GROSS DISCONTINUITIES IN THE MAGNETIZATION OF TYPE II SUPERCONDUCTORS

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Summary

Continuous records of magnetization as functions of field strength have been obtained for specimens of type II superconductors. The records show gross discontinuities which are attributed to an instability mechanism involving macroscopic induced currents. The supposed mechanism is described and certain theoretical predictions are shown to agree reasonably with the observations.

Introduction

A simple apparatus has been devised to continuously measure and record the magnetization of samples of high field type II superconductors when subjected to near-uniform fields up to 40 kG. The detailed behaviour of the magnetization of samples of Nb–Zr and Nb_3Sn placed in such fields showed numerous very sharp discontinuities, particularly at low fields in the range 0–5 kG. Recently Aron (1964) has also observed such behaviour. These discontinuities are not mere steps in the magnetization curve, which might be attributed to the entrance or exit of large flux "bundles", but are of a nature which suggests a gross instability mechanism. It will be shown that this behaviour appears to indicate the possibility of macroscopic induced currents having density distributions which are unstable.

Experimental Method

The apparatus consisted essentially of a superconducting magnet system which provided a slowly varying magnetic field having a constant gradient. The force on a sample in this field was measured by a specially made sensitive strain gauge. By means of a feedback system, small constant voltages were maintained at the superconducting terminals of the main field magnet thus causing a gradual steady change of field at the sample, typically of the order of 50 G/s. Signals proportional to the strain gauge output and the main field magnet current were made to operate an $X-Y$ recorder, so providing effectively a continuous record of magnetization as a function of field strength.

Samples of Nb–Zr and Nb_3Sn of various shapes and dimensions were used. The preparation of specimens varied widely and no attempt is made here to correlate the observations with specimen preparation. However, in many cases specimens differing only in size were compared, as for example when a large specimen was subsequently broken into smaller pieces. Wires prepared by the same manufacturer, but varying in diameter, offered another source where the size effect should be predominant.

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Observations and Discussion

Although only two examples of magnetization curves are presented, the general observations are based on the behaviour of a large number of specimens. Both the abundance and the size of the discontinuities depend markedly on the size of the specimens. In Figures 1(*a*) and 1(*b*) for example, the records are shown of a 0·25 mm diameter Nb–Zr wire and a 0·6 cm long by 0·6 cm diameter Nb–Zr cylinder. In each case the first magnetization loop was different from successive ones

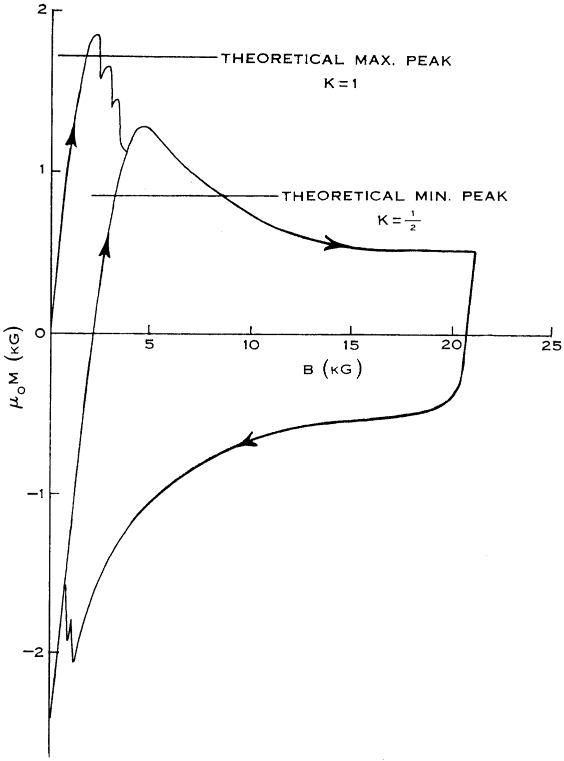

Fig. 1(*a*).—Magnetization of 0·25 mm diameter Nb–Zr wire with axis perpendicular to the field direction. Minimum $K \approx \frac{1}{2}$.

in its initial features. Successive loops repeated themselves and this was especially noticeable with the detailed features of the discontinuities. On the other hand, samples of 0·25 mm diameter Nb–Zr clad with 0·025 mm of copper very rarely showed any discontinuities, and then these were of almost imperceptible size.

The following remarks may be made:

(1) The marked size effect seemed to rule out the possibility that the discontinuities are a feature only of the microscopic properties of the material itself. It appeared instead that the observed magnetization was due largely to macroscopic induced "shielding" currents and that the discontinuities were due to abrupt changes in these currents.

(2) It might be suggested that the observed discontinuities were initiated by irregularities in the rate of change of the applied field. This, however, is unlikely because the nature of the electrical circuit ensured a smoothly varying magnet current. The field of the superconducting magnet itself was observed in an attempt

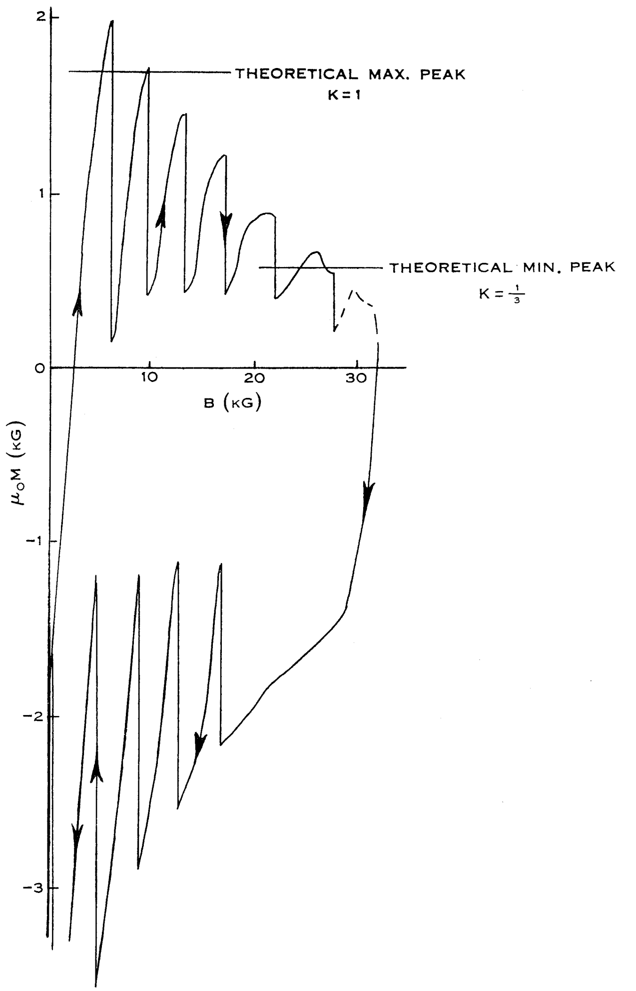

Fig. 1(*b*).—Magnetization of 0·6 cm long by 0·6 cm diameter Nb–Zr cylinder with axis parallel to the field direction. Minimum $K \approx \frac{1}{3}$.

to detect any irregularities that might be due to the magnetization of the magnet wire. No such irregularities were observed, however, owing no doubt to the fact that it was constructed of copper-clad wire.

(3) An explanation of the discontinuities based on the following arguments is suggested. According to Bean (1962) and Kim, Hempstead, and Strnad (1963), macroscopic currents of densities up to a critical value may be carried in type II

superconductors. Induced shielding currents should therefore occupy a layer which is adjacent to the surface and has a finite thickness and a current density verging on the critical value. The critical current density, besides being a function of field strength, is also a function of temperature. Therefore a slight perturbing increase in temperature would result in a partial or complete change to the normal (non-superconducting) state. The current density would then decay throughout the original current-carrying layer and the thickness of the layer would increase. The time scale of these changes would depend on the sample size, the current density distribution, and the normal resistivity of the material. The process would continue until everywhere the current density were again just verging on the critical value. Throughout the time of the process, however, ohmic heating would take place, so within the process there is a regenerative mechanism. It is possible therefore that, in some conditions of magnetization, current density distributions might occur which are unstable in that a slight disturbance in any parameter would lead to a sudden release of magnetization energy and the establishment of a new, stable, current density distribution.

With regard to the effect of copper cladding on the $0 \cdot 25$ mm diameter samples, it is worth noting that without cladding the time scale of the current redistribution process is estimated to be of the order of 2×10^{-8} s, whereas the time required for any redistribution of temperature throughout the sample is of the order of 3×10^{-5} s. Thermal conduction would therefore play no significant part in the process in this case. However, when samples are copper clad, the electrical redistribution (which requires redistribution of flux within the cladding) requires times that are much longer than for the case without cladding, because of the close inductive coupling between superconductor and cladding, and because the decay time-constant for the cladding is itself of the order of 2×10^{-5} s. Thus the electrical redistribution time is of the same order as the thermal redistribution time, so that thermal conduction, e.g. to the middle of the sample or to the cladding itself, cannot be ignored. Thermal conduction would, of course, inhibit the regenerative process because it would reduce temperature changes in the current-carrying region. One might then expect copper cladding to have the effect that has been observed.

It is difficult to evolve and apply a satisfactory mathematical interpretation of the suggested instability process at this stage, because of the uncertain nature of such a process and the uncertainties of the values of some of the physical constants. However, one may show that simple analytical arguments lead to predictions of the correct order of magnitude. For instance, consider the process involved in establishing a shielding current layer following the application of a field difference ΔB (MKS units) between the surface and the middle of a superconducting sample. Suppose the layer finally grows to depth δ in time Δt, and assume finite critical current densities in the layer. There will then be a monotonic variation in B throughout δ and the added flux in the layer will be of the order of $\frac{1}{2}\Delta B\delta$. The average induced e.m.f. during Δt will therefore be

$$E \approx \tfrac{1}{2}\Delta B\delta/\Delta t.$$

This e.m.f. will have opposed the ohmic voltage necessary during the growth of the layer. (There cannot of course be any changes in B or in the current density j within the sample unless the material ceases to be superconducting during the period of the change.) The energy released as ohmic heating (per unit surface area) during the growth of the layer is therefore of the order of

$$\tfrac{1}{2}\delta(\Delta B)^2/\mu_0.$$

Since thermal redistribution times are quite small, almost certainly this energy will cause no significant temperature rise during the comparatively slow application of ΔB. The currents in the layer will therefore be verging on critical values after the dissipation of the above energy.

So far we have assumed that any unstabilizing processes have been inhibited. Let us now remove this inhibition and examine the effect of a slight perturbing temperature rise. Such a rise will cause the currents in the layer to be in excess of their critical values, i.e. the resistance will become finite. Ohmic heating will again occur and the resistance will assume the normal state value ρ_n. At the same time a new conducting layer adjacent to the first will grow, this time at an extremely fast rate. It is easily shown by consideration of the inductance and resistance involved, that the growth time of the second layer is of the order of

$$\mu_0\,\delta^2/\rho_n.$$

On the other hand, the time required for heat transfer from these layers to the middle of the sample or to the surface is of the order of

$$\delta^2\gamma c/k,$$

where γ, c, and k are specific gravity, specific heat, and thermal conductivity respectively. Hence, if

$$\rho_n\,\gamma c/\mu_0\,k \gg 1,$$

heat conduction may be ignored. This is so for bare samples, typical values for Nb–Zr being of the order of 10^3. Hence the temperature rise in the second layer will be approximately

$$(\Delta B)^2/2\mu_0\,\gamma c.$$

If this temperature rise causes the second layer temperature to approach the critical temperature, then δ for the second layer will increase to the dimensions of the sample (radius r) in endeavouring to satisfy the relation

$$\Delta B = \mu_0\,j\delta.$$

The time involved in this decay process would be of the order

$$\tau_e \approx \mu_0 r^2/\rho_n.$$

The condition for the process to be regenerative in the manner outlined is therefore

$$\Delta B \geqslant \{2\mu_0\,\gamma c(T_c - T)\}^{\frac{1}{2}}.$$

The ΔB involved in each discontinuity is directly related to the magnetization (we may ignore shape effects such as variations in demagnetization factor). We may approximately relate average (negative) magnetization M and ΔB by

$$\mu_0 M = K\Delta B,$$

where $K = 1$ when the macroscopic shielding current skin depth is relatively small, and K decreases as the skin depth increases.

Following from the models of Bean (1962) and of Kim, Hempstead, and Strnad (1963), the maximum slope of the magnetization curve must correspond to such thin current layers. Thus the maximum slope may be used to establish the $\mu_0 M$ ordinate. The instability relation for $\mu_0 M$ is then

$$\mu_0 M = K\{2\mu_0 \gamma c(T_c - T)\}^{\frac{1}{2}}.$$

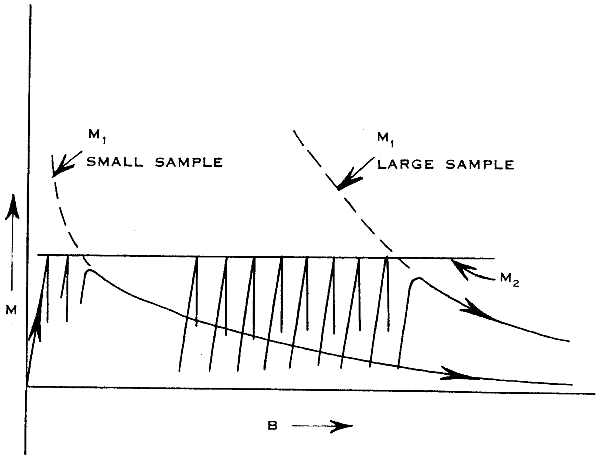

Fig. 2.—Illustrating the effect of the two limiting magnetizations M_1 and M_2 on the observed magnetization. M_1 is dependent on size and critical current density (that is, $M_1 = f(r, j_c)$). $M_2 = K\{2\gamma c(T_c - T)/\mu_0\}^{\frac{1}{2}}$ is the stability limit.

Values of K other than 1 are difficult to establish precisely. However, following from the models of Bean and of Kim, Hempstead, and Strnad, one would not expect K to be much less than $\frac{1}{3}$. The minimum value would correspond to the drooping parts of the magnetization curve at higher fields. Hence, the peak $\mu_0 M$, just prior to a discontinuity, should decrease as higher fields are applied. The maximum peak should approach

$$\{2\mu_0 \gamma c(T_c - T)\}^{\frac{1}{2}},$$

and the minimum peak should be approximately one third of this.

Reasonable values of γ, c, and $(T_c - T)$ are 8×10^3 k/m^3, 3×10^{-1} J degK^{-1} kg^{-1}, and 5°K respectively. With $K = 1$, $\frac{1}{3}$ respectively, the corresponding values of $\mu_0 M$ would then be of the order of $1 \cdot 7$ and $0 \cdot 56$ kG, which are not markedly different from those suggested by the observations. In attempting to interpret the observations one must remember that specific heat, besides being a function of temperature, is also a function of magnetic field (Morin *et al.* 1962). It is thought that "unstable" regions of magnetization curves reported by others, e.g. Morin *et al.* for Va–Ga, are probably due to similar discontinuities brought about by the process outlined here.

To attempt to explain the multiplicity of discontinuities in the 0·6 cm diameter sample, it must be remembered that the magnetization (M_1) may be limited also by the critical current density and the size of the sample. One can assume that the critical current density is independent of size. The limiting values M_1 should then be proportional to the scale factor. On the other hand, the maximum stable magnetization (M_2) is more or less independent of scale except for the factor K. If the latter limit is less than the former, we should then expect discontinuities to repeat as the field is raised until the two limiting values of M coincide, the number of repetitions being greater for larger samples. This process is illustrated in Figure 2, where K has been assumed constant for simplicity.

Conclusions

Magnetization curves derived from a number of discrete measurements, even when found to repeat many times at specific values of field, can give a very inadequate picture of the true nature of the relationship. It seems reasonable to suspect that a major part of the magnetic behaviour of many samples of type II superconductors, even of small size, is due to macroscopic effects which are additional to the effects attributable to the microscopic behaviour of the material, usually the subject of investigation.

The instability mechanism outlined here may throw light on the behaviour of many types of superconducting solenoids whose maximum capabilities are often unpredictable.

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