SHORT COMMUNICATIONS

THE LEVEL STRUCTURE OF MASS-5 AND MASS-3 NUCLEI*

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The known energy levels of the mass-5 nuclei (Lauritsen and Ajzenberg-Selove 1966) consist of a broad ground state and first excited state, which have been clearly demonstrated to have a structure ($^4_2\mathrm{He}+1p_{3/2}$ nucleon) and ($^4_2\mathrm{He}+1p_{1/2}$ nucleon) respectively, and a pair of states at high excitation whose parity is even. These latter states are at 16·7 MeV ($\frac{3}{2}^+$) and 20 MeV approximately ($\frac{3}{2}^+$ or $\frac{5}{2}^+$). The reduced widths of these states indicate that they have a nature consistent with the cluster model prediction, namely, that of a deuteron–triton configuration in ^{5}He. In the single-particle picture, this corresponds to the excitation of a single particle from the $1s$ shell into the $1p$ shell.

Thus it is interesting to see if the features of the ^{5}He level spectrum can be given on the basis of a single-particle calculation. The purpose of this note is to report the results of such an attempt, using the particle–hole model. In this framework, the α-particle is taken as the physical vacuum and even parity states of ^{5}He are envisaged either as excitation of the $1p_{3/2}$ neutron to the $2s$–$1d$ shell or as excitation of a $1s_{1/2}$ nucleon to the $1p$ shell; that is, the excitations are characterized either as single-particle excitations or as 2-particle–1-hole excitations respectively. The even parity states of mass-3 nuclei may be treated by considering the excitation of a $1s_{1/2}$ particle to the $1p$ shell, giving a 2-hole–1-particle state.

One is encouraged to pursue the study of these light nuclei by the particle–hole technique because of the apparent success of this model in describing the odd parity states of the mass-4 nuclei (Spicer 1966). Again, it should be emphasized that the numerical results obtained can be taken only as a guide, because the role of the residual interaction is such an important one in calculations on such light nuclei. The small number of particles present makes it difficult to envisage a self-consistent field in this context, and the zero-range force used to approximate the residual interaction is necessarily a poor approximation. However, the symmetry properties of the states given by the calculation should be similar to the results that may be expected from a more careful calculation. Thus the compositions of the eigenstates obtained are of as much interest as their energies.

The formulation of the energy matrix for the 2-particle–1-hole (or 2-hole–1-particle) case follows the general procedure set out in Eisenberg, Spicer, and Rose (1965). A more detailed discussion of it will be given separately (Fraser, Garnsworthy, and Spicer, to be published). Spurious states were eliminated by first constructing them explicitly, according to the prescription of Baranger and Lee (1961), and then using the Schmidt orthogonalization technique to remove them.

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The unperturbed level spectrum used was in part the same as that used for the study of ^{4}He (Spicer 1966). Other necessary information was the energy to be assigned to the $1d_{5/2}$, $2s_{1/2}$, and $1d_{3/2}$ states. Only a lower limit could be placed on the unperturbed energy of these states, and this was obtained by requiring that all the calculated states below 24 MeV excitation have small or zero single-particle components in their wave functions. This is required by the results reported by Hoop and Barschall (1966) on the elastic scattering of neutrons by α-particles. This experiment demonstrates that there is no large, even parity, single-particle phase shift for ^{5}He levels below 24 MeV. The $\frac{3}{2}^+;\frac{1}{2}$ state at 16·7 MeV has, according to Hoop and Barschall, a small $1d_{3/2}$ single-particle component in its wave function, and this is borne out qualitatively by the calculation. In the results reported here, all three

TABLE 1

EXCITED STATES OF MASS-5 NUCLEI

Theoretical		Experimental		Theoretical	
Energy (MeV)	Spin, Parity, and Isospin	Energy (MeV)	Spin, Parity, and Isospin	Energy (MeV)	Spin, Parity, and Isospin
15·7	$\frac{3}{2}^+;\ \frac{1}{2}$	16·67	$\frac{3}{2}^+;\ \frac{1}{2}$	29·9	$\frac{5}{2}^+;\ \frac{3}{2}$
20·7	$\frac{1}{2}^+;\ \frac{1}{2}$	20	$\frac{3}{2}^+$ or $\frac{5}{2}^+$	30·5	$\frac{3}{2}^+;\ \frac{3}{2}$
22·7	$\frac{7}{2}^+;\ \frac{1}{2}$			30·8	$\frac{5}{2}^+;\ \frac{3}{2}$
24·4	$\frac{5}{2}^+;\ \frac{1}{2}$			31·1	$\frac{3}{2}^+;\ \frac{1}{2}$
25·1	$\frac{1}{2}^+;\ \frac{1}{2}$			32·7	$\frac{3}{2}^+;\ \frac{3}{2}$
25·6	$\frac{5}{2}^+;\ \frac{1}{2}$			33·3	$\frac{1}{2}^+;\ \frac{1}{2}$
26·1	$\frac{1}{2}^+;\ \frac{3}{2}$			34·3	$\frac{1}{2}^+;\ \frac{1}{2}$
26·6	$\frac{3}{2}^+;\ \frac{1}{2}$			35·8	$\frac{1}{2}^+;\ \frac{3}{2}$
27·5	$\frac{5}{2}^+;\ \frac{1}{2}$			36·6	$\frac{3}{2}^+;\ \frac{3}{2}$
27·8	$\frac{3}{2}^+;\ \frac{1}{2}$			38·3	$\frac{5}{2}^+;\ \frac{1}{2}$
29·1	$\frac{3}{2}^+;\ \frac{1}{2}$			40·0	$\frac{1}{2}^+;\ \frac{3}{2}$
29·4	$\frac{1}{2}^+;\ \frac{1}{2}$			41·4	$\frac{3}{2}^+;\ \frac{1}{2}$

$1d$–$2s$ states have been replaced at an unperturbed energy 45 MeV above the $1p_{3/2}$ state, this being the smallest value that satisfied the condition required by Hoop and Barschall's experiment. It may be noted that the effect of the residual interaction in this calculation was to push the $2s_{1/2}$ single-particle strength to an energy well below its unperturbed value, and, even with a $2s_{1/2}$—$1p_{3/2}$ energy difference of 45 MeV, a substantial fraction of the $2s_{1/2}$ single-particle strength resides in the $\frac{1}{2}^+$ state predicted at 25·1 MeV. It is of interest to note that the s-wave phase shift given by Hoop and Barschall (1966) was rising markedly towards the end of the energy range over which they measured.

It is also interesting to observe that the $2s$–$1d$ unperturbed levels considered are some 48 MeV above the zero of energy, i.e. into the continuum. It is perfectly reasonable to query the implicit assumption that energy levels can exist at these energies; our only justification for including them is that their presence is required by experimental results.

The results for the mass-5 nuclei are shown in Table 1, for the choice of the free parameter $V_0\alpha^3/4\pi = 15$ MeV, where V_0 is the strength of the two-body interaction, taken to have zero range, and a is the harmonic oscillator range parameter. The spin dependence of the two-nucleon interaction was taken to be the Soper mixture ($\eta = 0\cdot 135$).

The value of the residual interaction is lower than that used for ^{4}He (Spicer 1966), and this may be explained by the considerations that the particle–particle matrix elements (which occur in the 2-particle–1-hole calculation, but not in the 1-particle–1-hole calculation) are important, and that the zero-range force may overestimate these in comparison with the particle–hole matrix elements. This choice of parameter places the lowest even parity excited state at too low an energy (15·7 MeV, compared with the experimental value of 16·70 MeV in ^{5}He). However, the energy of this state is quite sensitive to the strength of the residual interaction and a small adjustment of about 1 MeV to the latter will give the energy of this state correctly. The major configurations represented (j–j coupling basis) in the wave function of this state are $(1s_{1/2})^{-1}\{(1p_{3/2})^2, J_p = 1^+, T_p = 0\}$ and $(1s_{1/2})^{-1}\{(1p_{3/2}\,1p_{1/2}), J_p = 1^+, T_p = 0\}$. These are consistent with the cluster-model view of this state, which is seen as a ^{3}H cluster plus a ^{2}H cluster in a relative s-wave.

The second excited state found is at about 20 MeV, and has been assigned spin $\frac{3}{2}^+$ or $\frac{5}{2}^+$ (Lauritsen and Ajzenberg-Selove 1966). The assignment comes from the observations by Tombrello (personal communication) that this state in ^{5}Li is formed in the elastic scattering of deuterons by ^{3}He, and that the scattering resonance is due to d-wave scattering. This conclusion alone gives the possible assignments $\frac{1}{2}^+$, $\frac{3}{2}^+$, $\frac{5}{2}^+$, $\frac{7}{2}^+$ for this state. The $\frac{1}{2}^+$ assignment is ruled out by the results of Bame and Perry (1957) on the angular distributions of neutrons from the reaction ^{3}H(d, n)^{4}He. These data indicate that the coefficient of the $P_2(\cos\theta)$ term in the angular distribution is resonant for a deuteron bombarding energy corresponding to an excitation energy of 20 MeV in ^{5}He. This cannot be the case if the spin of the intermediate state is $\frac{1}{2}^+$. It seems that the possibility of a $\frac{7}{2}^+$ assignment for this state cannot be ruled out. If this latter assignment is allowed, one would match this state with the one calculated to be at 22·7 MeV. There are insufficient experimental data to decide the energy of the $\frac{1}{2}^+;\frac{1}{2}$ state calculated to be at 20·7 MeV.

Another point of interest arises from the energies of the calculated $T = \frac{3}{2}$ states, and these results impinge on the question of the stability or otherwise of $^5_1$H. The ground state of $^5_1$H is expected to be an even parity state, the parity being given by the odd $1s_{1/2}$ proton. The neutrons would be expected to form the configuration $(1s_{1/2})^2(1p_{3/2})^2$. The first $T = \frac{3}{2}$ state is calculated to be at an excitation of 26·1 MeV above the ground state of $^5_2$He (or $^5_3$Li). Since $^5_2$He has a mass excess of 11·45 MeV, this implies a mass excess of 37·5 MeV for $^5_1$H. This in turn means that $^5_1$H would be unstable against disintegration into ^{3}H+2n, the disintegration energy being 6·3 MeV. These values are rather larger than those given by Lauritsen and Ajzenberg-Selove (1966). However, the data available to them were sufficient to allow only the estimation of a lower limit for a mass excess of $^5_1$H. It seems certain, from these results, that $^5_1$H is unstable against decay into ^{3}H+2n by some 6 MeV.

In a similar calculation, with a similar residual interaction strength ($V_0 a^3/4\pi = 15$ MeV), the possible states of the mass-3 nuclei were calculated. These are displayed in Table 2. No excited states of ^3_1H or ^3_2He are known experimentally. The present model gives three (probably broad) excited states for ^3_2He (or ^3_1H) in the excitation energy region 16–19 MeV. It is known that it is difficult to fit the radius and binding energy of ^3_2He (or ^3_1H) and its photodisintegration cross section simultaneously; however, calculations carried out so far have not treated carefully such details as tensor forces and final state interactions (Fetisov, Gorbunov, and Varfolomeev 1965). The existence of the predicted states may be of relevance in the solution of this difficulty.

TABLE 2

EXCITED STATES OF MASS-3 NUCLEI

Energy (MeV)	16·7	17·9	18·9	22·3	23·1	26·4	29·0
Spin, parity, and isospin	$\frac{5}{2}^-;\ \frac{1}{2}$	$\frac{3}{2}^-;\ \frac{1}{2}$	$\frac{1}{2}^-;\ \frac{1}{2}$	$\frac{3}{2}^-;\ \frac{1}{2}$	$\frac{1}{2}^-;\ \frac{1}{2}$	$\frac{3}{2}^-;\ \frac{3}{2}$	$\frac{1}{2}^-;\ \frac{3}{2}$

Of more quantitative interest is the prediction of the calculation regarding the stability of the trineutron (^3n). The structure of the trineutron is expected to be $(1s_{1/2})^2(1p_{3/2})$ and thus to be, in its lowest state, a $J=\frac{3}{2}^-$; $T=\frac{3}{2}$ state. The lowest $T=\frac{3}{2}$ state predicted is indeed of this nature, and has a calculated energy 26·4 MeV above the $T=\frac{1}{2}$ ground state. Since ^3_2He has a mass excess of 14·93 MeV, this leads to a mass excess for the trineutron of 41·3 MeV, and an available energy for disintegration into three neutrons of 17 MeV. This value is in agreement with the report of Jänecke, and in disagreement with the remarks of Donovan, both of which follow a paper by Wilson (1964) on the status of the few-nucleon problem.

This discussion on the heavy isotopes of light elements may be completed by using the results of Spicer (1966) to examine the stability of ^4_1H. Its most stable state is expected to have a proton in the $1s_{1/2}$ state and a neutron configuration $(1s_{1/2})^2(1p_{3/2})$, so that the total angular momentum is expected to be 1^- or 2^-. Spicer's calculation predicts a $2^-;1$ state at 25·2 MeV, and Tombrello's (1966) examination of available experimental data places this state at 24·0 MeV. If one takes this latter value, ^4_1H is unstable against decay into ^3H+n by approximately 3·4 MeV.

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