AXIAL RATIO OF THE PROJECTED POLARIZATION ELLIPSE*†

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The axial ratio of the polarization ellipse of a radio wave is often evaluated using a crossed-dipole antenna system placed in a horizontal plane. The axial ratio of the ellipse as deduced from the responses of the two antennas should correspond to the axial ratio of the ellipse in the wave-front as projected onto the horizontal plane. When one of the two dipoles is situated in the plane of incidence of the radio wave, the axial ratio of the projected polarization ellipse is a function of the angle of incidence and the orientation of the polarization ellipse in the wave-front. A convenient parameter with which to relate axial ratio and orientation is the angle made by the major axis with the line of intersection of the wave-front with the horizontal plane. This line would actually correspond to the direction of one of the dipoles when the other is situated in the incident plane.

In his paper "Polarization measurements of Jupiter radio bursts at 10·1 Mc/s", Dowden (1963§) investigated the extent of variation of the axial ratio of the projected ellipse with the angle of incidence and with orientation. In the equations presented by him for evaluating the projected axial ratio (Dowden p. 401), there appears to be an omission; the curves shown in his Figure 3 could not be reproduced by using the equations he gives. The purpose of this communication is to present a set of equations from which these curves can be reproduced. A set of curves, drawn by using equations (1)–(4) as presented below, is shown in Figure 1.

Dowden has defined θ in his equations as the "angle between the major axis of the ellipse and the *horizontal plane* (or the inclination of the minor axis)". This interpretation for θ cannot be correct. When the wave is incident at an angle i, the angle made by the major axis in the wave-front would vary from 0° to i and not from 0° to 90° as shown in his Figure 3. Since θ is varied from 0° to 90° it is to be interpreted as the angle between the major axis and the *horizontal direction in the wave-front*, which in turn coincides with the direction of one of the two antennas, in the particular case when the other antenna is in the plane of incidence. In this special case, the projected axial ratio can conveniently be evaluated by resolving the ellipse into linear components parallel to the directions of the two antennas. For example, when $\theta = 0°$ the major axis (of magnitude a) lies along one antenna and the minor axis (of magnitude b) makes an angle i with the other antenna, its component being $b \cos i$. Since the phase difference between the two components is 90°, they give rise to an ellipse of axial ratio $(b \cos i)/a$. This coincides with the starting point for the curves in Dowden's Figure 3, which are the same as the curves shown in the

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§ DOWDEN, R. L. (1963).—*Aust. J. Phys.* **16**: 398–410.

present paper in Figure 1. When $\theta = 90°$, the minor axis lies along one antenna and the major axis makes an angle i with the other antenna, giving rise to a component of $(a \cos i)$ along that antenna. The axial ratio therefore becomes $b/(a \cos i)$, which is the end point for the curves. Dowden's equation does not reduce to these limiting values when $\theta = 0°$ and $\theta = 90°$ are substituted. When the major axis takes any other orientation, it is necessary to find the amplitudes of the two components along the antennas and also their phase difference in order to evaluate the projected axial ratio. The equations below supply all the information regarding the projected ellipse.

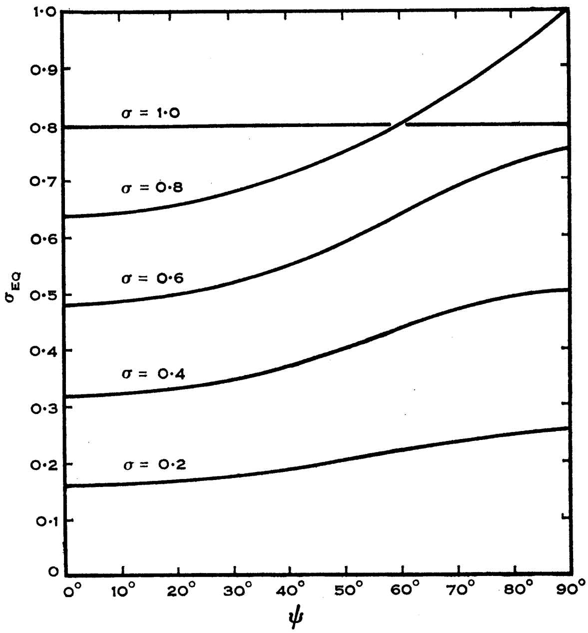

Fig. 1.—Equivalent projected axial ratio σ_{EQ} as a function of the inclination of the major axis ψ for various actual axial ratios σ and for an angle of incidence of 37°, when one of the antennas is in the incident plane.

If σ_{EQ} is the equivalent projected axial ratio, then

$$\sigma_{\mathrm{EQ}} = \left(\frac{\rho^2 \tan^2 \theta' - \cos^2 i}{\tan^2 \theta' - \rho^2}\right)^{\frac{1}{2}}, \tag{1}$$

where

$$\rho = \left(\frac{\sigma^2 \tan^2 \theta + 1}{\tan^2 \theta + \sigma^2}\right)^{\frac{1}{2}}, \tag{2}$$

$$\theta' = \tfrac{1}{2} \arctan\left(\frac{2\rho \cos \phi \cos i}{\rho^2 - \cos^2 i}\right), \tag{3}$$

$$\phi = \arctan\left(\frac{2\sigma}{1-\sigma^2} \operatorname{cosec} 2\theta\right). \tag{4}$$

In these equations ρ corresponds to the ratio of the amplitudes of the components along the aerials and ϕ to their phase difference when the actual ratio of axes is σ. The angle θ' corresponds to the inclination of the major axis of the projected ellipse with the horizontal direction in the wave-front. It should be noted that this angle is not the same as the angle made by the major axis when projected onto the horizontal plane. In polarization studies the phrase "projected polarization ellipse" is often used; however, it should be remembered that the polarization ellipse in the wave-front does not give rise to a geometrical ellipse of the same specifications as the evaluated projected ellipse when projected onto the plane of the antennas. By projected polarization ellipse is meant a polarization ellipse in the plane of the antennas that would produce the same response in the two antennas as the polarization ellipse in the wave-front. For this reason it is appropriate to designate it the "equivalent axial ratio" of the projected ellipse.

In the above discussion it is assumed that one of the antennas is situated in the plane of incidence. In polarization studies using radio waves from astronomical bodies, the plane of incidence changes with time relative to the orientation of the fixed, crossed-dipole antenna system, and the above equations cannot be applied in such situations.

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Corrigenda

Volume 17, Number 3, Pages 389–403

"Average radiation-pressure forces produced by sound fields"

Page 397, *Equations (56) should read:*

$$\left.\begin{aligned}\mathbf{u}_r' &= -\nabla\phi_r^*,\\ p_r' &= -\mathrm{i}\omega\rho_0\phi_r^*,\end{aligned}\right\} \tag{56}$$

Page 397, *After equation (57) insert:*

Further suppose each of the ϕ_r to be of constant phase over S, in which case it is permissible to remove the complex conjugates of equations (56) when describing the fields on S.