SOME ASPECTS OF IRREGULAR DIFFRACTION STUDIED BY MEANS OF ULTRASONIC WAVES

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Summary

Ultrasonic waves of frequency 32 kc/s are allowed to pass through a rising stream of hot turbulent air. Spaced microphones are used to investigate the diffraction pattern formed beyond the air stream, and spaced thermistors are used to record the temperature fluctuations in the air stream. The arrangement is used to study various aspects of random diffraction and to verify assumptions often made in the investigation of ionospheric irregularities. It is shown that the results are broadly in agreement with theory, in spite of various uncertainties about the applicability of the theories. In particular, reliable measurements of the velocity of the air stream and of the scale of the temperature fluctuations can be obtained from observations made at a few fixed points in the diffraction pattern.

I. Introduction

In observations of the irregularities of electron density frequently present in the ionosphere, use is made of radio waves, either reflected from the ionosphere or transmitted through it. In both cases, an irregular "diffraction pattern" is formed over the ground, and this is investigated by the use of radio receivers, which record the time variations of the field strength at fixed points on the ground. Usually only the amplitude is recorded, and from the records information is obtained about the movements and dimensions of the ionospheric irregularities. For example, ionospheric drifts are determined from the time displacements between the fading records at three spaced receivers (e.g. Mitra 1949; Krautkrämer 1950). In this case, the radio waves originate from a pulse transmitter, usually close to the receivers. In the study of radio star scintillations, information about the sizes, shapes, and motions of irregularities in the F region of the ionosphere is obtained by the use of spaced interferometers observing the same radio star (e.g. Hewish 1951; Spencer 1955). By observing scintillations of radio waves transmitted from a satellite, similar information can be obtained and it is also possible to determine the height of the irregularities (e.g. James 1962).

All these methods depend upon a knowledge of the relation between the diffraction pattern formed over the ground and the ionospheric irregularities that produce it. The irregularities can be imagined as forming a "diffracting screen", and the object of the experiments is to find out as much as possible about the screen from observations of the diffraction pattern. Thus we need to know how the velocity

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of the pattern over the ground is related to the velocity of the irregularities and how the "scale size" of the irregularities in the diffraction pattern is related to the dimensions of the ionospheric irregularities.

The theoretical problems of "random" diffraction have been reviewed by Ratcliffe (1956). The theories that have been used make a number of assumptions and approximations, which may or may not be valid in the actual problem. Also, the complete diffraction pattern is never observed, but rather it is sampled at a few points only. This necessitates further assumptions and makes the estimation of the errors very difficult, since the irregularities themselves are inaccessible, and the results cannot be checked by direct observation.

In view of these points, it was considered desirable to perform a laboratory experiment to verify some of the assumptions. The aim was to construct an irregular diffracting screen whose velocity and statistical properties could be measured directly. Then, the relevant parameters could also be deduced from observations made at a few points in the diffraction pattern and the results could be compared with those obtained by direct measurement.

There are two main considerations in the choice of such a laboratory experiment. The wavelength used, of course, must be much less than in ionospheric work, in order to reduce the system to laboratory scale. A wavelength of about 1 cm is convenient, and this suggests the use of either centimetre radio waves or ultrasonic waves propagating in air. Secondly, it must be possible to make an analogue of the ionospheric irregularities. For ultrasonic waves, the variations of wave velocity arising from temperature fluctuations in air provide a close analogy. For example, a heater element produces a rising stream of turbulent air, which appears to simulate quite closely the drifting pattern of irregularities in the ionosphere. As no simple device of this kind appears to be readily available for centimetre radio waves, it was decided to use ultrasonic waves. The transmission of waves through such a turbulent air stream is analogous to the experiments on radio star and satellite scintillations. It is not directly analogous to experiments in which radio waves are reflected from the ionosphere, although many of the features, such as the derivation of the velocity from observations at spaced receivers, are very similar.

In the present experiment, direct measurements on the diffracting screen are made by the use of thermistors, which enable records to be made of the temperature fluctuations at fixed points in the air. Provided these temperature fluctuations are small compared with the mean temperature, they are analogous to the fluctuations of electron density in the ionosphere, as is shown in Section II.

A diffracting screen of this type modifies only the phase of the waves transmitted through it. As the wave propagates beyond the screen, however, amplitude fluctuations appear and become larger as the distance from the screen increases. It is of interest to verify the theories of this phenomenon using ultrasonic waves, and this is done in Section IV.

The velocity of the rising air stream can be obtained from the thermistor readings, and this is compared with the velocity determined from the diffraction pattern in Section V. The scale sizes of the irregularities in the screen are compared with those in the diffraction pattern in Section VI.

II. Theory

Consider a region of length l, in which the wave velocity is $c+\Delta c$, embedded in a uniform medium in which the velocity is c. For a wave of angular frequency ω passing through this region, the phase difference $\Delta\phi$, relative to the phase that would exist if the irregularity were not present, is given by

$$\Delta\phi = \omega l\left(\frac{1}{c}-\frac{1}{c+\Delta c}\right) \simeq (\omega/c^2)(l\,.\,\Delta c)\,. \tag{1}$$

For radio waves in the ionosphere, if the Earth's magnetic field is neglected, the wave velocity c_{r} is given by

$$c_{\mathrm{r}}^2 = c_0^2(1-Ne^2/\epsilon_0 m\omega^2)^{-1}, \tag{2}$$

where c_0 is the velocity of electromagnetic waves *in vacuo*, N is the electron density, and e and m are the charge and mass of the electron. Thus, if we consider a region of electron density $N+\Delta N$ embedded in a region of mean ionization density N, we have from equation (2)

$$\frac{\Delta c_{\mathrm{r}}}{c_{\mathrm{r}}^2} = \left(1-\frac{Ne^2}{\epsilon_0 m\omega^2}\right)^{-\frac{1}{2}}\left(\frac{e^2}{2c_0\,\epsilon_0 m\omega^2}\right)\Delta N\,, \tag{3}$$

and, from equations (1) and (3), we find for the phase difference $\Delta\phi_{\mathrm{r}}$ in the radio case

$$\Delta\phi_{\mathrm{r}} = \left(\frac{e^2}{2c_0\,\epsilon_0 m\omega}\right)\left(1-\frac{Ne^2}{\epsilon_0 m\omega^2}\right)^{-\frac{1}{2}}\left(l\,\Delta N\right). \tag{4}$$

In the case of ultrasonic waves in air, the wave velocity c_{u} is given by

$$c_{\mathrm{u}} = (\gamma RT/M)^{\frac{1}{2}}\,, \tag{5}$$

where γ is the ratio of the specific heats, R is the gas constant, T is the absolute temperature, and M is the mean molecular weight. Now consider a volume of air at a temperature $T+\Delta T$ embedded in air at a mean temperature T. From (5)

$$\Delta c_{\mathrm{u}}/c_{\mathrm{u}}^2 = \tfrac{1}{2}(M/\gamma RT^3)^{\frac{1}{2}}\Delta T\,, \tag{6}$$

and thus the phase difference $\Delta\phi_{\mathrm{u}}$ for the ultrasonic waves is given by

$$\Delta\phi_{\mathrm{u}} = \tfrac{1}{2}\omega(M/\gamma RT^3)^{\frac{1}{2}}(l\,\Delta T). \tag{7}$$

A comparison of equations (4) and (7) shows that the phase difference is proportional to $(l\,\Delta N)$ for the radio waves in the ionosphere, and to $(l\,\Delta T)$ for the ultrasonic waves in air. Therefore, fluctuations of temperature in a moving air stream may be regarded as analogous to the fluctuations of electron density in the ionosphere. We assume that in both cases the fluctuations are small compared with the mean.

An ultrasonic wave transmitted normally through a plane sheet containing random temperature fluctuations emerges with random variations of phase across its wavefront. We shall refer to such a sheet as a "phase screen". The important parameters for such a screen are the root-mean-square phase deviation (ϕ_0) that it produces, and the autocorrelogram that describes the spatial structure of the phase variations across the wavefront. The autocorrelogram is related to the three-dimensional autocorrelogram of the temperature fluctuations, which can be investigated by means of the spaced thermistors. These ideas are developed further in Section IV.

III. Apparatus

It was apparent from the outset that considerable difficulties might arise from reflections of the ultrasonic waves from the walls and furniture of the laboratory. These reflected waves would interfere with the direct wave passing through the phase screen, and modify the diffraction pattern being investigated. For this reason, it was decided to use short pulses of waves, so that the unwanted signals could be separated from the main wave by their greater time delay.

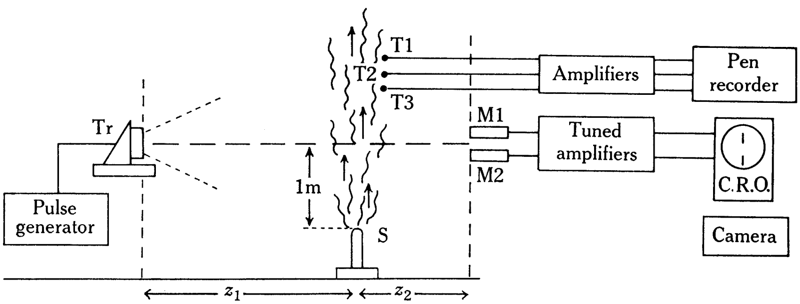

Fig. 1.—Arrangement of apparatus, showing relative positions of the transducer Tr, phase screen S, microphones M1 and M2, and thermistors T1, T2, and T3.

The general arrangement of the apparatus is shown in Figure 1. A pulse generator produces $0 \cdot 3$ msec pulses of 32 kc/s electrical signal, at a repetition rate of 50 c/s, and these pulses are applied to an electrostatic transducer, which is the source of ultrasonic waves. The transducer, similar in construction to an electrostatic loudspeaker, consists of a tightly stretched piece of aluminized Mylar film about 10 cm square, separated from a flat metal back plate by a sheet of thin perforated cardboard. The electrical pulses, of about 800 V peak-to-peak amplitude, are applied between the back plate and the Mylar film, causing the film to vibrate and produce the ultrasonic wave. To prevent electrostatic attraction on both half cycles of the signal, and consequent frequency doubling, the transducer is polarized by applying in series with the signal a d.c. voltage of about 1 kV. The polar diagram of the transducer is such that the width of the main beam is sufficient to fill the whole of the phase screen.

The phase screen consists of a stream of hot turbulent air rising from a heating element made up of six 1 kW radiator bars, connected end to end, and placed near the floor of the room, about 1 m below the direct path between the transducer and detecting system. The screen thus formed is relatively thin, and about 2 m wide.

The signals are picked up on a pair of condenser microphones, which were

ULTRASONIC DIFFRACTION

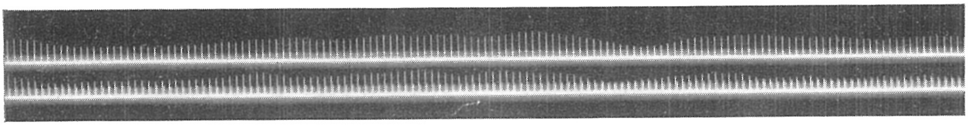

Fig. 1.—Typical microphone record, taken with a separation of 14 cm, showing time delay between signals at upper and lower microphones. The lower trace shows an unwanted reflected pulse, separated from the main signal by a small time delay.

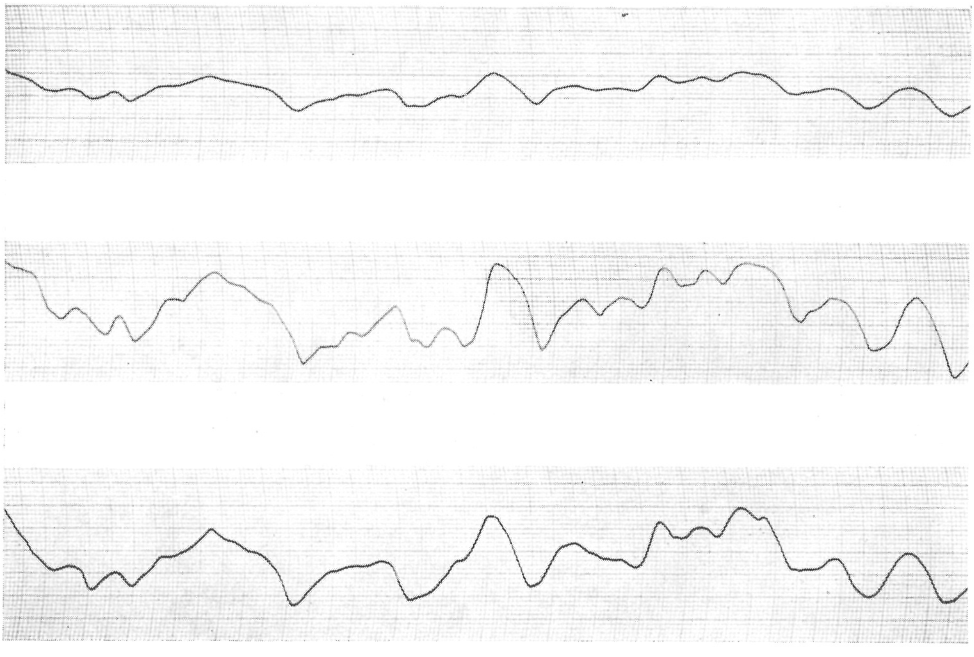

Fig. 2.—Typical thermistor records, taken with vertical and horizontal separations of 6 cm. The time delay between the upper two traces, corresponding to the vertical pair, is clearly visible.

constructed especially for this experiment, since the requirements of small size (to allow investigation of fine structure in the diffraction pattern) and adequate sensitivity in the ultrasonic region could not be met by any readily available commercial units. The microphone cell and a preamplifier are mounted together in the form of a probe, approximately 1 in. diameter and 5 in. long. The diaphragm of the microphone is about $0 \cdot 6$ in. diameter.

The signal from each microphone passes via coaxial cable to a tuned amplifier. Since the microphones are sensitive to a wide band of frequencies, the passband of the amplifiers is limited to a bandwidth of about 4 kc/s, in order to discriminate against random room noises, and thus improve the signal-to-noise ratio. After amplification, the pulses are rectified and filtered, and the two resulting pulse envelopes are applied to a double-beam oscilloscope, with the time base disconnected. Thus, there are two vertical lines on the oscilloscope screen, one above the other, the height of each being proportional to the instantaneous signal strength at the corresponding microphone. When the oscilloscope screen is photographed on a continuously moving film, a record such as that shown in Plate 1, Figure 1, results. The microphones in this particular example were separated vertically by 14 cm. A time delay can clearly be seen between the two signal envelopes; variations at the lower microphone are repeated later at the upper one.

Direct measurements of the temperature fluctuations in the phase screen are carried out by using three thermistors, arranged in a right-angled triangle, with equal separations of 6 cm between vertical and horizontal pairs. The thermistors are supplied with current from a high impedance source, and the voltage developed across each, which is proportional to its resistance, is amplified. The three resulting signals are recorded on a three-channel pen recorder. A typical record is shown in Plate 1, Figure 2. The time delay between the traces corresponding to the vertical pair is clearly visible; there is no obvious time delay between the horizontal pair.

Since it is desirable that the thermistors should follow the temperature fluctuations in the phase screen as faithfully as possible, it is necessary that their time constant should be short compared with the period of the fastest fluctuations present in the screen. The time constant of the thermistors used was estimated by measuring the thermistor's response to a step temperature change. This was produced by heating the thermistor electrically, and then plotting on a recorder chart the cooling curve when the current was switched off. By this means it was found that the time constant of the smallest available thermistor was about $1 \cdot 5$ sec, which was too long. The time constant was therefore shortened by reducing the size, and hence the thermal capacity, of the thermistor bead by careful grinding of material from the bead. In this way the time constant was reduced to $0 \cdot 12$ sec. However, evidence discussed in Section IV shows that a still shorter time constant would have been desirable for completely satisfactory results.

IV. The Increase of Scintillation Depth with Distance from the Screen

The way in which amplitude fluctuations appear and increase as the wave travels beyond a phase screen has been considered by Mercier (1962), Wagner (1962), and others. It is necessary to define a suitable measure of the amount of fluctuation or scintillation depth, and different authors have used different definitions. The four most frequently used measures, which may be denoted by S_1, S_2, S_3, and S_4, are defined as follows:

$$S_1 = (1/\bar{R})\overline{|R-\bar{R}|}\,, \tag{8}$$

$$S_2 = (1/\bar{R})\{\overline{(R-\bar{R})^2}\}^{\frac{1}{2}}\,, \tag{9}$$

$$S_3 = (1/\bar{R}^2)\overline{|R^2-\bar{R}^2|}\,, \tag{10}$$

$$S_4 = (1/\bar{R}^2)\{\overline{(R^2-\bar{R}^2)^2}\}^{\frac{1}{2}}\,, \tag{11}$$

where R is the amplitude and the bars denote time averages.

The relation between these measures will depend on the probability distribution of R, which is in general not known. Briggs and Parkin (1963) have investigated the relation empirically for satellite scintillations, showing that in this case S_1, S_2, S_3,

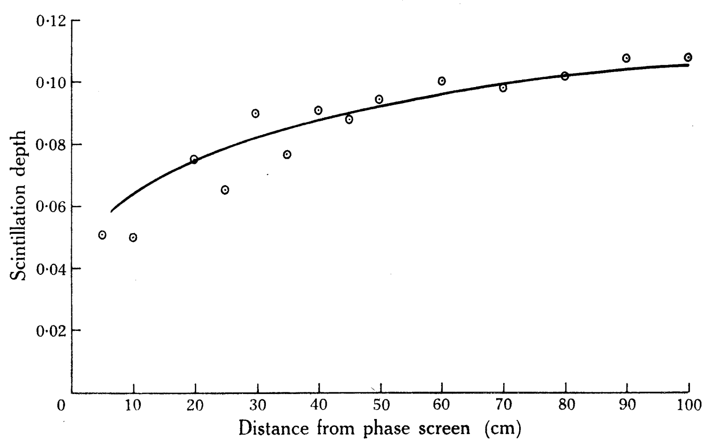

Fig. 2.—Experimentally determined points and theoretical curve (full line) for increase of scintillation depth S_2 with distance from the phase screen.

and S_4 are proportional to each other, with proportionality constants close to those predicted theoretically for a Rayleigh distribution of R, namely,

$$S_1 = 0\cdot 42S_4, \qquad S_2 = 0\cdot 52S_4, \qquad S_3 = 0\cdot 73S_4.$$

Wagner (1962) has shown that the manner in which the amplitude fluctuations build up with increasing distance beyond the phase screen depends markedly on the form of the spatial correlogram of the phase fluctuations produced by the screen. This may be denoted by $\rho_\phi(\xi)$, where $\rho_\phi(\xi)$ is the correlation between the phases at two points separated by a distance ξ in the wavefront as it emerges from the screen. In Figure 2 the points are the experimentally measured values of S_2, and these are compared with Wagner's theoretical curve (full line) for an exponential correlogram

$$\rho_\phi(\xi) = \exp(-|\xi/\xi_0|)\,.$$

This was found to fit the measured variations more closely than the other forms considered by Wagner. The parameters used were the scale factor ξ_0, as determined in Section VI, and the r.m.s. phase variation ϕ_0, which was chosen for best fit of the theoretical curve to the observed points. (The actual values used were $\xi_0 = 5 \cdot 6$ cm and $\phi_0 = 0 \cdot 21$ rad.)

The assumption of an approximately exponential form for $\rho_\phi(\xi)$ is supported by the form of the temporal correlogram of the diffraction pattern formed at some distance beyond the screen. This is also approximately exponential (see Fig. 6). As the time variations arise mainly from a pattern drifting past the microphones, the spatial correlogram must be of similar form. On the other hand, the correlogram determined by the thermistors situated in the phase screen itself appears to be closer to a Gaussian than to an exponential curve. The explanation of this discrepancy appears to lie in the limited frequency response of the thermistors. The time constant of the thermistors used was 0·12 sec, indicating a loss of response for frequencies above about 1·5 c/s. Frequencies higher than this can, however, be seen on typical records and it is clear that these must have been severely attenuated. The effect of the high frequency attenuation on the correlograms will be most marked near the origin, tending to make an exponential curve into a more rounded, Gaussian-like form.

In principle, it should be possible to correct the correlograms for the limited frequency response of the thermistors. Fourier transformation of the correlogram would yield the power frequency spectrum of the corresponding signal, and this would be passed through a mathematical "filter", whose bandpass would be the inverse of the squared frequency response curve of the thermistor, obtained by Fourier analysis of the thermistor's response to a step function. Further Fourier transformation would then yield the corrected correlogram. Attempts to correct the correlograms in this way, however, proved unsuccessful, due to the high level of noise present in the thermistor records. The noise becomes proportionately more important at the higher frequencies, where the original signal has suffered attenuation, and leads to distortion of the corrected correlogram.

As a result, an alternative method was adopted. Exponential correlograms were assumed, with various scale factors, and the correlograms that would have been obtained if the corresponding signal had been passed through a filter with bandpass equal to that of the thermistors were calculated. These were then compared with the actual correlograms obtained experimentally.

It was found that when an exponential function, with scale such that it fell to 0·5 of its initial value in a time of 0·17 sec, was modified by a filter with time constant of 0·12 sec, a function was obtained that resembled the observed autocorrelogram quite closely for correlation values greater than 0·5. Figure 3 shows both the assumed exponential and the derived Gaussian-like functions, plotted with the autocorrelograms observed at each of the three thermistors. The divergence between the curves for smaller correlation values can be accounted for by sampling errors, and is not inconsistent with the assumption that all the curves belong to the same population (Awe 1964).

A further point that should be stressed is that the autocorrelograms referred to in Wagner's theory relate to the phase variations in the wavefront emerging from the screen, whereas the autocorrelograms obtained in the present experiment are of the temperature fluctuations producing the phase variation. Theory shows that these are of the same form for the particular case of a Gaussian autocorrelogram, but this will not necessarily be true in the general case. The results of the experiment do, however, indicate that the autocorrelograms of the fluctuations in the phase screen are closer to an exponential than to a Gaussian form.

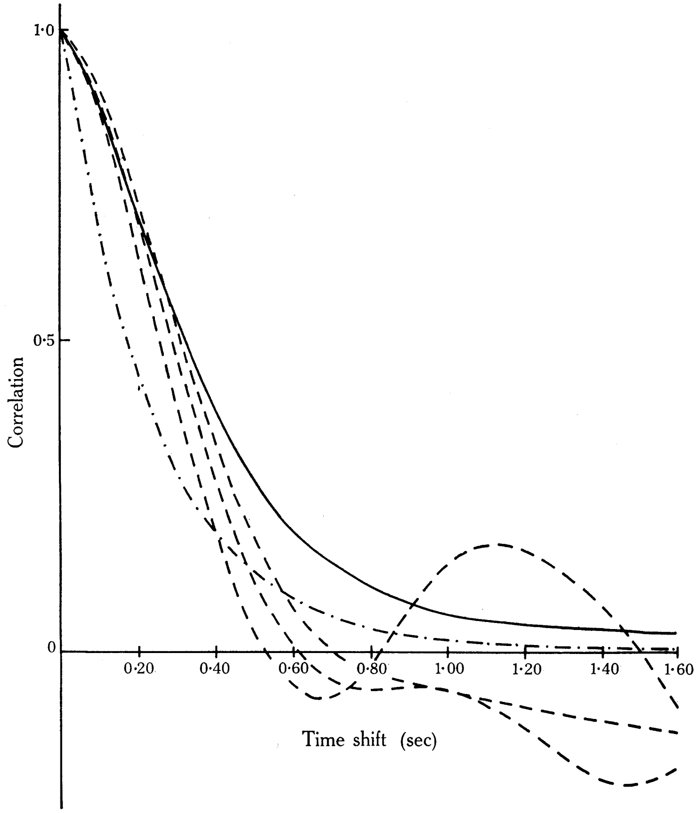

Fig. 3.—Theoretical and observed autocorrelograms of temperature fluctuations in the phase screen; ---- observed correlograms, -·-·-· assumed exponential function, ——— corrected exponential function.

In order to investigate the relationships between S_1, S_2, S_3, and S_4, these quantities were calculated at different distances from the phase screen. As the scintillation depth is changing, the probability distribution of the amplitude R is also changing, and this provides a test of the dependence of the relationships on the probability distribution.

In Figure 4 the measured values of S_1, S_2, and S_3 have been plotted against S_4. It can be seen that the indices are very nearly proportional to one another. Furthermore, the slopes of the lines of best fit drawn through the observed points produce the following relationships:

$$S_1 = 0\cdot 40 S_4, \qquad S_2 = 0\cdot 50 S_4, \qquad S_3 = 0\cdot 79 S_4.$$

The constants of proportionality are quite close to those obtained for a Rayleigh distribution, indicating that the relationships between the various scintillation indices are not strongly dependent on the probability distribution of R.

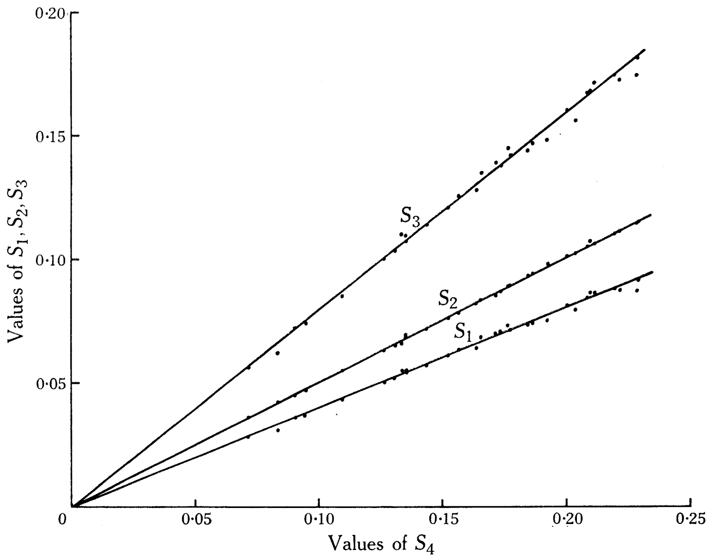

Fig. 4.—Relationships between scintillation indices S_1, S_2, S_3, and S_4.

V. The Air Velocity determined from Direct Measurements and from Measurements on the Diffraction Pattern

In view of the extensive use of measurements of this type in the investigation of ionospheric drifts, it is particularly important to estimate the accuracy that can be attained, bearing in mind that observations are limited to a few points in the diffraction pattern.

As the rising air stream is turbulent, it would be expected to change in form as it drifts. The records obtained by two detectors in line with the direction of drift will then not be exactly similar after removing the time shift. This applies both to the thermistors situated in the air stream itself and to the microphones that sample the diffraction pattern. Under these conditions, an accurate value of the drift velocity cannot be obtained by dividing the detector separation by the time shift that gives maximum correlation between the records. It is necessary to evaluate the auto- and cross-correlograms of the records, and to use a method of calculation described by Briggs, Phillips, and Shinn (1950). This method is briefly described below; for the justification of the method, reference must be made to the original paper.

Suppose two detectors are situated in line with the direction of drift. Then the auto- and cross-correlograms of the corresponding records may be expected to have the general form shown in Figure 5. The autocorrelogram is obtained by evaluating the correlation between one record shifted relative to itself by a time τ. It may be denoted by $\rho(0,\tau)$, to indicate that there is zero space separation and a time shift τ. The functions $\rho(0,\tau)$ produced from each detector should be found to be the same, within the errors of measurement, and the curve used may be the average of the two. The cross-correlogram is obtained by evaluating the correlation between the two records for various values of time shift τ. It may be denoted by $\rho(\xi_0,\tau)$ where ξ_0 is the spatial separation of the two detectors. If $\rho(\xi_0,\tau)$ shows a maximum for some value of τ, this indicates a tendency for the pattern to drift past the two detectors, so that values obtained at one detector tend to be repeated at the other after a certain time delay.

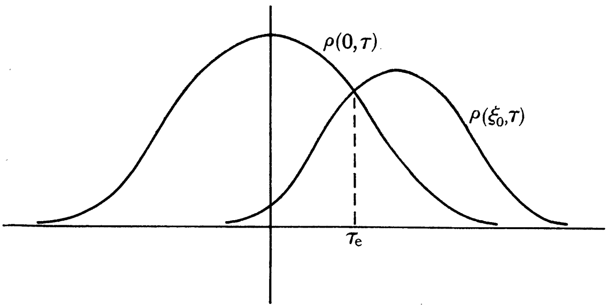

Fig. 5.—Idealized auto- and cross-correlograms, showing the method of finding τ_e used in velocity determinations.

Briggs, Phillips, and Shinn (1950) describe various methods by which the true velocity of drift may be determined from these functions. The simplest method is to determine the time delay τ_e such that $\rho(0,\tau_e) = \rho(\xi_0,\tau_e)$, i.e. the crossing point of the two curves. The true velocity of drift is then given by

$$V = \xi_0/2\tau_e\,. \tag{12}$$

This procedure can be applied to the thermistor records and to the records produced by the microphones and is illustrated in Figure 6 with typical auto- and cross-correlograms obtained from the microphone records. Since the thermistor records have not been corrected for the frequency response of the thermistors, these measurements will only refer to the motion of the larger irregularities, but this is sufficient, if it is assumed that both the large- and small-scale components of the phase screen move with the same mean velocity.

The velocities calculated were the apparent velocity, obtained by dividing the detector separation by the time shift that gave maximum correlation between records, and the corrected velocity, calculated by means of equation (12). The results are set out in Table 1. It can be seen that the two measures of the velocity differ significantly, indicating that there is considerable evolution in the phase screen structure as it moves.

The interesting question then is how the velocity of the diffraction pattern is related to the velocity of the phase screen itself. In considering this it is important to note that diffraction theory predicts a "geometrical magnification" of the velocity when a point source is used, so that if V_S is the velocity of the phase screen, then the velocity of the diffraction pattern is given by

$$V=\left(\frac{z_2+z_1}{z_1}\right)V_S, \tag{13}$$

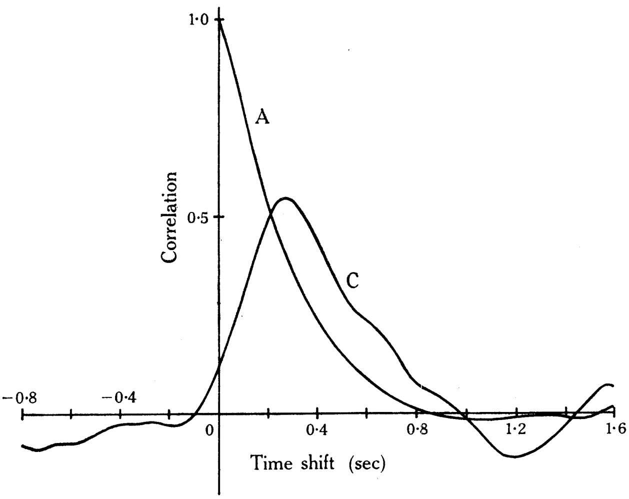

Fig. 6.—Typical microphone correlograms taken at 3 m from the phase screen; A autocorrelogram, C cross-correlogram.

TABLE 1

VELOCITY OF PHASE SCREEN AND DIFFRACTION PATTERN

Measurement	Apparent Velocity (cm/sec)	Corrected Velocity (cm/sec)	Predicted Velocity of Phase Screen (cm/sec)
Diffraction pattern at 3 m	50 ± 6	32 ± 4	25 ± 3
Diffraction pattern at 1 m	40 ± 6	29 ± 4	26 ± 4
Temp. fluctuations in screen	39 ± 6	27 ± 4	—

where z_1 is the distance from the phase screen to the source and z_2 is the distance from the phase screen to the plane in which the pattern is observed (Fig. 1). The last column of Table 1 shows the velocity of the phase screen, calculated from the velocity of the diffraction pattern by allowing for the appropriate geometrical magnification. It can be seen that the velocity of the phase screen deduced by this means is in each case in good agreement with the value obtained by direct measurement.

VI. The Scale Size determined from Direct Measurements and from Measurements on the Diffraction Pattern

In deducing the scale sizes of ionospheric irregularities, many authors have assumed, at any rate as an order of magnitude, that the scale size is the same as that in the diffraction pattern on the ground, apart from the geometrical magnification $(z_1+z_2)/z_1$ that would be expected when a point source is used (see Fig. 1). Diffraction theory shows that this is approximately true, provided that the phase deviations introduced by the phase screen are less than 1 rad.

In order to make the discussion more precise, it is necessary to introduce a quantitative definition of scale size, and in most existing theories this is done by assuming a definite form for the autocorrelogram of the irregularities introduced by the phase screen. Usually a Gaussian function is chosen, as this introduces considerable theoretical simplifications, but there is, of course, no guarantee that this will apply in practice, and it is precisely the restrictive nature of this type of assumption that is to be investigated in the present experiments.

With the assumption that the screen produces a phase variation having a Gaussian spatial correlogram

$$\rho_\phi(r) = \exp(-r^2/r_0^2)\,,$$

Bowhill (1961) has shown that the amplitude pattern formed far from the screen also has a Gaussian correlogram, and, for plane-wave illumination, has the same value of the parameter r_0. Closer to the screen the spatial correlogram of the amplitude pattern is not Gaussian. If a Gaussian function is chosen with the same curvature at the origin as the actual correlogram very close to the screen, its parameter is $r_0/\sqrt{3}$. In other words, the scale size of the pattern close to the screen is considerably less than for the pattern formed at some distance beyond the screen.

As explained in Section IV, there is, in general, no simple relationship between the correlogram for the phase variations across the emerging wavefront and the correlogram of the temperature fluctuations as determined by the thermistors. However, in the ionospheric problem it is the analogue of the latter that we wish to deduce from observations of the diffraction pattern. We therefore proceed now to compare the thermistor and the microphone observations, without making any specific assumption about the form of the correlograms. For this purpose we introduce a scale size as the distance at which the correlation falls to $0 \cdot 5$, regardless of the actual shape of the correlogram. We denote this distance by l for the amplitude pattern, and by l_T for the temperature fluctuations; our object is to study the relation between l and l_T.

The results of the measurements are summarized in Table 2. Three separate methods were used to estimate the scale sizes from the observed records. In the first and most direct method, the separation of the detectors is varied and the correlation at each separation is calculated. The points at which the resultant vertical and horizontal correlograms fall to $0 \cdot 5$ give the respective scale sizes. Results obtained by this method are listed in the second column of the table. Only the average of the vertical and horizontal scale sizes at each distance is presented, since no significant difference was detected between these. The phase screen thus appears to be isotropic.

The remaining two methods make use of a fixed detector separation, and are often used in ionospheric work. The autocorrelogram $\rho(0,\tau)$ obtained from one of the detectors (or the average from all three) is assumed to have the same form as the spatial correlogram. The scale factor needed to convert the temporal to the spatial correlogram is then obtained either from the cross-correlation value between the respective pairs of detectors at the separation used, or from the known velocity of drift of the pattern. The last method can obviously only be used to estimate the scale size in the direction of drift. The results obtained by these two methods are listed in the third and fourth columns of Table 2. The average scale size at each distance is listed in the fifth column.

TABLE 2

SCALE SIZES OF IRREGULARITIES IN PHASE SCREEN (l_T) AND DIFFRACTION PATTERN (l)

Measurement	Scale Sizes				
	Variable Spacing Method (cm)	Correlation Scaling Method (cm)	Velocity Scaling Method (cm)	Average (cm)	Calculated from l_T (cm)
l at 3 m	$7\cdot0\pm0\cdot5$	$7\cdot4\pm1$	$7\cdot0\pm0\cdot9$	$7\cdot1$	$7\cdot3$
l at 1 m	$3\cdot5\pm0\cdot8$	$3\cdot2\pm1$	$5\cdot5\pm0\cdot8$	$4\cdot1$	$6\cdot2$
l_T	—	$6\cdot6\pm1\cdot5$	$4\cdot6\pm1$	$5\cdot6$	—

It can be seen that all three methods give essentially the same results. The measurements made at 1 m from the phase screen are inherently less reliable than those made at 3 m, since the smaller scintillation depth at this distance allows noise to play a proportionately more important part. The discrepancy between the results obtained using thermistors can be accounted for by the indirect method used to correct the thermistor correlograms. The variable spacing method was not used to measure the scale size of the irregularities in the air stream because of the delicate nature of the modified thermistors.

Using the average value of $5\cdot6$ cm for l_T, the scale size of the irregularities in the air stream, the scale sizes expected for the diffraction pattern at 1 and 3 m from the phase screen were calculated using the geometrical magnification factors, and these are listed in the last column of Table 2. It can be seen that at 1 m from the phase screen, where the irregularities have not fully built up, the actual scale size is much smaller than predicted, as expected from diffraction theory. At 3 m from the screen, however, the observed and predicted scale sizes are in good agreement, indicating that the fluctuations are fully developed.

It appears that reasonable estimates of the sizes of the irregularities of refractive index can be made from observations of the diffraction pattern, in spite of the uncertainties in the form of the correlograms, provided that the observations are made at a large distance from the phase screen, where the amplitude fluctuations have fully developed.

VII. Conclusions

It has been shown that ultrasonic waves diffracted by a rising stream of hot turbulent air provide a useful analogy to various effects observed with radio waves in the ionosphere. As the air stream is accessible to direct observation by thermistors, various features of irregular diffraction may be investigated quantitatively. The results so far obtained are broadly in agreement with theory, within the limitations to which the theories may be expected to apply to an actual experiment.

In particular, it has been shown that both the scale size and the velocity of the irregularities in the "phase screen" can be determined from observations of the amplitude fluctuations, or scintillations, at a large distance from the screen. The increase of scintillation depth beyond the screen appears to agree with theory if the phase fluctuations across the wavefront are assumed to have an exponential correlogram. The commonly used measures of scintillation depth have been shown to be very nearly proportional to one another.

The inadequate frequency response of the thermistors used to investigate the phase screen has been an experimental limitation. A more satisfactory temperature sensor would allow further aspects of irregular diffraction to be investigated. In particular, diffraction by "thick" phase screens and the increase of scintillation depth in waves of two different frequencies transmitted simultaneously through the screen are two problems that could be investigated.

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IX. References

Awe, O. (1964).—*J. atmos. terr. Phys.* **26**, 1257.
Bowhill, S. A. (1961).—*J. Res. natn. Bur. Stand.* D **65**, 275.
Briggs, B. H., and Parkin, I. A. (1963).—*J. atmos. terr. Phys.* **25**, 339.
Briggs, B. H., Phillips, G. J., and Shinn, D. H. (1950).—*Proc. phys. Soc.* B **63**, 106.
Hewish, A. (1951).—*Proc. R. Soc.* A. **209**, 81.
James, P. W. (1962).—*J. atmos. terr. Phys.* **24**, 237.
Krautkrämer, J. (1950).—*Arch. elekt. Übertr.* **4**, 133.
Mercier, R. P. (1962).—*Proc. Camb. phil. Soc. math. phys. Sci.* **58**, 382.
Mitra, S. N. (1949).—*Proc. Instn elect. Engrs*, Pt 3, **96**, 441.
Ratcliffe, J. A. (1956).—*Rep. Prog. Phys.* **19**, 188.
Spencer, M. (1955).—*Proc. phys. Soc.* B **68**, 493.
Wagner, L. S. (1962).—*J. geophys. Res.* **67**, 4195.