DIFFUSION OF METEOR TRAILS IN THE EARTH'S MAGNETIC FIELD

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Summary

The diffusion equation is solved for isotropic diffusion in the absence of external forces. The asymmetry due to an external magnetic field is introduced and solutions of the diffusion equation are given for the two cases of magnetic field parallel and perpendicular to the diffusing column.

Scattering formulae for an underdense meteor trail are derived and the effect of the Earth's magnetic field is shown to be large were it not for the early onset of ambipolar diffusion, which is shown to be the controlling factor in the decay of meteor echoes. However, a factor of two is shown to exist between the cases of parallel and perpendicular magnetic field.

Finally, the effect of wind currents in the meteor region is briefly discussed.

I. Introduction

The theory of radio reflections from meteor trails is now well known. The radar equation for a column of ionized air in the atmosphere was derived by Lovell and Clegg (1948) on the assumption that the column was narrow compared to the wavelength used and that the incident wave could penetrate the column without significant modification. A very comprehensive treatment has been given by Kaiser (1953) for both low density ($< 10^{14}$/m) and high density trails and mention is made here of the possible effects of the Earth's magnetic field which will modify the shape of the expanding ionized column.

An alternative treatment by Huxley (1952) treats the reflections as occurring not throughout the column but at the surface where the electron density is critical for the radio wave. In this paper the theory of ambipolar diffusion is applied to the spreading of a meteor trail.

Weiss (1955), in discussing the measurement of diffusion coefficients from the rate of decay of meteor echoes, suggests that the Earth's magnetic field can account for some of the scatter in the experimental results obtained and he shows by physical arguments that the reflected signal can be reduced by a factor of two when account is taken of the magnetic field.

In what follows, the diffusion equation for meteor particles is solved formally by Green's method for the case of isotropic diffusion. Account is then taken of the Earth's field by introducing a "magnetic diffusion coefficient" and the resulting

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equation is solved by the same method. The two cases of the meteor trail parallel and perpendicular to the magnetic field are discussed and these two solutions are used to find the reflected signal from a ground-based radio transmitter.

II. The Diffusion Equation

A meteor in passing through the atmosphere will leave in its wake a long column of ionized air. Since the majority of meteors are physically small in relation to the mean free path of the atmospheric molecules, it is reasonable to assume that the initial concentration of ionized particles will lie along a line. The volume density of the ionized particles in the trail subsequently decreases as the trail expands radially owing to diffusion of the particles from the region of higher concentration.

When no external forces act, diffusion is isotropic and the diffusion equation for the particles—electrons and ions—can be written

$$\partial n/\partial t = D \nabla^2 n, \tag{1}$$

where n = number density and D = diffusion coefficient. Equation (1) may be solved by regarding the meteor trail initially as an accumulation of point sources. For such a source located at the point (x_0, y_0, z_0) at $t = t_0$ the Green's function for (1), $g(x, y, z, t;\ x_0, y_0, z_0, t_0)$, satisfies

$$-\partial g/\partial t + D \nabla^2 g = -\delta(x-x_0)\delta(y-y_0)\delta(z-z_0)\delta(t-t_0).$$

By writing

$$g = (2\pi)^{-3/2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{d}\alpha \mathrm{d}\beta \mathrm{d}\gamma \{\exp -[\alpha(x-x_0)+\beta(y-y_0)+\gamma(z-z_0)]\}$$

$$\times F(\alpha, \beta, \gamma, t),$$

then

$$\mathrm{d}F/\mathrm{d}t + D(\alpha^2+\beta^2+\gamma^2)F = (2\pi)^{-3/2}\delta(t-t_0),$$

whence

$$F = (2\pi)^{-3/2} \exp -\mathrm{t}D(\alpha^2+\beta^2+\gamma^2), \qquad \text{for } t > t_0,$$

and

$$g = (2\pi)^{-3} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{d}\alpha \mathrm{d}\beta \mathrm{d}\gamma \exp\{\mathrm{i}[\alpha(x-x_0)+\beta(y-y_0)+\gamma(z-z_0)]$$

$$-tD(\alpha^2+\beta^2+\gamma^2)\}$$

$$= \frac{1}{8(\pi t D)^{3/2}} \exp -[(x-x_0)^2+(y-y_0)^2+(z-z_0)^2]/4tD.$$

Therefore

$$n = \frac{N_0}{8(\pi t D)^{3/2}} \exp -[(x-x_0)^2+(y-y_0)^2+(z-z_0)^2]/4tD,$$

where N_0 is the initial number density.

For an initial line source of infinite extent parallel to the z_0 axis

$$n = \frac{N_0}{4\pi tD} \exp -[(x-x_0)^2+(y-y_0)^2]/4tD. \tag{2}$$

III. Diffusion of Charged Particles in a Magnetic Field

A charged particle moving with velocity $\mathbf{V}$ in a magnetic field $\mathbf{B}$ will experience a force $(\mathbf{V}\times\mathbf{B})$. The effect of this will be to reduce the rate of diffusion normal to the magnetic field. The reduction can be described by a "magnetic" coefficient of diffusion M so that for $\mathbf{B}$ parallel to z there is a flow of particles in the x-direction given by

$$-D\,\partial n/\partial x+M\,\partial n/\partial x-M\,\partial n/\partial y.$$

In the y-direction the flow is given by

$$-D\,\partial n/\partial y+M\,\partial n/\partial y-M\,\partial n/\partial x.$$

The flow in the z-direction is not modified by the magnetic field so that the diffusion equation becomes

$$\partial n/\partial t = (D-M)\,\partial^2 n/\partial x^2+(D-M)\,\partial^2 n/\partial y^2+D\,\partial^2 n/\partial z^2. \tag{3}$$

By writing $\epsilon = \{D/(D-M)\}^{\frac{1}{2}}\,x$, $\eta = \{D/(D-M)\}^{\frac{1}{2}}y$, equation (3) becomes

$$\partial n/\partial t = D\,\partial^2 n/\partial \epsilon^2+D\,\partial^2 n/\partial \eta^2+D\,\partial^2 n/\partial z^2,$$

i.e. the equation is transformed to principal axes. The method of Section II may now be used to obtain the solution

$$\begin{aligned} n &= \frac{N_0}{4\pi tD} \exp -(\epsilon^2+\eta^2)/4tD \\ &= \frac{N_0}{4\pi t\,(D-M)} \exp -(x^2+y^2)/4t(D-M). \end{aligned} \tag{4}$$

For $\mathbf{B}$ parallel to y the diffusion equation is

$$\partial n/\partial t = (D-M)\partial^2 n/\partial x^2+D\,\partial^2 n/\partial y^2+(D-M)\partial^2 n/\partial z^2,$$

and in the same way as for (3) the solution for a line source parallel to z is given by

$$n = \frac{N_0}{4\pi tD^{\frac{1}{2}}\,(D-M)^{\frac{1}{2}}} \exp -\left[\frac{x^2}{4t(D-M)}+\frac{y^2}{4tD}\right]. \tag{5}$$

IV. Scattering of Electromagnetic Waves by a Meteor Trail

The scattered field strength at range R from a single electron moving in the x direction with acceleration $\ddot{x}$ is, in MKS units, $\mu_0 e\ddot{x}\sin\alpha/4\pi R$ where μ_0 is the permeability of free space and α is the angle between the z-direction and the direction to the field point. The acceleration of a free electron due to an incident electromagnetic wave is $E_{\text{inc}}\,e/m$ (omitting the exponential time variation).

Considering only backscattering so that $\sin \alpha = 1$, then

$$\frac{E_{\mathrm{sc}}}{E_{\mathrm{inc}}} = \frac{-\mu_0 e^2}{4m} \frac{1}{\pi R}.$$

For a cloud of electrons with number density n in a region v,

$$\frac{E_{\mathrm{sc}}}{E_{\mathrm{inc}}} = -\int_v \mathrm{d}v \, \frac{\mu_0 e^2 n}{4\pi m R}$$

$$= \frac{-\mu_0 e^2}{4m} \frac{1}{\pi R_1} \int_v \mathrm{d}v \, n \exp 2\mathrm{i}KR_2, \tag{6}$$

where K is the wave number $= 2\pi/\lambda$, R_1 is the range to a reference point in the cloud, and R_2 is the range to each electron. (See Fig. 1.)

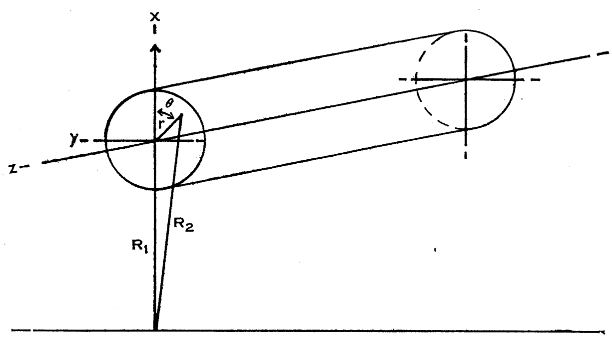

Fig. 1.—The geometry of a meteor trail.

By inserting the expressions for n given by (4) and (5) into (6) and carrying out the integrations there results for **B** parallel to z, i.e. the magnetic field parallel to the meteor longitudinal axis,

$$\frac{E_{\mathrm{sc}}}{E_{\mathrm{inc}}} = \frac{-\mu_0 e^2 N_0 \lambda^{\frac{1}{2}}}{8m\pi R_1^{\frac{1}{2}}} \exp -[2t(D-M)K^2], \tag{7}$$

and for the magnetic field perpendicular to the meteor trail,

$$\frac{E_{\mathrm{sc}}}{E_{\mathrm{inc}}} = -\frac{\mu_0 e^2 N_0 \lambda^{\frac{1}{2}} (D-M)^{\frac{1}{2}}}{8m\pi R_1^{\frac{1}{2}} D^{\frac{1}{2}}} \exp -[2t(D-M)K^2]. \tag{8}$$

In obtaining the results (7) and (8) it is assumed that the meteor trail is infinitely long and for (8) that D is large compared with M. This last assumption is only approximately true and prevails under conditions of ambipolar diffusion discussed in Section VI.

The scattered signal power

$$P_{sc} = P_{inc} \times \left(\frac{E_{sc}}{E_{inc}}\right)^2$$

$$= \frac{P_t\, G}{4\pi R_1^2} \times \left(\frac{E_{sc}}{E_{inc}}\right)^2,$$

where P_t = transmitted signal power and G = transmitting antenna gain. The power available at the receiver terminal is given by $P_R = P_{sc} \times$(capture area of receiving antenna).

That is,

$$P_R = P_{sc} \times G\lambda^2/4\pi.$$

Hence,

$$P_R = \frac{P_t\, g^2\lambda^2}{16\,\pi^2 R_1^2} \times \left(\frac{E_{sc}}{E_{inc}}\right)^2. \tag{9}$$

Using (7) and (8) this gives for the meteor trail parallel to **B**

$$P_R = \left[\frac{\mu_0 e^2}{4m}\right]^2 \frac{N_0^2\, P_t\, G^2\lambda^3}{64\pi^4 R_1^3} \exp -\left[\frac{16\pi^2 t(D-M)}{\lambda^2}\right], \tag{10}$$

and for the meteor trail perpendicular to **B**

$$P_R = \left[\frac{\mu_0 e^2}{4m}\right]^2 \frac{N_0^2\, P_t\, G^2\lambda^3(D-M)}{64\pi^4 R_1^3\, D} \exp -\left[\frac{16\pi^2 t(D-M)}{\lambda^2}\right]. \tag{11}$$

By setting $M = 0$ in (10) and (11) the result is identical to that given by Kaiser (1953) in which the Earth's magnetic field is neglected.

V. The "Magnetic" Diffusion Coefficient

A charged particle in motion parallel to a uniform magnetic field will be unaffected by the field, while a charged particle moving normal to the magnetic field will tend to spiral around the field lines. It is then apparent that electrons moving with the velocity of diffusion under the action of a concentration gradient will tend to spread out more slowly in directions normal to the magnetic field than along the field.

This effect is discussed by Chapman and Cowling (1952) where it is shown that the velocity of diffusion normal to the magnetic field is reduced in the ratio $1 : (1+\omega^2/\nu^2)$. Here ω = gyromagnetic frequency and ν = collisional frequency. The diffusion coefficient can then be said to be reduced in this ratio so that

$$D-M = D\nu^2/(\nu^2+\omega^2). \tag{12}$$

ω is given by eB/m with e and m the particle charge and mass respectively and B the magnetic field strength. The magnitude of the Earth's magnetic field is taken to be $0{\cdot}6\times10^{-4}$ Wb/m^2 at ground level. The field strength reduces with height by about 6 mG per 40 km but is slightly modified by atmospheric charges so that at an altitude of 95 km, the magnitude is $0{\cdot}586\times10^{-4}$ Wb/m^2. This height is typical for meteors.

Taking $e = 1{\cdot}6\times10^{-19}$ C and $m = 9{\cdot}1\times10^{-31}$ kg, ω for electrons $= 10^7$ c/s.

The collisional frequency for electrons in the atmosphere is difficult to measure directly. Deductions have been made from measurements made in pure nitrogen at low electron mean energies and an empirical formula, quoted by Crompton, Huxley, and Sutton (1953), has been derived. A later result by Dr. Crompton (personal communication) is used here. This is $\nu=1\cdot2\times10^{8}p$ where p is the atmospheric pressure in millimetres of mercury. $p=1\cdot6\times10^{-4}$ at 100 km, $0\cdot9\times10^{-3}$ at 95 km, and $1\cdot0\times10^{-2}$ at 80 km. Hence, $\nu=2\cdot0\times10^{4}$, $1\cdot0\times10^{5}$, and $1\cdot2\times10^{6}$ s^{-1} at these heights respectively.

Using (12)

$$\begin{aligned} D-\mathrm{M} &= D\times4\cdot0\times10^{-6} \text{ at } 100 \text{ km} \\ &= D\times1\cdot0\times10^{-4} \text{ at } 95 \text{ km} \\ &= D\times1\cdot4\times10^{-2} \text{ at } 80 \text{ km.} \end{aligned}$$

Thus electron diffusion is strongly influenced by the Earth's magnetic field right through the meteor region. The explanation is found in consideration of the diffusion process. In the absence of external forces, particles moving with initial thermal velocities will tend, after encounters with other particles, to move from regions of high concentration to regions of lower concentration and travel in straight lines between collisions. When a magnetic field is present, the charged particles tend to spiral the field lines and in the region considered are describing about 100 orbits between collisions. The effect of a collision will be to disturb the orbit slightly; the tendency being to increase the radius of the orbit, again following the concentration gradient. That is, the rate of diffusion normal to the magnetic field is greatly reduced compared to diffusion parallel to the field.

This means that the meteor trail could attain a high degree of ellipticity and the effect on the radio signal power reflected would be great. By choosing the value of ν corresponding to 95 km altitude so that $D-M=D\times10^{-4}$, equation (11) shows that P_{R} is reduced by a factor of 10^4 when the meteor is travelling normal to the Earth's field compared with the parallel case. This variation in the reflected signal is much larger than that indicated by practical results. The positive ions, although ineffective as scatterers of the radio waves, are likely to exert considerable influence on the motion of the electrons and so modify these results.

VI. Ambipolar Diffusion

The diffusion coefficient for electrons in the atmosphere at 95 km is about 10^4 m^2/s, while that for ions is about 10 m^2/s, i.e. 1000 times smaller than for electrons. Immediately after formation a meteor trail will start to spread out with electrons moving out more rapidly than ions. The separation of the charges will produce an electrostatic field tending to oppose the separation. This problem has been discussed by Francey (1963) who showed that, for the duration of a meteor radio echo, conditions of ambipolar diffusion prevail such that electrons and ions diffuse at the same rate giving rise to a coefficient of ambipolar diffusion D_{A}.

Kaiser (1953) has shown that in a meteor trail, when the Earth's magnetic field is neglected, $D_{\mathrm{A}}=2D_{\mathrm{i}}$ where D_{i} is the coefficient of diffusion for ions alone.

In directions normal to the magnetic field, as shown in Section V, the diffusion coefficient for electrons is reduced and at a height of 95 km $D_{\mathrm{e}} \ll D_{\mathrm{i}}$ if the ions are assumed not to be effected by the magnetic field. It can be concluded that in this case $D_{\mathrm{A}} = D_{\mathrm{i}}$. Hence

$$D-M = D_{\mathrm{i}},$$

$$D = 2D_{\mathrm{i}}.$$

The results of Section IV now show that the Earth's magnetic field has no effect on the reflected signal power when the meteor trail is parallel to the field direction. When the trail is normal to the field there is a reduction in the reflected power by a factor of two. This reduction should be observable and it is expected that experimental data being analysed at Adelaide may give some confirmation of this.

VII. The Effect of Winds

When there are wind currents in the meteor region it is to be expected that the reflected signals from a meteor trail will be modified. When the wind vector **W** is parallel to the magnetic field acting in the y-direction, the diffusion equation is

$$\partial n/\partial t = (D-M)\,\partial^2 n/\partial x^2 + D\,\partial^2 n/\partial y^2 - \mathrm{W}\,\partial n/\partial y + (D-M)\,\partial^2 n/\partial z^2, \tag{13}$$

and the solution for a line source along z is

$$n = \frac{N_0}{4\pi t\,(D-M)^{\frac{1}{2}}\,D^{\frac{1}{2}}} \exp -\left[\frac{x^2}{4t(D-M)} + \frac{(y-Wt)^2}{4tD}\right],$$

$$P_{\mathrm{R}} = \left[\frac{\mu_0 e^2}{4m}\right]^2 \frac{N_0^2\,P_{\mathrm{t}}\,G^2\lambda^3(D-M)}{64\pi^4 R_1^3\,D} \exp\left[\frac{tW^2 - 64\pi^2 t(D-M)^2}{4(D-M)\,\lambda^2}\right]. \tag{14}$$

In obtaining (14) a number of approximations were made, with the result that it is not possible to distinguish the case of **W** perpendicular to **B** from the above. In any case with winds of the order of 100 m/s this gives only a minor correction to the formula (11).

VIII. Discussion

The method using Green's functions is very powerful for obtaining solutions of the diffusion equation and may be extended to include effects other than the Earth's magnetic field. The integrals arising in the scattering formulae are difficult to evaluate, and it has been possible, in this paper, to obtain results only after considerable approximation. However, the results so obtained confirm those of earlier workers.

It is evident that ambipolar diffusion is the dominant factor in controlling the expansion of meteor trails and hence the duration of a radio echo. The effect of the Earth's magnetic field, which could be large under conditions of differential diffusion, is reduced to a factor which should still be observable.

IX. Acknowledgment

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