ON WIND-INDUCED FLOWS IN CLOSED CHANNELS

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Summary

The mean velocity and shear-stress profiles for a wind-induced flow in a closed channel with constant surface slope are first derived using a turbulent viscosity constructed by dimensional arguments. Using a perturbation analysis based on the fact that the surface slope is small, these results are extended to the case where the surface slope, and hence flow conditions generally, are allowed to vary in a downwind direction. Explicit results are obtained for the velocity and shear-stress profiles and also for the surface slope as a function of distance downwind. The results agree quite well with experiment, although better agreement would probably be obtained by using a more elaborate turbulent viscosity.

I. Introduction

This paper deals with the action of an air stream, which is in general turbulent, on the free surface of a body of water of finite depth and large surface extent and refers particularly to the effects of wind blowing over a lake or large water storage. As is well known, the action of the air-flow is twofold—waves are generated on the surface of the lake, and mass transport is induced within the water body through the action of surface stress which, because of the presence of the boundaries of the lake, produces a set-up, or rise in level, to the leeward side. The precise nature of the interaction between the air and the water is largely unknown, and no exact mathematical analysis can be given. Not only are the surface processes largely unknown, but also little can be said about the flow in the water, except that it will be turbulent in general and that the turbulence will decay with depth except perhaps near the bottom, where the boundary layer on the bottom, even if it starts off laminar, must separate as it approaches the windward shore. It is clear, then, that any analysis must be an approximate one based upon plausible assumptions, the value of the analysis resting on a comparison of the predicted results with the experimental ones.

An approximate analysis of the response of a lake or closed channel to surface stresses generated by the wind has been given by Keulegan (1951) and further analysed by Ursell (1956). Keulegan considers a region near the centre of the channel well away from the ends and, assuming the flow to be laminar, derives an expression for the velocity profile using the flow equation, neglecting inertial effects. He then uses an integrated form of the momentum equation to derive an expression for the surface slope appropriate to this flow. By introducing a "surface coefficient" Φ_s he generalizes the expression for the surface slope to include the case of turbulent flow and is able to obtain reasonable agreement with experiment. He also discusses the effects of waves by splitting the set-up into two parts, one due to the waves and the other due to surface stress.

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Fitzgerald and Mansfield (1965) have analysed the response of a closed channel to surface stresses for the case of turbulent flow in the water. Assuming that the air produces a (constant) shear stress at the water surface, considered smooth, they construct a turbulent velocity profile by assuming the flow across a vertical section of the channel to be divided into two parts about the point of zero shear stress over each of which an essentially logarithmic profile operates. They then choose a number of adjustable constants so that the total profile is continuous and so that the condition of zero total mass flow across the section is satisfied; they obtain good agreement with the observed turbulent flow profiles (see Fitzgerald and Mansfield 1965, Fig. 3).

In this paper we present an analysis, based essentially on a perturbation process, for a wind-induced flow in a long channel with small surface slope. Neglecting the direct effects of surface roughness and using fairly simple assumptions about the structure of the turbulent flow, we first derive a velocity profile in a long channel appropriate to constant surface slope. We then introduce a change of variable in this profile and, making certain assumptions about the similarity properties of a flow with nonconstant surface slope, expand the various flow functions in terms of the surface slope, neglecting second- and higher-order terms. We are then able to discuss in terms of this first-order theory the variation in flow properties in both a horizontal and vertical direction and, in particular, are able to determine the surface height as a function of distance along the surface. This is in contradistinction to both Fitzgerald and Mansfield and Keulegan who assume the flow conditions to be constant with distance. The presence of the channel ends implies a variation in flow quantities in a downwind direction, although in the region considered, well away from the ends, this variation will be small. Nevertheless, the evaluation of this variation is not without interest.

The turbulent profile derived here is based on very simple assumptions; a more elaborate profile would probably yield better results. As it is, the results compare favourably with experiment, and, since the purpose of the present paper is to indicate how one might calculate the surface set-up in a consistent and more rigorous manner than has been done hitherto, the profile is sufficient for this purpose. The properties of more elaborate profiles in relation to surface set-up will be discussed in a later paper.

II. Flow Equations

In this section we derive the particular form of the flow equations we require in our analysis. Consider the action of a flow of air over the surface of a body of water of finite depth. The water bottom is flat and horizontal and the length of the water body is large compared to its depth so that effects due to the end-walls need not be taken into account explicitly. The air flow, which may be laminar or turbulent, will produce a set-up, or change in height with distance in the water surface, owing to the action of the end-walls and, further, will induce in the water an overall circulatory flow which itself may be laminar or turbulent. We assume that this circulatory flow lies, in a mean sense at least, in the vertical plane. Taking the origin of coordinates at a point on the bottom, well away from the end-walls, let x denote distance in a downwind direction measured from this origin and z denote distance vertically upward. In general, we write the equation to the surface as $z = H(x, t)$, where t is the time coordinate, but as we consider flows that are steady, at least in a mean sense, we write $z = H(x)$, and this defines the mean height of the water surface.

If the flow in the water be steady in the mean, the flow equations take the form

$$\rho\left(u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z}\right) = \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xz}}{\partial z}, \tag{2.1}$$

$$\rho\left(u\frac{\partial w}{\partial x} + w\frac{\partial w}{\partial z}\right) = \frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{zz}}{\partial z} - \rho g, \tag{2.2}$$

$$\rho\left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right) = 0, \tag{2.3}$$

assuming no mean motion in the y direction normal to x and z; u and w are velocity components and σ_{xx}, σ_{xz}, σ_{zz} are stress components, all these quantities being time-averaged; the density ρ is constant.

Introducing an eddy viscosity ϵ we write

$$\sigma_{xx} = -p + 2\epsilon\frac{\partial u}{\partial x}, \qquad \sigma_{xz} = \epsilon\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right), \qquad \sigma_{zz} = -p + 2\epsilon\frac{\partial w}{\partial z}, \tag{2.4}$$

assuming the Reynolds number to be large enough so that the effects of molecular viscosity can be neglected; p is the pressure.

If we multiply (2.3) by u, add to (2.1), and integrate with respect to z over $\{0, H(x)\}$ for fixed x we obtain an integrated form of the x-momentum equation. Thus,

$$2\rho\int_{z=0}^{z=H(x)} u\frac{\partial u}{\partial x}\,\mathrm{d}z + \rho uw\Big|_{z=0}^{z=H(x)} = \int_{z=0}^{z=H(x)} \frac{\partial \sigma_{xx}}{\partial x}\,\mathrm{d}z + \sigma_{xz}\Big|_{z=0}^{z=H(x)}.$$

Applying the boundary conditions $w = 0$ when $z = 0$, $x > 0$, and $w = w_s(x)$ when $z = H(x)$, $x > 0$, we have

$$2\rho\int_{z=0}^{z=H(x)} u\frac{\partial u}{\partial x}\,\mathrm{d}z + \rho\, u_s(x)\, w_s(x) = \int_{z=0}^{z=H(x)} \frac{\partial \sigma_{xx}}{\partial x}\,\mathrm{d}z + \tau_s(x) - \tau_0(x). \tag{2.5}$$

Here $u_s(x)$ and $w_s(x)$ are the values of u and w at the surface and $\tau_s(x)$, $\tau_0(x)$ are the shear stresses at the surface and bottom respectively. In general, these quantities are all functions of x. A similar equation can be obtained for the z-momentum equation.

We now make the assumption that the surface slope is small compared to unity and that the surface curvature is of a higher order than the slope. That is, we assume that the square of $H'(x)$ and higher powers and $H''(x)$ are negligible compared with $H'(x)$ itself. An immediate consequence of this, which derives from the z-momentum equation (2.2), is that we may assume the pressure to be hydrostatic and write

$$p = \rho g\{H(x) - z\} + p_a, \tag{2.6}$$

where p_a is the external air pressure.

A further consequence is that the boundary condition of zero mass flux at the outer surface takes the form $w_s(x) = u_s(x)\,H'(x)$, and so the surface velocity is $u_s(x)$ to the first order of small quantities. It is interesting to note that if the normal velocity is not zero so that mass transfer is taking place, for example by evaporation, then the surface slope $H'(x)$ and the mass flux could be of the same order of magnitude, and, consequently, the whole flow pattern would be affected by the evaporation. This aspect will be discussed in a later paper.

III. Velocity Profile in a Closed Channel with Constant Surface Slope

We now derive a velocity profile appropriate to wind-induced flow in a long closed channel with a smooth surface and constant small surface slope.

Assuming the pressure hydrostatic we have $\partial\tau/\partial z$ constant throughout this flow. It follows, then, that the shear stress τ is a linear function of the depth, and we write

$$\tau = \tau_s - (\tau_s - \tau_0)\,(1 - z/H),$$

or

$$\tau = \tau_s\{1 - \alpha(1 - z/H)\}, \tag{3.1}$$

where τ_s and τ_0 are the (constant) mean shear stresses at the surface and bottom respectively, $\alpha = 1 - \tau_0/\tau_s$, and H is the depth of the flow.

It is convenient at this stage to introduce the surface and bottom friction speeds u_{*s}, u_{*0}, defined by $u_{*s} = (\tau_s/\rho)^{\frac{1}{2}}$ and $u_{*0} = (-\tau_0/\rho)^{\frac{1}{2}}$, the minus sign appearing since τ_0 is negative.

To construct the velocity profile we use the fact that the bottom flow is a wall flow and that the flow near the surface is not unlike it. Neglecting the effects of molecular viscosity and assuming that the eddy viscosity near the surface is determined by the flow conditions near the surface, we may use a dimensional argument to assert that

$$\epsilon = k_s\,u_{*s}(H - z). \tag{3.2}$$

It would be better to use u_* in place of u_{*s}, but for the present u_{*s} will suffice.

Near the bottom we will have

$$\epsilon = k_0\,u_{*0}\,z; \tag{3.3}$$

k_s and k_0 are constants, the "mixing constants" for the surface and bottom flows respectively; k_0 will be von Kármán's constant for wall flow with a value of around 0·4, and k_s will be an analogous quantity for the surface flow; k_s will not necessarily have the same value as k_0, since k_0 refers to flow near a rigid wall and k_s refers to flow near a free surface.

Combining (3.2) and (3.3) and introducing a linear factor, we may approximate the eddy viscosity in the entire flow by

$$\epsilon/(k_0\,u_{*0}\,H) = (z/H)\,(1 - z/H)\,(1 + \beta\,z/H). \tag{3.4}$$

In order that (3.4) may reduce to (3.2) when $z \sim H$ and to (3.3) when $z \sim 0$, we take

$$1 + \beta = (k_s/k_0)\,(u_{*s}/u_{*0}) = (k_s/k_0)\,(\alpha - 1)^{-\frac{1}{2}}.$$

To determine the velocity profile we use (3.1) and (3.4) in

$$\tau = \rho\epsilon(\mathrm{d}u/\mathrm{d}z), \tag{3.5}$$

u being the mean horizontal velocity. Setting $\zeta = z/H$ we have

$$\frac{\mathrm{d}(u/u_{*\mathrm{s}})}{\mathrm{d}\zeta} = \frac{(1-\alpha)\beta}{k_{\mathrm{s}}} \frac{1+\gamma\zeta}{\zeta(1-\zeta)(1+\beta\zeta)}, \tag{3.6}$$

where $\gamma = \alpha/(1-\alpha)$.

Integrating (3.6) we have

$$\frac{u}{u_{*\mathrm{s}}} = G(\zeta)+C, \tag{3.7}$$

where
$$G(\zeta) = \frac{(1-\alpha)\beta}{k_{\mathrm{s}}}\left\{\ln\zeta - \frac{1+\gamma}{1+\beta}\ln(1-\zeta) - \frac{\beta-\gamma}{1+\beta}\ln(1+\beta\zeta)\right\} \tag{3.8}$$

and C is a constant of integration.

On examining (3.6), (3.8) we see that the velocity profile becomes logarithmically infinite at $\zeta = 0$ and $\zeta = 1$, this being a consequence of the nonvanishing of the shear stress at these points. It follows that the flow near the surface and near the bottom must go over into viscous sublayers, for only then can the boundary condition $u = 0$ at the bottom and $u = u_{\mathrm{s}}$ at the surface be satisfied. Neglecting the transition regions between the linear velocity distribution region in the viscous sublayer and the logarithmic distribution valid outside, we write

$$\frac{u}{u_{*\mathrm{s}}} = \begin{cases} -(\alpha-1)^{\frac{1}{2}} R_* \zeta & R_*\zeta < C_0, \quad (3.9\mathrm{a}) \\ \dfrac{R_{\mathrm{s}}(\alpha-1)^{\frac{1}{2}}}{R_*} - \dfrac{R_*(1-\zeta)}{(1-\alpha)^{\frac{1}{2}}} & \dfrac{R_*(1-\zeta)}{(1-\alpha)^{\frac{1}{2}}} < C_{\mathrm{s}}, \quad (3.9\mathrm{b}) \\ G(\zeta)+C & \text{otherwise.} \quad (3.9\mathrm{c}) \end{cases}$$

The Reynolds numbers R_*, R_{s} are defined by $R_* = u_{*0}H/\nu$, $R_{\mathrm{s}} = u_{\mathrm{s}}H/\nu$, where u_{s} is the mean surface velocity and ν is the kinematic viscosity. The constants C_0, C_{s} are assumed known and probably have values in the range 5–20.

To fix the numerical content of the velocity profile (3.9), we prescribe the surface Reynolds number R_{s} and determine the constants α, R_*, and C so that $u(\zeta)$ is everywhere continuous in (0,1) and that the total flow across the channel is zero. The constants k_0, k_{s} are assumed known. The friction speeds $u_{*\mathrm{s}}$, u_{*0} and also u_{s} cannot be computed until the depth H and the kinematic viscosity ν are known.

Setting $\zeta_0 = C_0/R_*$ and $\zeta_{\mathrm{s}} = 1-C_{\mathrm{s}}(\alpha-1)^{\frac{1}{2}}/R_*$, the two continuity conditions yield

$$-C_0(\alpha-1)^{\frac{1}{2}} = G(\zeta_0)+C, \tag{3.10}$$

$$(\alpha-1)^{\frac{1}{2}}\frac{R_{\mathrm{s}}}{R_*} - C_{\mathrm{s}} = G(\zeta_{\mathrm{s}})+C, \tag{3.11}$$

while the condition of zero total mass flow yields

$$\mathscr{M}(\zeta_0) - \mathscr{M}(\zeta_s) + C\left\{1 - \frac{C_s(\alpha-1)^{\frac{1}{2}}+C_0}{R_*}\right\} + C_s\,R_s\,\frac{\alpha-1}{R_*^2} - (C_0^2+C_s^2)\frac{(\alpha-1)^{\frac{1}{2}}}{2R_*} = 0, \tag{3.12}$$

where
$$\mathscr{M}(\zeta) = \frac{1-\alpha}{k_s}\left\{\mathscr{F}(\zeta) + \frac{1+\gamma}{1+\beta}\mathscr{F}(1-\zeta) - \frac{\beta-\gamma}{\beta(1+\beta)}\mathscr{F}(1+\beta\zeta)\right\}, \tag{3.13}$$

with $\mathscr{F}(\zeta) = \zeta(\ln\zeta - 1)$.

Combining (3.10) and (3.11) we have

$$(R_s/R_* + C_0)(\alpha-1)^{\frac{1}{2}} - C_s = G(\zeta_s) - G(\zeta_0). \tag{3.14}$$

This is an equation relating R_* and α.

The solution method for obtaining R_*, α, C is as follows. For given R_s choose a value of α and solve (3.14) for R_* and then obtain C from (3.10). Substitute for α, R_*, C in (3.12) and adjust α until (3.12) is satisfied to within some specified error.

The assumptions used in deriving this profile are all open to question. First and foremost, the interaction of the water flow with the air flow should be expressed explicitly instead of just through R_s. Further, the form of the eddy viscosity can be questioned. However, the agreement with experiment is reasonable and the deduced profile sufficient for the present purpose. As far as the eddy viscosity is concerned it would be better to use completely local quantities in its derivation, and we will examine in a later paper profiles deduced in this way.

IV. Velocity Profile for Nonconstant Surface Slope

As we have already remarked, the presence of the ends of the channel must certainly cause a variation in flow quantities in a downwind direction, although in the region of the channel well away from the ends this variation will be small. Nevertheless, it is of interest, and necessary for completeness, to extend the above analysis to the case of varying flow conditions, the approximations in the structure of the turbulence through the eddy viscosity notwithstanding.

To obtain the velocity profile in an extended closed channel with nonconstant slope we use the profile for constant surface slope and, in effect, make a similarity transformation using the fact that the surface slope is small ($\ll 1$).

For constant surface slope we have

$$\frac{u(\zeta)}{u_{*s}} = F(\zeta) \qquad 0<\zeta<1, \tag{4.1}$$

where $F(\zeta)$ is given by (3.9).

To obtain the velocity profile for nonconstant surface slope we write

$$\frac{u(\xi,\zeta)}{u_{*s}^0} = \frac{1}{\mathscr{H}(\xi)}\;F\{\zeta/\mathscr{H}(\xi)\} \qquad \xi>0,\quad 0<\zeta<1, \tag{4.2}$$

where $\xi = x/H^0$ and $\mathscr{H}(\xi) = H(x)/H^0$, with $H^0 = H(0)$, the depth at the origin; u^0_{*s} is the surface friction speed at $x = 0$. When $\xi = 0$, (4.2) reduces to (4.1) and we see that once the flow at one point in the flow, namely $\xi = 0$, is prescribed the flow in the downwind region $\xi > 0$ is immediately known through the (similarity) transformation implied in (4.2). The derivation of this transformation rests largely on dimensional grounds, although it is probably the simplest transformation one could use in this way to account for the varying surface slope, especially since the surface slope is to be small. It also enables us to obtain an explicit expression for the surface slope, and thence the channel depth, as a function of distance downwind.

For the vertical component of velocity w we have from the continuity equation

$$\frac{w(\xi, \zeta)}{u^0_{*s}} = \frac{\mathscr{H}'(\xi)}{\{\mathscr{H}(\xi)\}^2}\,\zeta\, F\{\zeta/\mathscr{H}(\xi)\}. \tag{4.3}$$

In order to obtain an expression for the mean shear stress $\tau(\xi, \zeta)$, and so obtain a flow system as simply self-consistent as possible, we write for the eddy viscosity

$$\frac{\epsilon}{k_0\, u^0_{*0}\, H^0} = \mathscr{H}(\xi)\, \chi(1-\chi)(1+\beta\chi) \tag{4.4}$$

on setting $\chi = \zeta/\mathscr{H}(\xi)$. For $\tau(\xi, \zeta)$ we then have

$$\tau(\xi, \zeta) = \frac{\rho\epsilon u^0_{*s}}{H^0}\frac{\partial}{\partial\zeta}\left[\frac{u(\xi, \zeta)}{u^0_{*s}}\right] \tag{4.5}$$

or

$$\tau(\xi, \zeta) = k_0\, \rho\, u^0_{*s}\, u^0_{*0}\, \mathscr{N}(\chi)/\mathscr{H}(\xi), \tag{4.6}$$

where $\quad \mathscr{N}(\chi) = \chi(1-\chi)(1+\beta\chi)\, G'(\chi)$. $\quad$ (4.7)

For the surface and bottom stresses we have

$$\tau_s(\xi) = \tau^0_s/\mathscr{H}(\xi), \qquad \tau_0(\xi) = \tau^0_0/\mathscr{H}(\xi). \tag{4.8}$$

The parameter β is a function of the friction speeds at $\xi = 0$ and so should also be involved in the similarity transformation, but we shall neglect this particularly since k_0 and k_s also probably should vary.

In the next section we derive the form of the variation in surface slope and channel depth with distance downstream.

V. Form of the Water Surface

The integrated form of the equation for rate of change of momentum in the x direction is given by (2.5). In terms of the variables ξ, ζ, the first convection term T_1 on the left-hand side of (2.5) may be written

$$T_1 = 2\rho(u^0_{*s})^2\, \mathscr{H}(\xi) \int_{\chi=0}^{\chi=1} \frac{u(\xi, \zeta)}{u^0_{*s}}\frac{\partial}{\partial\zeta}\left[\frac{u(\xi, \zeta)}{u^0_{*s}}\right] d\chi,$$

which, in virtue of (4.2), (4.3), becomes

$$T_1 = -2\rho(u^0_{*s})^2\, \delta\, \mathscr{H}'(\xi)/\mathscr{H}(\xi), \tag{5.1}$$

where
$$\delta = \int_{\chi=0}^{\chi=1} F(\chi)\{F(\chi)+\chi F'(\chi)\}\,\mathrm{d}\chi. \tag{5.2}$$

The second convection term T_2 yields
$$T_2 = \rho(u^0_{*s})^2\{F(1)\}^2\mathscr{H}'(\xi)/\{\mathscr{H}(\xi)\}^3. \tag{5.3}$$

The term involving the normal stress in the present approximation of small surface slope reduces to
$$-\int_0^{H(x)}\frac{\partial p}{\partial x}\,\mathrm{d}z,$$
which yields
$$T_3 = -\rho g H^0\mathscr{H}(\xi)\mathscr{H}'(\xi). \tag{5.4}$$

For the shear stress terms we have
$$T_4 = k_0 u^0_{*s}\frac{(1-\alpha)\beta}{k_s}\frac{\gamma}{\mathscr{H}(\xi)}. \tag{5.5}$$

Combining these four terms in the sum $T_1+T_2+T_3+T_4 = 0$ we have, finally,
$$-2\delta\frac{\mathscr{H}'(\xi)}{\mathscr{H}(\xi)}+\{F(1)\}^2\frac{\mathscr{H}'(\xi)}{\{\mathscr{H}(\xi)\}^3}+\frac{1}{f^2}\mathscr{H}(\xi)\mathscr{H}'(\xi)-\frac{\alpha}{\mathscr{H}(\xi)} = 0, \tag{5.6}$$
where
$$F(1) = R_s - 1/R_*.$$

On rearrangement (5.6) yields
$$\mathscr{H}'(\xi) = \frac{\{\mathscr{H}(\xi)\}^2}{\delta_1\{\mathscr{H}(\xi)\}^4+\delta_2\{\mathscr{H}(\xi)\}^2+\delta_3}, \tag{5.7}$$
where
$$\delta_1 = 1/\alpha f^2, \qquad \delta_2 = -2\delta/\alpha, \qquad \delta_3 = \{F(1)^2\}/\alpha, \tag{5.8}$$
$f = (gH^0)^{-\frac{1}{2}}u^0_{*s}$ being the Froude number. Equation (5.7) is a differential equation in $\mathscr{H}(\xi)$ for $\xi > 0$ and is to be solved with the initial condition $\mathscr{H}(0) = 1$.

From (5.6) we have immediately that the surface slope at $\xi = 0$ is given by
$$\mathscr{H}'(0) = \frac{1}{\delta_1+\delta_2+\delta_3}. \tag{5.9}$$

Separating the variables in (5.6) and integrating, we have
$$\frac{\delta_1\{\mathscr{H}(\xi)\}^3}{3}+\delta_2\mathscr{H}(\xi)-\frac{\delta_3}{\mathscr{H}(\xi)}-\left(\frac{\delta_1}{3}+\delta_2-\delta_3\right) = \xi, \tag{5.10}$$
so that $\mathscr{H}(\xi)$ satisfies the fourth-degree equation
$$a_0\{\mathscr{H}(\xi)\}^4+a_2\{\mathscr{H}(\xi)\}^2+a_3\mathscr{H}(\xi)+a_4 = 0, \tag{5.11}$$
where
$$a_0 = \tfrac{1}{3}\delta_1, \qquad a_2 = \delta_2, \qquad a_3 = -(\tfrac{1}{3}\delta_1+\delta_2-\delta_3-\xi), \qquad a_4 = \delta_3. \tag{5.12}$$

VI. A Particular Example

We applied the present analysis to some results of Fitzgerald and Mansfield (1965), such as presented in their Figure 3. For these data $u_s^0 = 9 \cdot 1$ (cm/sec) and $H^0 = 13 \cdot 3$ (cm), so that $R_s = 12\,000$. Assuming $k_0 = 0 \cdot 41$ and $C_0 = 5$ we have the detailed results set out in Table 1.

Table 1

CHARACTERISTICS OF SOME THEORETICAL VELOCITY PROFILES

k_s	C_s	α	R_{*0}	R_{*s}	f	δ_1	δ_2	δ_3
0·4	5	1·1067	243·7	745·9	0·00491	$3 \cdot 746 \times 10^4$	1469·6	237·9
1·0	5	1·0649	257·9	1012·6	0·00667	$2 \cdot 113 \times 10^4$	3075·5	134·2
1·5	5	1·0589	257·1	1162·5	0·00765	$1 \cdot 628 \times 10^4$	3168·4	103·2
1·0	10	1·0714	201·3	753·2	0·00460	$3 \cdot 796 \times 10^4$	6722·0	241·0

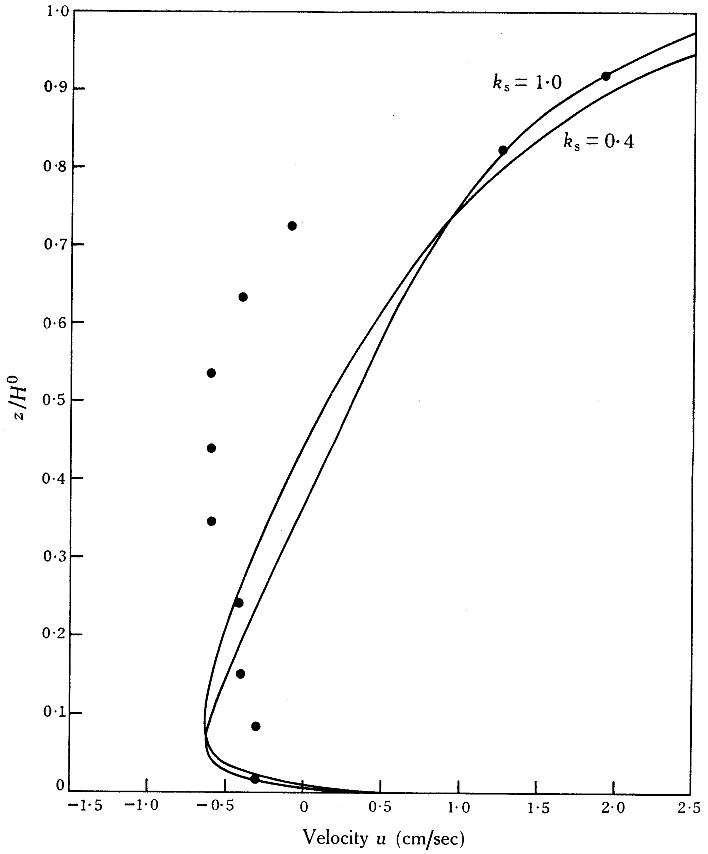

Fig. 1.—Comparison of experimental data (●) with theoretical velocity profiles for $u_s^0 = 9 \cdot 1$ cm/sec, $H^0 = 13 \cdot 3$ cm and $k_0 = 0 \cdot 41$, $C_0 = 5$ and $C_s = 5$.

The profiles for $k_s = 0 \cdot 4$, $C_s = 5$, and $k_s = 1 \cdot 0$, $C_s = 5$ are shown in Figure 1. The agreement is good near the bottom and near the surface, and in the central region is acceptable for the moment, since the experimental error at these speeds is $\pm 0 \cdot 3$ cm/sec. However, it is clear that in the central flow region a defect profile is required to describe the flow adequately. The computed velocity profile of Fitzgerald and Mansfield has this behaviour near the central region on account of the purely logarithmic profiles used, but of course the present profile is not a logarithmic one in the usual sense except near the bottom and near the surface.

The computed values of the surface set-up, defined by $S(x) = \{H(x) - H(0)\}/H(0)$, are quite distinct in the two cases—for $k_s = 1 \cdot 0$ and $x = 1$ cm, $S = 4 \cdot 1 \times 10^{-4}$, while for $k_s = 0 \cdot 1$ and $x = 1$ cm, $S = 1 \cdot 0 \times 10^{-4}$. There are no experimental results to check these.

It is clear that there is little to distinguish the various values of k_s, save that values near unity would appear to be more appropriate than smaller ones; that is, it appears that k_s is probably larger than k_0. This is not surprising, since k_s refers to the flow near a free surface while k_0 refers to flow near a rigid wall.

On examining the constants δ_1, δ_2, δ_3 we see that δ_1 and δ_2 are both large compared to δ_3 although δ_1 is always greater than δ_2. From the definition of these quantities given in (5.8) we see that for small Froude number, as for instance in shallow channels, the balance of forces is essentially between the surface stresses and the inertial forces, although it is of course the Froude effect that produces the set-up in this case. If the Froude number is large, i.e. for a deeper channel with the same surface friction speed, the balance includes the effects of surface flow.

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