ENERGY LEVEL SPECTRA FOR ^{118}Sn AND ^{122}Sn

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[*Manuscript received December* 15,1964]

Summary

Shell-model calculations have been made for the energy level spectra of ^{118}Sn and ^{122}Sn. Best fit with experimental data for ^{118}Sn is found for singlet s- and d-state forces of the same strength and range. Values for several unobserved levels of ^{122}Sn have been predicted.

I. INTRODUCTION

In the past few years, the powerful mathematical technique based on relative coordinates (Moshinsky 1959; Arima and Terasawa 1960; Balashov and Eltekov 1960; Lawson and Goeppert Mayer 1960; Mitra and Pandya 1960) has been widely used for shell-model calculations for the nuclear energy levels. Its application allows one to get useful knowledge about the effective two-body nuclear interaction, and in particular about the relative contributions due to states of different angular momenta. For instance, Shah and Pandya (1962) find from their analysis of energy level spectra for certain light nuclei that p-state forces are negligible. They had studied the effects of setting up a sort of non-locality for the even-state interactions. Also, Moszkowski (1960) has suggested that even the study of s-state interactions gives a valuable means of evaluating properties of nuclear spectra.

In the present paper, calculations have been made in the framework of the above-mentioned method based on relative coordinates to reproduce and to predict some energy levels of two isotopes of tin with mass numbers 118 and 122. For the sake of simplicity the two-body nuclear interaction has been chosen to be a central interaction.

It may be pointed out that the number of protons for each of the nuclear species is magic (namely 50) and hence their low-lying states may be characterized to a large extent by shell-model considerations. However, the two isotopes of tin have neutron numbers of 68 and 72, which are far from magic numbers, so they may be expected to exhibit some collective effects explainable on the basis of the vibrational model. Our results are in good agreement with the shell-model calculations, though the effects of weakly coupled vibrations present owing to collective motions cannot be ruled out. It will be useful to compare the predicted energy levels for ^{122}Sn with the experimental data when they become available.

II. RÉSUMÉ OF THE METHOD

Here we sketch very briefly the method for shell-model calculations as described by Moshinsky (1959). If we consider only two-body forces, the Hamiltonian matrix

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elements between the states

$$|(n_1 l_1 s_1) j_1, (n_2 l_2 s_2) j_2 : JM\rangle \quad \text{and} \quad |(n_1' l_1' s_1) j_1', (n_2' l_2' s_2) j_2' : JM\rangle$$

may be put for the central forces as:

$$\langle j_1 j_2 : JM | H_{12} | j_1' j_2' : JM\rangle = \sum_{\substack{LL'SS' \\ N\Lambda n l n' l'}} A \begin{pmatrix} l_1 & \frac{1}{2} & j_1 \\ l_2 & \frac{1}{2} & j_2 \\ L & S & J \end{pmatrix} A \begin{pmatrix} l_1' & \frac{1}{2} & j_1' \\ l_2' & \frac{1}{2} & j_2' \\ L' & S' & J \end{pmatrix}$$

$$\times B^{n_1 l_1 n_2 l_2}_{N\Lambda n l}(L)\, B^{n_1' l_1' n_2' l_2'}_{N\Lambda n' l'}(L')\, \delta_{LL'} \langle nl; S | H_{12} | n'l'; S\rangle \qquad (1)$$

by transforming the two-nucleon wave functions first from j–j coupling formalism to L–S coupling formalism, and then by changing over into centre-of-mass and relative coordinates with the help of transformation brackets. Here the A's are the LS–jj transformation coefficients tabulated by Kennedy and Cliff (1955), and the B's are the Moshinsky brackets given in the tables of Moshinsky and Brody (1960).

The reduced matrix elements, i.e. the integrals

$$I_{nl} = \langle nl \| V(r) \| nl\rangle = \int_0^\infty R_{nl}^2(r)\, V(r)\, r^2\, \mathrm{d}r \qquad (2)$$

for Gaussian potential $\exp(-r^2/r_0^2)$, can be evaluated as a function of $\lambda = r_0/r_l$ by using $B(nl, n'l', p)$ coefficients of Brody, Jacob, and Moshinsky (1960), where r_0 is the range of the two-body Gaussian potential, and r_l is the range of the harmonic oscillator wave function; r_l is known from electron-scattering experiments. Some of the values for matrix elements I_{nl} were tabulated earlier (Waghmare, Gupta, and Kumar 1964). These calculations have now been greatly extended. Calculations for previously tabulated values were checked again very carefully and a few errors were discovered. A complete set of values is given in Table 1.

III. Analysis

In ^{118}Sn and ^{122}Sn there are respectively two holes inside and two particles outside a complete subshell in $T = 1$ states. Thus the two cases may be treated alike, as hole configurations can be treated as equivalent to particle configurations (Pandya 1956). Configurations are considered to be of the type $(j)^2$, and so only the even J values may be included for this study as odd J values will not be allowed according to Pauli's exclusion principle.

Several values of $\lambda = r_0/r_l$ were tried for these calculations with different values of the potential-strength parameter. Contributions from s- and d-states for equal potential strength were included. Varying degrees of agreement were obtained between the calculated values and the experimental values (Ramaswamy *et al.* 1960) of the energy levels. Best fit was obtained for the forces of strength 50 MeV and for $\lambda = 0{\cdot}9$. The calculated and observed values are given in Table 2. There is slight disagreement for the 6^+ level. Better agreement is obtained if the strength of the

forces for this level is reduced. This reduction of potential strength for the 6^+ level may be attributed to the configuration dependence (Thankappan, Waghmare, and Pandya 1961) or to the angular momentum dependence (Waghmare 1962) of the effective two-body nuclear interaction.

TABLE 1

THE MATRIX ELEMENTS $I_{nl} = \langle nl||\exp -(r/r_0)^2||nl\rangle$ FOR DIFFERENT VALUES OF $\lambda = r_0/r_l$

λ / I_{nl}	0·5	0·6	0·7	0·8	0·9	1·0	1·1	1·2
I_{0s}	0·0894	0·1362	0·1886	0·2436	0·2994	0·3536	0·3816	0·4534
I_{1s}	0·0894	0·1200	0·1478	0·1730	0·1970	0·2210	0·2342	0·2725
I_{2s}	0·0798	0·0983	0·1190	0·1366	0·1542	0·1721	0·1839	0·2127
I_{3s}	0·0711	0·0870	0·1036	0·1193	0·1348	0·1500	0·1565	0·1793
I_{4s}	0·0639	0·0778	0·0903	0·1092	0·1208	0·1352	0·1396	0·1588
I_{5s}	0·0763	0·0793	0·0847	0·0786	0·0369	0·1234	0·1184	0·1237
I_{0p}	0·0179	0·0361	0·0620	0·0950	0·1340	0·1768	0·2007	0·2676
I_{1p}	0·0292	0·0513	0·0765	0·1028	0·1291	0·1547	0·1683	0·2052
I_{2p}	0·0354	0·0560	0·0766	0·0967	0·1165	0·1359	0·1471	0·1743
I_{3p}	0·0387	0·0566	0·0740	0·0895	0·1063	0·1227	0·1316	0·1552
I_{4p}	0·0348	0·0522	0·0710	0·0875	0·0913	0·1108	0·1206	0·1439
I_{0d}	0·0036	0·0095	0·0204	0·0370	0·0599	0·0884	0·1056	0·1580
I_{1d}	0·0083	0·0187	0·0344	0·0540	0·0761	0·0994	0·1122	0·1481
I_{2d}	0·0121	0·0251	0·0418	0·0603	0·0796	0·0988	0·1092	0·1367
I_{3d}	0·0163	0·0299	0·0447	0·0645	0·0803	0·0981	0·1048	0·1283
I_{4d}	0·0307	0·0377	0·0491	0·0504	0·0946	0·0935	0·0932	0·1062
I_{0f}	0·0007	0·0025	0·0067	0·0145	0·0268	0·0442	0·0556	0·0932
I_{1f}	0·0021	0·0064	0·0143	0·0264	0·0422	0·0608	0·0715	0·1029
I_{2f}	0·0038	0·0102	0·0207	0·0337	0·0498	0·0669	0·0767	0·1037
I_{3f}	0·0029	0·0120	0·0238	0·0338	0·0510	0·0646	0·0789	0·1039
I_{0g}	0·0001	0·0007	0·0022	0·0057	0·0120	0·0221	0·0292	0·0550
I_{1g}	0·0005	0·0020	0·0057	0·0124	0·0178	0·0359	0·0442	0·0700
I_{2g}	0·0016	0·0042	0·0093	0·0189	0·0302	0·0447	0·0522	0·0761
I_{3g}	0·0057	0·0070	0·0144	0·0183	0·0405	0·0485	0·0533	0·0729
I_{0h}		0·0002	0·0007	0·0022	0·0054	0·0110	0·0154	0·0324
I_{1h}	0·0001	0·0006	0·0022	0·0057	0·0117	0·0207	0·0267	0·0467
I_{2h}	0·0002	0·0011	0·0039	0·0101	0·0167	0·0279	0·0348	0·0555
I_{0i}		0·0001	0·0002	0·0009	0·0024	0·0055	0·0081	0·0192
I_{1i}	0·0001	0·0002	0·0009	0·0025	0·0060	0·0118	0·0158	0·0308
I_{2i}	0·0003	0·0003	0·0021	0·0041	0·0106	0·0170	0·0216	0·0376
I_{0j}			0·0001	0·0003	0·0011	0·0028	0·0043	0·0113
I_{1j}		0·0001	0·0003	0·0012	0·0029	0·0066	0·0093	0·0202
I_{2j}		0·0004	0·0010	0·0019	0·0050	0·0113	0·0147	0·0272

Table 3 gives the calculated energy levels of ^{122}Sn with forces of the same strength and range as in the case of ^{118}Sn. The assumption that the same potential strength may be used is justified by the fact that the two nuclear species are very close to each other. Experimentally, not much is known about the level structure of ^{122}Sn (National Academy of Sciences–National Research Council 1960) except for

its two levels 0^+ (ground state) and 2^+ (1·14 MeV). A detailed comparison of calculated and experimental values is, therefore, not possible. For the 2^+ level, the calculated value comes out to be at 1·14 MeV if the potential-strength parameter is $V_0 = -51$ MeV, and $\lambda = 0{\cdot}8$. For these values of the parameters, other calculated energy levels are found to be at 2·11 MeV (4^+ state), 2·41 MeV (6^+ state), 2·57 MeV

TABLE 2

CALCULATED AND OBSERVED LEVELS OF ^{118}Sn

Values in MeV

J	0^+	2^+	4^+	6^+
E observed	ground state	1·22	2·25	2·55
E calculated	ground state	1·22	2·25	2·63

(8^+ state), and 2·64 MeV (10^+ state). It should, however, be pointed out that the 2^+ level is not very sensitive to the value of λ, while this is not the case for other levels. When experimental data for some other energy levels of ^{122}Sn become available, it will be possible to get useful information about the effective two-body forces by comparing the experimental data with the calculated values.

TABLE 3

CALCULATED ENERGY LEVELS OF ^{122}Sn

Values in MeV

J / λ	0^+	2^+	4^+	6^+	8^+	10^+
0·5	ground state	0·93	1·21	1·30	1·35	1·38
0·6	ground state	1·04	1·49	1·64	1·71	1·75
0·7	ground state	1·11	1·79	2·01	2·12	2·17
0·8	ground state	1·12	2·07	2·36	2·52	2·59
0·9	ground state	1·11	2·33	2·70	2·90	2·99
1·0	ground state	1·10	2·59	3·04	3·28	3·42
1·1	ground state	1·08	2·72	3·21	3·48	3·64
1·2	ground state	1·05	3·12	3·71	4·04	4·29

Nealy and Sheline (1964) have reported the following values of energy levels for ^{122}Sn: 0, 1·141, 2·147, 2·247, 2·414, 2·494 in MeV. The authors have not made any spin and parity assignments. If the potential-strength parameter is -50 MeV and $\lambda = 0{\cdot}78$, our calculated values come out to be 0 (ground state), 1·12 MeV (2^+ state), 2·01 MeV (4^+ state), 2·27 MeV (6^+ state), 2·42 MeV (8^+ state), and 2·49 MeV (10^+ state). The agreement is quite good.

IV. Acknowledgment

The authors wish to thank Professor B. M. Anand for providing an opportunity to undertake this investigation and for continued encouragement.

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