simultaneously, is it likely that the radiation would be detected from isolated bunches.

On the other hand, the continuous hiss emissions sometimes appear to be made up of many hooks occurring in rapid succession. They are also known to extend over considerable ranges of latitude and longitude (Ellis 1961) during the one event, and the angular distribution of the external radiation would be expected to be spread by a corresponding amount. The need for an observer to be in a particular direction for observations at a given frequency would not then arise and it is therefore much more likely that the external radiation would be observable during the occurrence of hiss or v.l.f. noise storms. Normally these occur during geomagnetic storms or concurrently with magnetic bays (Ellis 1959, 1960). Simple methods are available for recording the v.l.f. emissions and hence for obtaining information on the times of the external radiation (Ellis 1963*b*).

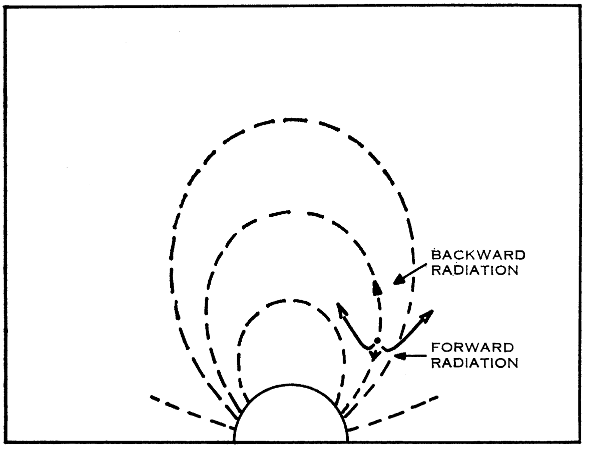

Fig. 8.—Ray paths of the backward and forward radiation.

The wave frequency for which the radiation is most intense is normally slightly greater than the X-mode cut-off frequency f_X at the point of emission. In most cases the value of f_X provides a useful indication of the minimum frequency for which observations will be possible at any given height in the exosphere.

References

Bekefi, G., Hirshfield, J. L., and Brown, S. C. (1961).—*Phys. Rev.* **122**: 1037.
Dowden, R. L. (1962*a*).—*J. Geophys. Res.* **67**: 1745.
Dowden, R. L. (1962*b*).—*Aust. J. Phys.* **15**: 490.
Dowden, R. L. (1962*c*).—*Aust. J. Phys.* **15**: 114.
Dowden, R. L. (1963).—*Planet. Space Sci.* **11**: 361.
Eidman, V. Ia. (1958).—*J. Exp. Theor. Phys. U.S.S.R.* **7**: 91.
Eidman, V. Ia. (1959).—*J. Exp. Theor. Phys. U.S.S.R.* **9**: 947.
Ellis, G. R. A. (1959).—*Planet. Space Sci.* **1**: 253.
Ellis, G. R. A. (1960).—*J. Geophys. Res.* **65**: 1705.
Ellis, G. R. A. (1961).—*J. Geophys. Res.* **66**: 19.
Ellis, G. R. A. (1962).—*Aust. J. Phys.* **15**: 344.
Ellis, G. R. A. (1963*a*).—*Aust. J. Phys.* **16**: 380.
Ellis, G. R. A. (1963*b*).—*Proc. Inst. Radio Engrs. Aust.* **24**: 204.
Ginsburg, V. L., and Zelezniakov, V. V. (1962).—*Sov. Astron.* **2**: 653.

Hansen, S. F. (1963).—*J. Geophys. Res.* **68**: 5925.
Helliwell, R. A. (1963).—*J. Geophys. Res.* (in press.)
Smith, R. L. (1960).—Tech. Report No. 6. Stanford University, Radioscience Laboratory.
Twiss, R. Q. (1958).—*Aust. J. Phys.* **11**: 564.
Wild, J. P., Smerd, S. F., and Weiss, A. A. (1963).—*Annu. Rev. Astr. Astrophys.* **1**: 291.