THE RESPONSE OF CLOSED CHANNELS TO WIND STRESSES

By L. M. FITZGERALD*† and W. W. MANSFIELD*

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Summary

The surface velocity, surface slope, and velocity profile produced by the application of a wind stress to the smooth surface of a closed channel have been determined by adapting the empirical laws of flow in smooth tubes. The estimated responses agree well with the available experimental data.

It is shown that the response of a channel with a rough water surface may usefully be predicted by separating the wind stress into two parts, one producing liquid flow and the other, surface waves. This formal division was suggested by Keulegan (1951).

I. INTRODUCTION

A wind blows along a closed channel of a liquid and exerts a surface stress τ_s (dyn/cm^2). There results a slope $\mathrm{d}h/\mathrm{d}x$ of the water surface at a distance x (cm) down-wind, a flow u (cm/s) at a height z (cm) above the bed of the channel, and a shearing stress τ_0 (dyn/cm^2) at the bed. From Keulegan (1951), steady conditions are obtained when

$$\rho\frac{\mathrm{d}}{\mathrm{d}x}\int_0^{H+h} u^2\,\mathrm{d}z+\rho g(H+h)\frac{\mathrm{d}h}{\mathrm{d}x} = \tau_s-\tau_0, \tag{1}$$

where ρ (g/cm^3) is the liquid density, g (cm/s^2) is the gravitational acceleration, and H (cm) is the undisturbed depth of the liquid. Relation (1) assumes that any change of air pressure along the channel is negligible or incorporated as a correction to the surface slope. It is applicable to either turbulent or laminar flow; when the former prevails appropriate mean values of u, h, and τ are intended.

Normally, excepting regions close to the windward and leeward boundaries, the acceleration term within (1) may be neglected. Thus over the major portion of the channel, for the additional condition of $h \ll H$,

$$\mathrm{d}h/\mathrm{d}x = (\tau_s-\tau_0)/\rho g H. \tag{2}$$

A further requirement for steady conditions is that

$$\int_0^H u\,\mathrm{d}z = 0. \tag{3}$$

* Division of Physical Chemistry, CSIRO, Chemical Research Laboratories, Melbourne.

† Present address: Explosives Research and Development Establishment, Waltham Abbey, Essex, England.

The response of the channel to the wind stress has been determined by Keulegan (1951) for the condition that flow everywhere is controlled by molecular viscosity. The velocity profile reads

$$u/u_s = 3(z/H)^2 - 2(z/H), \tag{4}$$

to the level of the approximations of (2) and (3). The surface velocity is

$$u_s = \tau_s H/4\eta, \tag{5}$$

η (poises) being the liquid viscosity, and the surface slope is

$$\mathrm{d}h/\mathrm{d}x = 3\tau_s/2\rho g H, \tag{6}$$

since the induced stress at the bed is numerically one-half the surface stress.

Keulegan (1951) showed experimentally that laminar flow prevails within a closed channel provided the Reynolds number $R = u_s H/\nu$, ν (cm²/s) being the dynamic viscosity of the liquid, is less than about 600. For all normal water storages in the open air, this limiting Reynolds number is usually exceeded greatly, so that flow within the storage is turbulent. For turbulent flow it is common to assume that $-\tau_0 \ll \tau_s$, giving

$$\mathrm{d}h/\mathrm{d}x \doteqdot \tau_s/\rho g H, \tag{7}$$

a relation that has been used to estimate wind stresses over water surfaces; a recent review of these measurements is given by Deacon and Webb (1962). Van Dorn (1953) measured the shearing stress at the bottom of an exposed pond 2 m deep as $|\tau_0| < \frac{1}{10}\tau_s$. Measurements of wind stress by (7) agree reasonably well with values estimated by other methods, such as analysis of the wind velocity profile (Keulegan 1951; van Dorn 1953; Francis 1953; Fitzgerald 1963). Although the neglect of τ_0 obviously is not a serious error, some knowledge of the applicability of (7) is desirable.

The surface velocities induced when wind blows over closed channels or water storages in turbulent flow have been measured by Keulegan (1951), van Dorn (1953), Francis (1953), Vines (1962), and Fitzgerald (1964). All these results may be summarized in the form

$$u_s = \alpha V, \tag{8}$$

where α is a coefficient varying slightly with the Reynolds number, and V (cm/s) is the wind velocity measured at some reference height above the water surface. A feature of these results is that the coefficient is affected negligibly by the roughness of the water surface, which may be diminished at any given wind velocity by the addition of detergents to the water. When the water surface is smooth, either because the wind velocity is sufficiently low or because of the presence of an appropriate surface film, (8) may be replaced by

$$u_s = \theta u_s^*, \tag{9}$$

where θ varies slowly with Reynolds number, and $u_s^* = (\tau_s/\rho)^{\frac{1}{2}}$ is the friction velocity at the water surface. The magnitude and variation of θ is significant in the maintenance of evaporation-retarding surface films on water storages (Mansfield 1959).

In the present paper the reactions to applied wind stresses of closed channels in turbulent flow, but maintaining smooth surfaces, are estimated semi-empirically. All parameters determined are shown to agree well with measurements reported previously or in the present work. The applicability of the results to channels with waves generated at the surface is then discussed.

II. Estimates of Channel Response

The flow across any vertical section of a channel may be divided formally into two sections, the flow within each section being regarded as one-half of the symmetrical flow obtained between two smooth plates separated vertically. Thus, in the lower region of depth z_c the stress increases from τ_0 (negative) at the bed to zero, and the flow velocity decreases from zero at the bed to u_c (negative). In the upper region of depth $(H-z_c)$ the stress declines from τ_s to zero, the bounding wall of the region moves at a velocity u_s, and the velocity declines from u_s to u_c.

Levich (1962) summarizes the observed velocity profiles for turbulent flow over smooth plates as

$$\left.\begin{aligned} u/u^* &= u^*l/\nu, && 0 \leqslant u^*l/\nu \leqslant 5;\\ u/u^* &= 10\tan^{-1} u^*l/10\nu + 1\cdot 2, && 5 \leqslant u^*l/\nu \leqslant 30;\\ u/u^* &= 5\cdot 5 + 2\cdot 5 \ln u^*l/\nu, && u^*l/\nu \geqslant 30, \end{aligned}\right\} \tag{10}$$

where l (cm) is the distance from the plate. The use of (10) to describe flow between plates is unsound basically, since the requirement of zero velocity gradient at the axis is not fulfilled. Goldstein (1938), however, notes that (10) gives good agreement with experimental data on flow in tubes and between plates right up to the axis. Accordingly we assume that (10) is satisfactory for the present purpose after introducing appropriate modifications. Thus for the lower region $u^* = u_0^* = (-\tau_0/\rho)^{\frac{1}{2}}$, and all velocities are reckoned negative. For the upper region $u^* = u_s^* = (\tau_s/\rho)^{\frac{1}{2}}$, and actual flow velocities are obtained by subtracting velocities estimated from (10) from u_s, the surface velocity.

The routine of calculation is straightforward. For a given value of an alternative Reynolds number $R^* = u_s^* H/\nu$, a trial value of

$$k = (-\tau_0/\tau_s)^{\frac{1}{2}}$$

is selected. Since τ varies linearly with z,

$$z_c/H = k^2/(1+k^2),$$

giving the values of u^*l/ν at $z = z_c$ as $k^3R^*/(1+k^2)$ for the lower region and $R^*/(1+k^2)$ for the upper region. From (10), the value of u_s giving equality of flow at $z = z_c$ may then be determined, and this value is introduced to evaluate $\int_0^H u\,dz$. By successive approximation the value of k giving zero total flow is obtained readily. There results a set of corresponding values of R^*, k, and u_s/u_s^*.

In Figure 1 the dependence of u_s/u_s^* on R^* is compared with experimental data. The region C covers the spread of about 110 measurements by Keulegan (1951) and 9 measurements by Fitzgerald (1964); those measurements of Fitzgerald's which are

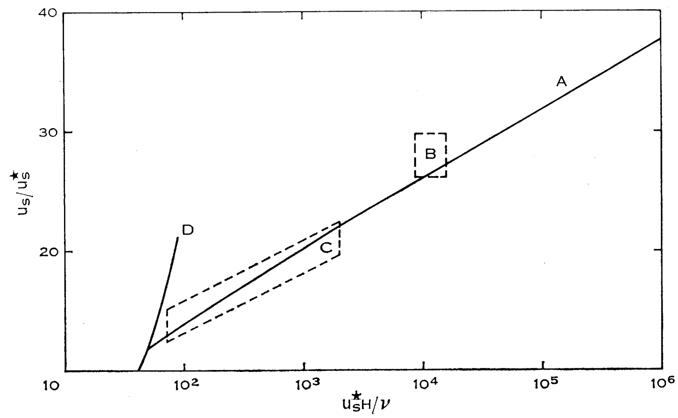

Fig. 1.—The influence of the Reynolds number u_s^*H/ν on surface velocity. A, the estimated relation; B, the experimental results of van Dorn (1953); C, the experimental results of Keulegan (1951) and Fitzgerald (1964); D, the relation for laminar flow.

influenced by surface acceleration (Mansfield, unpublished data) are omitted. The region B spans 5 measurements by van Dorn (1953) on a channel in the open air and 220 m long. For all these experiments smooth surfaces were obtained by adding

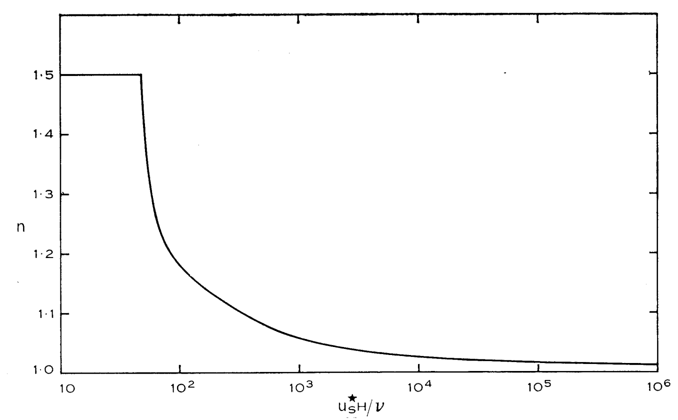

Fig. 2.—The significance of the bed stress as a function of the Reynolds number u_s^*H/ν.

detergent to the water. The estimated relation A accords well with these data, and intersects the relation indicative of laminar flow at $R \sim 580$, in very good agreement with Keulegan's experimental intercept of $R \sim 600$.

Relation (2) may be written as

$$\mathrm{d}h/\mathrm{d}x = n\tau_s/\rho gH, \qquad n = 1+k^2.$$

The estimated relation between n and R^* is given in Figure 2, showing that n differs from unity by less than 5% provided R^* exceeds about 10^3. For water storages about 10 m deep, this condition is satisfied provided the wind velocity at 2 m height is greater than only about 10 cm/s. Clearly the approximation $n = 1$ is entirely acceptable for all normal experiments with smooth surfaces.

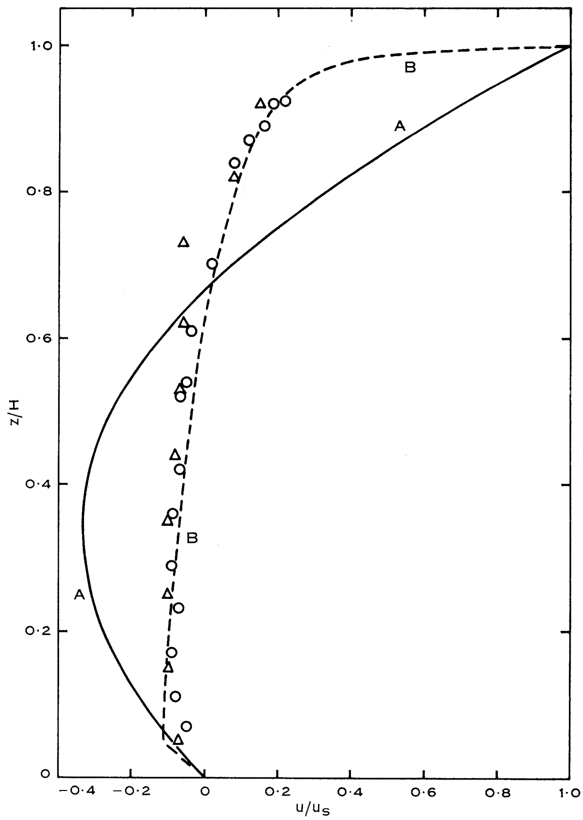

Fig. 3.—The variation of flow velocity through a vertical section of the channel. AA, estimated for $u_s^*H/\nu < 50$; BB, estimated for $u_s^*H/\nu = 10^3$. ○ Experimental values obtained with smooth surface at $u_s^*H/\nu = 1\cdot6\times10^3$. △ Experimental values obtained with rough surface at $u_s^*H/\nu = 1\cdot3\times10^3$.

In turbulent flow, velocities decline beneath the water surface much more rapidly than in laminar flow. Through the necessity of zero total flow, return velocities found towards the bed of the channel are reduced relatively. These trends are illustrated in Figure 3, which compares the estimated profiles for $R^* < 50$ and $R^* = 10^3$. An experimentally determined profile for $R^* = 1\cdot6\times10^3$ is included.

Since at each relative depth the standard deviation of each measurement of u/u_s is quite high at about $0 \cdot 03$, it is allowable to neglect the differences in R^* and note that agreement is satisfactory. Although the differences are not actually significant, the experimental profile does not appear to follow the unnaturally sharp change in $\mathrm{d}u/\mathrm{d}z$ at $z/H \sim 0 \cdot 05$ shown by the estimated profile. This is to be expected, and indicates that any actual value of $|\tau_0|$ should be somewhat less than that estimated. Thus the assumption of $n = 1$ should be even more general.

From this and previous work, both the surface velocity and the velocity profile remain sensibly invariant with x, except near the windward and leeward boundaries. To the accuracy of the experimental data, the acceleration term of (1) may be neglected over most of the channel. Nevertheless, it is desirable to define the conditions for which such neglect is unsound, even in the central region of the channel length.

If H is replaced by $(H+h)$ in relations (2) to (6), one finds that for laminar flow

$$E = \left(\frac{\mathrm{d}}{\mathrm{d}x}\int_0^{H+h} u^2\,\mathrm{d}z\right)\bigg/ g(H+h)\frac{\mathrm{d}h}{\mathrm{d}x} = (u_s^*\,H/\nu)^4 \cdot \nu^2/40gH^3.$$

Direct substitution in this way gives

$$\int_{x_1}^{x_2} w\,\mathrm{d}x \neq 0,$$

where w (cm/s) is the mean vertical flow velocity, and x_1 (cm) and x_2 (cm) are distances along the channel, between which $\mathrm{d}h/\mathrm{d}x$ is constant. Assuming that the resultant inbalance of vertical momentum is adjusted near the boundaries, E is an approximate measure of the ratio of the acceleration and surface slope terms. For $R^* = 50$, the limiting value for the laminar flow, E is of the order of 6×10^{-5} for 10 cm of water, and of the order of 6×10^{-2} for 1 cm of water. Thus the acceleration term is negligible except for very shallow channels. Ursell (1956) concluded differently after unfortunately introducing turbulent flow data into a similar laminar flow approximation.

From the velocity profiles estimated in Section II one may determine

$$m = \int_0^H u^2\,\mathrm{d}z/u_s^2 H.$$

For turbulent flow m decreases steadily with R^*, declining from the constant value of 2/15 found for laminar flow; for example, $m \sim 0 \cdot 019$ for $R^* = 10^3$. Additionally, from Figure 1,

$$\theta \sim 10(u_s^* H/\nu)^{0 \cdot 1}.$$

Proceeding as before, it may be estimated that for $R^* = 10^3$, $E \sim 10^{-3}$ for 10 cm of water, and $E \sim 1$ for 1 cm of water. Again the acceleration term may be neglected except for abnormally shallow channels.

III. The Effect of Surface Waves

Unless damped by suitable surface films, waves form upon water surfaces except at quite low wind velocities. It has been noted earlier that at a given wind velocity the surface velocity remains sensibly independent of the state of the surface. In Figure 3 it is shown that the velocity profile found throughout the body of water is similarly insensitive. At the same time the increased shearing stress obtained over a rough water surface is appropriately recorded as an increase in surface slope (Keulegan 1951; van Dorn 1953; Fitzgerald 1963).

Two matters of practical interest follow immediately. Firstly, measurement of surface velocity is not a sound method of measuring wind stress, since it is influenced by what one might describe as the smooth flow fraction of the stress. Secondly, since the bed shearing stress is determined by only part of the wind stress exerted on a rough surface, the conditions for which the approximation $n = 1$ is acceptable are even more general than for water bodies with smooth surfaces. (Throughout we have assumed a smooth bed; the influence of a rough bed remains to be determined.)

Keulegan (1951) suggested that a wind stress may be divided into two parts, one associated with surface traction and the other with the form resistance of waves. The present work reinforces the utility of this concept, and provides further evidence that at any given wind velocity the surface traction may be estimated approximately by assuming that this velocity lies on the smooth flow profile. Since the characteristics of waves are determined not only by the wind velocity but also by such factors as the duration of the wind and the fetch of the water surface, the shearing stress found over a water surface is necessarily the result of interaction between the wind and the body of water. This notion has been stressed by Stewart (1961).

IV. General

(a) *Conversion of Data*

Keulegan (1951) found that, for $R < 600$,

$$u_s/V = 7 \cdot 6 \times 10^{-4}(u_s H/\nu)^{\frac{1}{2}},$$

with V the mean wind velocity within the tunnel. Comparison with relation (5) gives

$$u_s^* = 1 \cdot 52 \times 10^{-3} V.$$

Introduction of this factor allowed ready conversion of the experimental data to the form required for Figure 1.

Van Dorn (1953) measured surface slope as a function of wind velocity. With water surfaces smoothed by the continuous addition of detergent, plots of surface slope against V^2 were sensibly linear, since the conditions were such that the approximation $n = 1$ was sound. From the experimental data and relation (7),

$$u_s^* = 1 \cdot 72 \times 10^{-3} V$$

for velocities measured at 25 cm above the surface. This factor was used as before.

(b) *Experimental*

Velocity profiles were determined in the tunnel described by Fitzgerald (1963) using a miniature current flowmeter (Sir W. G. Armstrong Whitworth Aircraft Co., U.K.). For some of the measurements the meter was operated at the limit of its sensitivity, but with scrupulous attention to cleanliness of the spindle bearings sufficiently accurate data were obtained.

V. Acknowledgment

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