EFFECT OF GRAVITATION IN THE ORIGIN OF COSMIC PARTICLES

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Summary

Following on Infeld's (1963) proposal of the dependence of the physical constants on the gravitational field, we apply this idea to quantum mechanics and other physical phenomena and suggest a possible explanation of the origin of cosmic rays.

I. Introduction

Gravitation can always play a significant role in the explanation of so many phenomena because it is universal. The cause of the very large number of cosmic particles in our Galaxy has yet to be properly explained. We consider the possible effects of gravitation in the origin of these particles.

If we consider a gravitational field such that in the metric

$$ds^2 = g_{00}\,dt^2 - g_{kl}\,dx^k\,dx^l$$

g_{00} and g_{kl} are independent of time, we call it static. In such a case we have to modify Planck's constant and the inertial mass of the particle (Infeld 1963) as

$$\hbar = \hbar_0/g_{00} \tag{1}$$

and

$$\mu = \mu_0/g_{00}, \tag{2}$$

where the velocity of light is assumed to be unity and $\hbar_0$ and μ_0 correspond to Planck's constant and the inertial mass of the particle when $g_{00} = 1$. The mass of the particle is such that it does not affect the gravitational field.

Infeld (1963) has given a possible explanation of the red-shift by using these relations. We have extended the utility of the results to other physical phenomena.

II. Utility in Some Quantum Phenomena

(*a*) *Uncertainty Principle*

Heisenberg's uncertainty relation at a place where $g_{00} \neq 1$ becomes

$$\Delta x\,\Delta p_x \geqslant \hbar_0/2g_{00}. \tag{3}$$

Suppose that $g_{00} = 1-\chi$, then

$$\begin{aligned}\Delta x\,\Delta p_x &\geqslant \hbar_0/2(1-\chi)\\ &\geqslant \tfrac{1}{2}\hbar_0(1+\chi) \qquad \text{if } \chi \text{ is small.}\end{aligned}$$

We see that the uncertainty increases if the gravitation is large. If χ is comparable to unity, the uncertainty will increase so much that it will be observable under normal conditions. If $\chi \to 1$, then even if we find the momentum of a particle to a poor approximation it will be impossible to know where the particle is situated.

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(*b*) *Schrödinger's Wave Equation*

Let us first consider the one-dimensional Schrödinger wave equation

$$-\frac{\hbar^2}{2\mu}\frac{\mathrm{d}^2\psi(x,t)}{\mathrm{d}x^2}+V(x)\psi(x,t) = -\frac{\hbar}{\mathrm{i}}\frac{\mathrm{d}\psi}{\mathrm{d}t}. \tag{4}$$

If $\psi(x,t) = U(x)\phi(t)$, then from the expression $\phi = \exp(\mathrm{i}Wt/\hbar_0)$ and equation (1) we have

$$\frac{\mathrm{d}^2U}{\mathrm{d}x^2}+\frac{2\mu g_{00}}{\hbar_0^2}\left\{W \quad g_{00}\,V(x)\right\}U = 0,$$

and from (2)

$$\frac{\mathrm{d}^2U}{\mathrm{d}x^2}+\frac{2\mu_0}{\hbar_0^2}\left\{W-g_{00}\,V(x)\right\}U = 0. \tag{5}$$

In a similar manner we can find the equation in three dimensions. We see that the g_{00} term appears only with the potential. It may be pointed out that Einstein's theory of gravitation as a first approximation to Newton's theory leads to the derivatives of g_{00} as the intensity of the gravitational field, and to g_{00} itself as the gravitational potential. Hence equation (5) having the g_{00} term with the potential seems to be consistent with Einstein's theory.

(*c*) *Electric Potentials*

The effective potential for a particle in a static gravitational field becomes

$$V' = g_{00}\,V. \tag{6}$$

Relation (6) is of great importance, for it shows that gravitational potential has a dominating effect on all types of potentials encountered by charged particles. If a particle has to cross a potential barrier and it happens to be in a gravitational field, the effective barrier will become $g_{00}\,V$; that is, the probability of penetration for the same type of particle, the interaction between similar particles, and the escape of certain particles from a potential well will differ in different gravitational fields. If g_{00} becomes very small, the effective potential V' will be small whatever may be the value of V. In such a case, e.g. α-particle emission, we see that the probability of emission will greatly increase due to the effective reduction of the potential barrier. Similarly, other nuclear particles will also experience a loosening in such a gravitational field. If $g_{00} \to 0$, all types of particles will behave as free particles, even though the potential V may have any value. Schrödinger's equation will approach the relation

$$\frac{\mathrm{d}^2U}{\mathrm{d}x^2}+\frac{2\mu_0}{\hbar_0^2}\,WU(x) = 0,$$

which is the equation for a free particle.

III. Origin of Cosmic Rays

In order to illustrate the theory clearly, we must choose a particular field as a model. Although not necessarily the correct model, we choose the Schwarzschild interior field in isotropic coordinates, which gives the values of g_{00} and g_{kl} (Wyman 1946) as

$$g_{00} = \{2a-2m+m(4a-m)r^2/2a^3\}^2\times\{(2a+m)(1+mr^2/2a^3)\}^{-2}, \tag{7}$$

$$g_{kl} = \{1+m/2a\}^6\{1+mr^2/2a^3\}^{-2}.$$

Since both g_{00} and g_{kl} are independent of time, the results are applicable.

The value of $g_{00}\to 0$ at the centre as $m\to a$, and at $r = a$ the value of $g_{00} = 0\cdot 1$. At intermediate values of r, g_{00} will vary from 0 to $0\cdot 1$. If we assume homogeneity in a star with density ρ_c, we have under such conditions $\rho_c = 3/4\pi a^2$ when $g_{00} = 0$. Let us consider two extreme cases.

(i) *Stars with a Radius* a $\simeq 10^4$ *times that of the Sun.*—For such stars with $g_{00} = 0$ at the centre

$$\rho_c = \frac{3}{4\pi}\times 10^{-8}\simeq\frac{1}{30}\quad\text{times that of water}$$

(standard density of water is $6\cdot 665\times 10^{-8}\ \mathrm{s}^{-2}$; Synge 1960), that is, a star with such a large radius cannot have a density greater than $1/30\ \mathrm{g/cm^3}$ on average. When such a density is attained in this type of star, the value of $g_{00} = 0$ at the centre reduces the effective potential barriers to zero near the centre. The matter in this neighbourhood will be most unstable, and it will be only in the form of freely moving neutrons, protons, electrons, mesons, hyperons, etc., the so-called fifth and sixth states of matter. Near the surface, the effective potential is not zero but is reduced to one-tenth of its value. Here we have ionized masses (together with many nuclei, α-particles, etc.), the so-called fourth state of matter or plasma. It is very difficult to confine a plasma to this shape when the core of the star is filled with elementary particles, and the star breaks up due to the instability of the plasma and emits a very large number of cosmic particles.

(ii) *Stars with a Density of* $\sim 10^5$.—In such cases

$$a^2 = \frac{3\pi}{4}\times\frac{1}{6\cdot 665\times 10^{-3}} = (3\cdot 456)^2,$$

or

$$a\simeq 3\cdot 456,$$

that is, a star with such a high density cannot have a radius less than one and a half times that of the Sun.

According to prevailing theories, when the entire hydrogen of a star is consumed the star first expands and then contracts more and more until it ultimately loses its brightness and is lost.

Here we introduce the new idea that, as the star shrinks, it tends to attain a minimum size. Before this minimum size is reached the interior of the star is converted into freely moving elementary particles and, near the surface, into plasma.

This arrangement is very unstable and the star breaks up into ionized particles and the freely moving particles, which are scattered in the Galaxy. Thus we see that the star is not lost as a block but rather as a dust.

(iii) *The Origin of Cosmic Rays*.—We have now seen that when the stars decay the ultimate product is a large number of particles, which are scattered at random in the Galaxy. Some of these particles enter the Earth's atmosphere and fall in the form of cosmic rays. This is the proposed theory of the origin of cosmic rays.

IV. Illustrations

(a) α-Decay Process

Here we examine the process by which particles are emitted from the nucleus under the effect of gravitation. As an example we consider the process of α-decay. The probability of emission of an α-particle from the nucleus is given by

$$P = \mathrm{e}^{-x},$$

where

$$x = 4\pi(2\mu)^{\frac{1}{2}}\, h^{-1} \int_{R_0}^{R'} (V-E)^{\frac{1}{2}}\, \mathrm{d}R.$$

However, using the modified equation of the form (5), we obtain

$$x = 4\pi(2\mu_0)^{\frac{1}{2}}\, h_0^{-1} \int_{R_0}^{R'} (g_{00}\, V-E)^{\frac{1}{2}}\, \mathrm{d}R,$$

or

$$x = 4\pi(2\mu_0)^{\frac{1}{2}}\, h_0^{-1}(zZe^2/E^{\frac{1}{2}})\{\cos^{-1} y^{\frac{1}{2}} - y^{\frac{1}{2}}(1-y)^{\frac{1}{2}}\},$$

where $y = E/g_{00}B$ and $B = 2Ze^2/R_0$. R_0 is the radius of the nucleus and E is the energy of the α-particle.

We shall now calculate the period of emission of an α-particle in the case of nuclei that do not give detectable emission ordinarily, i.e. for $g_{00} = 1$. Bethe (1947), assuming that an α-emitter with a period greater than 10^{14} years would be undetectable, has computed the smallest energy expected for the emitted particles for various values of Z as:

Z	10	30	50	70	90
E	0·13	0·8	1·7	2·7	3·7

The period T $(= T_0/P$, T_0 constant) of an α-emitter, taking $Z = 50$ and $E = 1{\cdot}7$, has been calculated from the above formulae for values of g_{00} of 0·6, 0·4, 0·2, 0·1. The results are shown in Table 1 together with calculated values of the disintegration constant λ $(= \lambda_{\mathrm{f}} P$, λ_{f} constant) for a Pa–Ac source.

If $g_{00} < 0{\cdot}1$, the energy E becomes greater than $g_{00} B$ and there is no question of penetration since the α-particle is free to emerge from the nucleus. Similar calculations can be made for any other value of Z and also for protons and other particles.

(b) *Solutions of the Einstein Field Equations*

Solutions of the Einstein field equations for contracting Schwarzschild static models are possible. Bondi (1964) has shown that the gravitating sphere contracts without radiating any energy. The limits to the contraction allowed by physical assumptions are

(1) $a > 9m/4$ for the central pressure to be finite, and

(2) $a > 18m/5$ for the central-energy invariant $(3p-\rho)$ to be positive.

TABLE 1

CALCULATED VALUES OF PERIOD AND DISINTEGRATION CONSTANT FOR VARIOUS VALUES OF g_{00}

g_{00}	Period T (for $Z = 50$, $E = 1 \cdot 7$)	Disintegration Constant λ for Pa–Ac (s^{-1})
$1 \cdot 0$	$\simeq 10^{14}$ yr	$5 \cdot 5 \times 10^{-13}$
$0 \cdot 6$	$\simeq 10^{9}$ yr	$5 \cdot 5 \times 10^{-8}$
$0 \cdot 4$	$\simeq 10^{2}$ yr	$2 \cdot 57 \times 10^{-2}$
$0 \cdot 2$	$\simeq 10^{4}$ s	$2 \cdot 5 \times 10^{18}$
$0 \cdot 1$	$\simeq 10^{-6}$ s	$E > g_{00} B$

If the limiting values of a are substituted in the expression for g_{00} (equation (7)) in the Schwarzschild internal solutions, we see that the limiting values of g_{00} are

$$
\begin{array}{lll}
(1)\ a > 9m/4 & \text{at } r = 0 & g_{00} \simeq 0 \cdot 2 \\
 & \text{at } r = a & g_{00} \simeq 0 \cdot 4 \\
(2)\ a > 18m/5 & \text{at } r = 0 & g_{00} \simeq 0 \cdot 4 \\
 & \text{at } r = a & g_{00} \simeq 0 \cdot 6
\end{array}
$$

In Table 1 we have seen the effect of these values of g_{00} on α-decay. After attaining the limiting size the sphere cannot contract further and the material composing it is mostly in the form of unstable nuclei that give spontaneous emission of particles.

(c) *Further Reactions*

For radiative contraction no such limit has been proposed, and a large number of phenomena await investigation. The value of g_{00} may go down to $0 \cdot 1$ or even less. It may be that, owing to contraction, the value of g_{00} is reduced and gives rise to radiation of nuclear particles. Calculations for other nuclear reactions are beyond the scope of the present paper.

V. CONCLUSION

We have presented here a possible solution for the origin of cosmic rays. The theory put forward may find its proper application in explaining various phenomena in cosmology. By taking some suitable metric, detailed calculations could be made.

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