

AN INVESTIGATION OF INTERFERENCE AND RESONANCE ASYMMETRY EFFECTS IN THE THEORY OF NEUTRON ABSORPTION IN REACTORS

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Summary

The nuclear effects that lead to asymmetric behaviour of neutron resonance cross sections are investigated and general expressions for cross sections developed. These are shown to be of a form suitable for calculation of escape probabilities, and the functions that arise from them are tabulated.

I. INTRODUCTION

In the standard A.A.E.C. treatment of resonance absorption in reactors, developed by Cohen and Goldstein (1962), and modified by McKay, Keane, and Pollard (1965), the basic resonance cross section shape is always taken to be the symmetric Breit-Wigner form. In the present paper, we extend the work of Rothenstein (1958) on interference scattering to cover all types of interference processes, as well as other factors that may lead to resonance asymmetry.

The types of asymmetry are first reviewed and classified in the standard Doppler-broadened cross section formulation. The resulting expressions are incorporated into the most accurate form to date of the effective resonance integral, and the corrected functions that develop are tabulated. It is found that, in practical applications, interference processes lead to deviations from symmetric resonance behaviour that are less than the usual errors in resonance parameters and those inherent in the resonance theory approximations.

II. RESONANCE ASYMMETRY

The asymmetries of a resonance can be placed in two broad categories:

- (a) that due to asymmetrizing energy-dependent factors, and
- (b) that due to interference between levels.

(a) *Energy-dependent Factors*

The (n, γ) cross section in the vicinity of a level of orbital angular momentum l may be written

$$\sigma_c(E) = 4\pi(\lambda/2\pi)^2 \frac{\Gamma_n^l \Gamma_\gamma}{\Gamma^2} \frac{1}{1+x^2}, \quad (1)$$

where $\lambda/2\pi$ is the neutron wavelength; $\Gamma_n^l = \gamma_n^l \theta_l E^{\frac{1}{2}}$; γ_n^l is the reduced neutron width, which is constant; θ_l is the angular momentum barrier transmission

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coefficient; Γ_γ is the radiation width, which is constant; E is the neutron energy; $x = (2/\Gamma)(E_r - E)$; E_r is the energy at exact resonance; $\Gamma = \Gamma_\gamma + \Gamma_f + \Gamma_n$; and Γ_f is the fission width, which is constant. Writing the energy dependence explicitly, we find

$$\sigma_c(E) = \frac{2 \cdot 608 \times 10^6 \gamma_n^l \theta_l(E) \Gamma_\gamma}{E^{\frac{1}{2}} \Gamma^2(E)} \frac{1}{1+x^2(E)}. \quad (2)$$

Expanding all terms in a Taylor series about $E = E_r$, except for the term $(E_r - E)$ in x , we obtain

$$\sigma_c(E) = \sigma_0 \frac{\Gamma_\gamma}{\Gamma(E_r)} \left\{ \frac{1}{1+x^2} + \frac{(a_E + a_l)x}{1+x^2} + O\left(\left(\frac{\Gamma x}{2E_r}\right)^2\right) \right\}, \quad (3)$$

where $a_E = -\Gamma/4E_r$ is the asymmetric contribution arising from the $E^{-\frac{1}{2}}$ factor in (2), and $a_l = \frac{1}{2}\Gamma\theta'_l(E_r)/\theta_l(E_r)$ is the asymmetry produced by the orbital angular momentum dependence.

Note that $a_0 = 0$, $\Gamma = \Gamma(E_r)$, and $\sigma_0 = 2 \cdot 608 \times 10^6 \Gamma_n^l(E_r)/\Gamma E_r$. The same parameters arise in the expression for the fission cross section.

(b) Interference Between Levels

In resonance scattering (Dresner 1960) an interference term in the cross section occurs between resonance and potential scattering such that the scattering cross section at 0°K is

$$\begin{aligned} \sigma_s(E) &= \sigma_0(E) \frac{\Gamma_n^l(E)}{\Gamma(E)} \frac{1}{1+x^2} + 2 \left(\frac{\sigma_0(E) \sigma_p \Gamma_n^l(E)}{\Gamma(E)} \right)^{\frac{1}{2}} \frac{x}{1+x^2} + \sigma_p \\ &\simeq \sigma_0 \left\{ \frac{\Gamma_n^l(E_r)}{\Gamma(1+x^2)} + (b_l + b_i) \frac{x}{1+x^2} + O\left(\left(\frac{\Gamma x}{2E_r}\right)^2\right) \right\} + \sigma_p, \end{aligned} \quad (4)$$

where

$$b_l = \Gamma_n^l(E_r) \theta'_l(E_r) / \theta_l(E_r) \quad (b_0 = 0),$$

$$b_i = 2 \left(\frac{\sigma_p \Gamma_n^l(E_r)}{\sigma_0 \Gamma} \right)^{\frac{1}{2}},$$

and

$$\sigma_r(E) = 4\pi(\lambda/2\pi)^2 \Gamma_n^l(E) / \Gamma(E).$$

Furthermore, Cook (1966) has shown that the multilevel interference contribution to the i th resonance can be partly included in an asymmetry coefficient a_{fi} given by

$$a_{fi} = \sum_{j \neq i} S_j \left(\frac{\sigma_{0i} \sigma_{0j} \Gamma_{fi} \Gamma_{fj}}{\Gamma_i \Gamma_j} \right)^{\frac{1}{2}} \frac{\Gamma_j(E_i - E_j)}{(E_i - E_j)^2 + \frac{1}{4}(\Gamma_i + \Gamma_j)^2} \quad (5)$$

and partly by modifying σ_{0i} with a factor

$$\mu_i = 1 + \sum_{j \neq i} S_j \left(\frac{\sigma_{0j} \Gamma_{fj} \Gamma_i}{\sigma_{0i} \Gamma_{fi} \Gamma_j} \right)^{\frac{1}{2}} \frac{\Gamma_j}{2} \frac{\Gamma_i + \Gamma_j}{(E_i - E_j)^2 + \frac{1}{4}(\Gamma_i + \Gamma_j)^2}, \quad (6)$$

with $S_j = \pm 1$, the sign of which must be determined from experiment. The sums over j in (5) and (6) represent those over all adjacent levels. The fission cross section therefore becomes

$$\sigma_f(E) \simeq \sigma_0 \frac{\Gamma_f}{\Gamma} \left(\frac{\mu}{1+x^2} + (a_E + a_{fi}) \frac{x}{1+x^2} \right). \quad (7)$$

The total cross section becomes

$$\begin{aligned} \sigma_t(E) &= \sigma_c(E) + \sigma_f(E) + \sigma_s(E) \\ &= \sigma_0 \left\{ \frac{1}{1+x^2} \left(1 - (1-\mu) \frac{\Gamma_f}{\Gamma} \right) + a \frac{x}{1+x^2} \right\} + \sigma_p, \end{aligned} \quad (8)$$

where

$$a = \{(\Gamma_f + \Gamma_\gamma)/\Gamma\}(a_E + a_i) + a_{fi} + b_i + b_i$$

is the asymmetry parameter describing the net effect upon the resonance of all partial asymmetries.

III. EFFECT OF ASYMMETRIES IN RESONANCE ABSORPTION THEORY

The method of McKay, Keane, and Pollard (1965) for calculating the resonance escape probability in a reactor is an improvement of the Hill and Schaefer (1962) analysis. In this revised method, one defines the value of p such that $0 \leq p \leq 1$ by putting

$$p = \exp(-\rho I / \xi \sigma_M) = \text{the escape probability}, \quad (9)$$

where σ_M is the total scattering cross section per absorber atom, $\rho = 2/(1+p)$, and

$$I = (\Gamma_a \sigma_0 / E_r) f_\lambda J(t, f_\lambda) \quad (10)$$

is the effective resonance integral, with

$$f_\lambda = \frac{\sigma_{M\lambda}}{\sigma_0} \frac{\Gamma}{\Gamma_a + \lambda_K \Gamma_n}, \quad (11)$$

$$\sigma_{M\lambda} = \sum_{S=1}^K \lambda_S \sigma_{MS},$$

$$\sigma_{MS} = N_S \sigma_{sS} / N_K,$$

$$\lambda_S = 1 - \alpha_S \frac{C_1 + C_\lambda}{x_S} \tan^{-1} \left(\frac{x_S}{C_1 + f_S C_\lambda} \right),$$

$$x_S = (2E_r / \Gamma)(1 - \alpha_S),$$

$$C_\lambda = (1 + 1/f_\lambda)^{\frac{1}{2}},$$

and

$$\Gamma_a = \Gamma_\gamma + \Gamma_f.$$

Here K refers to the resonance nuclide, and S ranges from 1 to $K-1$, being the number of moderator nuclides. Also α_S is the neutron fractional energy loss per collision with nuclide S , N_S is the individual nuclide concentration, λ_S is the Cohen

TABLE 1
RELATIVE DEVIATION IN J

a	Relative Deviation ΔJ for					
	$\beta = 10^{-3}$	10^{-2}	10^{-1}	1	10	10^2
$t = 10^4$						
0.001	1.89(-6)*	8.94(-8)	—	—	—	—
0.005	4.89(-5)	1.79(-6)	—	—	—	—
0.010	1.95(-4)	7.23(-6)	1.04(-7)	—	—	—
0.050	4.95(-3)	1.82(-4)	2.67(-6)	1.49(-8)	—	—
0.100	—	7.27(-4)	1.09(-5)	1.04(-7)	—	—
0.500	—	—	2.72(-4)	2.80(-6)	4.47(-8)	—
1.000	—	—	—	1.14(-5)	1.49(-7)	—
5.000	—	—	—	—	2.89(-6)	4.47(-8)
10.000	—	—	—	—	1.16(-5)	1.34(-7)
$t = 2.5 \times 10^3$						
0.001	4.35(-6)	2.53(-7)	—	—	—	—
0.005	1.11(-4)	5.51(-6)	8.94(-8)	—	—	—
0.010	4.45(-4)	2.17(-5)	4.47(-7)	—	—	—
0.050	1.15(-2)	5.41(-4)	1.12(-5)	5.96(-8)	—	—
0.100	—	2.17(-3)	4.50(-5)	4.32(-7)	—	—
0.500	—	—	1.13(-3)	1.26(-5)	1.49(-7)	—
1.000	—	—	—	5.08(-5)	4.77(-7)	—
5.000	—	—	—	—	1.30(-5)	1.49(-8)
10.000	—	—	—	—	5.16(-5)	3.73(-7)
$t = 4.0 \times 10^2$						
0.001	1.56(-5)	7.30(-7)	—	4.47(-8)	—	—
0.005	3.90(-4)	1.81(-5)	5.81(-7)	2.98(-8)	—	—
0.010	1.56(-3)	7.24(-5)	2.43(-6)	1.34(-7)	—	—
0.050	4.47(-2)	1.82(-3)	6.01(-5)	9.24(-7)	—	—
0.100	—	7.35(-3)	2.40(-4)	3.37(-6)	—	—
0.500	—	—	6.06(-3)	8.08(-5)	7.00(-7)	—
1.000	—	—	—	3.23(-4)	3.23(-6)	—
5.000	—	—	—	—	8.35(-5)	8.35(-7)
10.000	—	—	—	—	3.34(-4)	3.34(-6)
$t = 10^2$						
0.001	4.29(-5)	1.39(-6)	4.47(-8)	—	—	—
0.005	1.08(-3)	4.15(-5)	1.65(-6)	—	—	—
0.010	4.35(-3)	1.66(-4)	7.15(-6)	—	—	—
0.050	1.57(-1)	4.19(-3)	1.80(-4)	2.94(-6)	—	—
0.100	—	1.72(-2)	7.20(-4)	1.19(-5)	—	—
0.500	—	—	1.85(-2)	2.98(-4)	3.20(-6)	—
1.000	—	—	—	1.19(-3)	1.29(-5)	—
5.000	—	—	—	—	3.20(-4)	3.16(-6)
10.000	—	—	—	—	1.28(-3)	1.30(-5)

* Each number in parentheses is the common logarithm of the multiplier; 1.89(-6) means 1.89×10^{-6} .

TABLE 1 (Continued)

a	Relative Deviation ΔJ for					
	$\beta = 10^{-3}$	10^{-2}	10^{-1}	1	10	10^2
$t = 25$						
0.001	8.62(-5)	3.99(-6)	4.17(-7)	—	—	—
0.005	2.16(-3)	9.31(-5)	4.49(-6)	5.96(-8)	—	—
0.010	8.77(-3)	3.73(-4)	1.82(-5)	1.94(-7)	4.47(-8)	—
0.050	4.09(-1)	9.50(-3)	4.57(-4)	1.00(-5)	1.64(-7)	—
0.100	—	4.07(-2)	1.83(-3)	3.96(-5)	3.43(-7)	—
0.500	—	—	4.94(-2)	9.93(-4)	1.14(-5)	5.96(-8)
1.000	—	—	—	3.99(-3)	4.52(-5)	5.66(-7)
5.000	—	—	—	—	1.13(-3)	1.13(-5)
10.000	—	—	—	—	4.54(-3)	4.57(-5)
$t = 4$						
0.001	1.17(-4)	8.48(-6)	4.92(-7)	0.0	—	—
0.005	2.94(-3)	2.12(-4)	1.19(-5)	3.13(-7)	—	—
0.010	1.19(-2)	8.49(-4)	4.75(-5)	1.54(-6)	5.96(-8)	—
0.050	5.92(-1)	2.21(-2)	1.19(-3)	3.75(-5)	5.51(-7)	—
0.100	—	1.02(-1)	4.80(-3)	1.50(-4)	1.94(-6)	—
0.500	—	—	1.52(-1)	3.77(-3)	4.84(-5)	5.22(-7)
1.000	—	—	—	1.53(-2)	1.94(-4)	2.04(-6)
5.000	—	—	—	—	4.89(-3)	5.00(-5)
10.000	—	—	—	—	2.00(-2)	2.00(-4)
$t = 1$						
0.001	1.23(-4)	1.10(-5)	—	—	—	—
0.005	3.09(-2)	2.77(-4)	1.96(-5)	8.49(-7)	—	—
0.010	1.25(-2)	1.11(-3)	7.76(-5)	3.03(-6)	—	—
0.050	6.21(-1)	2.90(-2)	1.94(-3)	7.67(-5)	9.09(-7)	—
0.100	—	1.36(-1)	7.85(-3)	3.08(-4)	4.40(-6)	—
0.500	—	—	2.98(-1)	7.77(-3)	1.13(-4)	—
1.000	—	—	—	3.23(-2)	4.51(-4)	—
5.000	—	—	—	—	1.15(-2)	—
10.000	—	—	—	—	—	—
$t = 0.25$						
0.001	1.24(-4)	1.20(-5)	9.09(-7)	—	—	—
0.005	3.13(-3)	3.00(-4)	2.49(-5)	1.07(-6)	—	—
0.010	1.27(-2)	1.20(-3)	9.98(-5)	4.55(-6)	—	—
0.050	6.30(-1)	3.15(-2)	2.51(-3)	1.18(-4)	2.03(-6)	—
0.100	—	1.48(-1)	1.02(-2)	4.72(-4)	7.79(-6)	—
0.500	—	—	4.35(-1)	1.20(-2)	1.94(-4)	1.95(-6)
1.000	—	—	—	5.09(-2)	7.74(-4)	8.18(-6)
5.000	—	—	—	—	1.99(-2)	2.07(-4)
10.000	—	—	—	—	8.79(-2)	8.27(-4)
$t = 0.04$						
0.001	1.25(-4)	1.23(-5)	1.18(-6)	5.96(-8)	—	—
0.005	3.14(-3)	3.08(-4)	2.78(-5)	1.42(-6)	2.98(-8)	—
0.010	1.27(-2)	1.23(-3)	1.11(-4)	5.87(-6)	1.34(-7)	—
0.050	6.32(-1)	3.23(-2)	2.79(-3)	1.47(-4)	2.65(-6)	—
0.100	—	1.52(-1)	1.13(-2)	5.90(-4)	1.05(-5)	1.34(-7)
0.500	—	—	5.06(-1)	1.51(-2)	2.61(-4)	2.86(-6)
1.000	—	—	—	6.48(-2)	1.05(-3)	1.13(-5)
5.000	—	—	—	—	2.72(-2)	2.83(-4)
10.000	—	—	—	—	1.25(-1)	1.13(-3)

and Goldstein (1962) iterative parameter, and $t = 4E_r kT/A\Gamma^2$, with T the temperature in degrees absolute, k the Boltzmann constant, and A the mass number.

The function $J(t, \beta)$ occurring in (10) has the form

$$J(t, \beta) = \int_{-\infty}^{\infty} \psi(t, x)/(\psi(t, x) + \beta) dx, \quad (12)$$

where $\psi(t, x)$ is the usual profile function (Dresner 1960)

$$\psi(t, x) = \frac{1}{2}(\pi t)^{-\frac{1}{2}} \int_{-\infty}^{\infty} \exp\{-(x-y)^2/4t\} (1+y^2)^{-1} dy. \quad (13)$$

Now by gathering together equations (3), (4), (7), and (8) and averaging each over a Maxwellian distribution of atomic speeds one arrives at the Doppler-broadened forms

$$\left. \begin{aligned} \sigma_c(E, T) &= \sigma_0(\Gamma_r/\Gamma)\{\psi(t, x) + (a_E + a_i)\phi(t, x)\}, \\ \sigma_f(E, T) &= \sigma_0[(\Gamma_f/\Gamma)\mu\psi(t, x) + \{(\Gamma_f/\Gamma)(a_E + a_i) + a_{fi}\}\phi(t, x)], \\ \sigma_s(E, T) &= \sigma_0\{(\Gamma_n/\Gamma)\psi(t, x) + (b_i + b_l)\phi(t, x)\} + \sigma_p, \\ \text{and} \quad \sigma_t(E, T) &= \sigma_0[1 + (\mu - 1)\Gamma_f/\Gamma]\psi(t, x) + a\phi(t, x) + \sigma_p, \end{aligned} \right\} \quad (14)$$

where
$$\phi(t, x) = \frac{1}{2}(\pi t)^{-\frac{1}{2}} \int_{-\infty}^{\infty} \exp\{-(x-y)^2/4t\} y(1+y^2)^{-1} dy. \quad (15)$$

Rothenstein (1958) used both the "narrow resonance" and "infinite mass" approximations to include resonance scattering interference corrections, obtaining an effective J function

$$J_{\text{eff}}(t, \beta, b_i) = \int_{-\infty}^{\infty} \frac{\psi(t, x) dx}{\psi(t, x) + b_i \phi(t, x) + \beta}. \quad (16)$$

He expanded the denominator of the integrand in (16) as powers of b_i to obtain

$$J_{\text{eff}}(t, \beta, b_i) \simeq J(t, \beta, 0) + b_i^2 \int_{-\infty}^{\infty} \frac{\psi(t, x) \phi^2(t, x) dx}{\{\psi(t, x) + \beta\}^3} \quad (17)$$

and he computed curves for the integrals in the first and second terms.

Working rigorously through the Cohen and Goldstein (1962) theory, one finds that the integral (12) becomes replaced by an effective J function

$$J_{\text{eff}}(t', \beta', a', b) = J(t, \beta', a') + bK(t, \beta', a'), \quad (18)$$

where

$$a' = a/[1 + (\mu - 1)\Gamma_f/\Gamma], \quad \beta' = \beta/[1 + (\mu - 1)\Gamma_f/\Gamma],$$

these two corrections being multilevel effects,

$$K(t, \beta, a) = \int_{-\infty}^{\infty} \frac{\phi(t, x) dx}{\psi(t, x) + a\phi(t, x) + \beta}, \quad (19)$$

and

$$b = (\Gamma_n/\Gamma)(a_E + a_i) + a_{fi}. \quad (20)$$

In practice, if one uses resonance parameters that have been fitted on a single-level basis, the value of Γ_n has included in it the correction μ_i to the resonance height. Hence with single-level parameters, we can assume that $\mu = 1$, and that a_{fi} computed from single-level parameters differs little from that calculated with multilevel parameters. Alternatively, the asymmetry parameters can be readily obtained by direct graphical inspection of the experimental resonance; a is just the asymmetry parameter for the total cross section, and b for the absorption cross section.

IV. THE J AND K FUNCTIONS

The functions (19) and (20) were computed by direct integration on the A.A.E.C. Site IBM 7040 computer. Spot checks agreed with the graphs of Rothenstein (1958). The relative deviation function

$$\{J(t, \beta, a) - J(t, \beta, 0)\} / J(t, \beta, 0) = \Delta J$$

is shown in Table 1. Approximate values of a and b for ^{235}U resonances are listed in Table 2. Clearly Rothenstein's assumption that the additive correction to J is proportional to a^2 holds for a much wider range of a than one would suspect from a simple Taylor expansion.

TABLE 2
SOME VALUES FOR ASYMMETRY PARAMETERS IN ^{235}U

E_r (eV)	a_E	a_i	b_i	b_t	a_{fi}	a	b
0.290	-0.110	0.0	0.0	0.10	+0.37	+0.36	+0.26
1.142	-0.033	0.0	0.0	0.19	-0.15	+0.01	-0.18
2.035	-0.014	0.0	0.0	0.31	+0.065	+0.36	+0.051

Notice that a and b rarely exceed 0.2, being largest for low energy resonances. By inspection of equation (18) and Table 1, it is found that all of the corrections never lead to a deviation of more than 2% for $\beta > 1$. This is much less than errors in the determination of parameters, which could be around $\pm 20\%$ for Γ_n (Stehn *et al.* 1965), and errors inherent in the resonance theory approximation, which may reach 5% (Cook and Wall 1966), for moderately large fuel concentrations. For absorption in a pure non-fissile isotope, such as ^{238}U , the values of a and β are such that

$$a \simeq b_i = 2(\sigma_p/\sigma_0)^{\frac{1}{2}} \quad \text{and} \quad \beta = \sigma_p/\sigma_0 = \frac{1}{4}b_i^2, \quad (21)$$

so that the very small values of β obtained ($\sim 10^{-4}$) lead to correspondingly small values of a ($\sim 10^{-2}$).

At $T = 0^\circ\text{K}$, one finds by direct integration

$$\left. \begin{aligned} J(0, \beta, a) &= \pi / \{\beta(\beta+1) - \frac{1}{4}a^2\}^{\frac{1}{2}}, \\ K(0, \beta, a) &= -\frac{a}{2\beta} \frac{\pi}{\{\beta(\beta+1) - \frac{1}{4}a^2\}^{\frac{1}{2}}} = -\frac{a}{2\beta} J(0, \beta, a). \end{aligned} \right\} \quad (22)$$

TABLE 3
VALUES FOR THE J AND K FUNCTIONS

α	Values of K for						
	$\beta = 10^{-3}$	10^{-2}	10^{-1}	1	10	10^2	10^3
	$J = 553$	199	29.6	3.12	0.314	0.0314	0.00314
$t = 10^4$							
0.001	-7.56	-0.143	-1.81(-3)*	-1.87(-5)	-1.97(-7)	—	—
0.005	-37.8	-0.714	-9.07(-3)	-9.37(-5)	-9.38(-7)	—	—
0.01	-75.6	-1.43	-1.81(-2)	-1.87(-4)	-1.89(-6)	—	—
0.05	-381	-7.15	-9.07(-2)	-9.38(-4)	-9.41(-6)	—	—
0.10	—	-14.3	-0.181	-1.88(-3)	-1.88(-5)	-1.88(-7)	—
0.50	—	—	-0.907	-9.38(-3)	-9.41(-5)	-9.41(-7)	—
1.0	—	—	—	-1.88(-2)	-1.88(-4)	-1.88(-6)	-1.88(-8)
5.0	—	—	—	—	-9.41(-4)	-9.41(-6)	-9.41(-8)
10.0	—	—	—	—	-1.88(-3)	-1.88(-5)	-1.88(-7)
	$J = 333$	150	28.0	3.10	0.314	0.0314	0.00314
$t = 2.5 \times 10^3$							
0.001	-12.4	-0.237	-3.50(-3)	-3.75(-5)	-3.84(-7)	—	—
0.005	-61.8	-1.185	-1.75(-2)	-1.88(-4)	-1.90(-6)	—	—
0.01	-124	-2.37	-3.50(-2)	-3.76(-4)	-3.79(-6)	—	—
0.05	-633	-11.9	-1.75(-1)	-1.88(-3)	-1.89(-5)	-1.90(-7)	—
0.10	—	-23.8	-3.50(-1)	-3.76(-3)	-3.79(-5)	-3.79(-7)	—
0.50	—	—	-1.75	-1.88(-2)	-1.89(-4)	-1.89(-6)	—
1.0	—	—	—	-3.76(-2)	-3.79(-4)	-3.79(-6)	—
5.0	—	—	—	—	-1.89(-3)	-1.89(-5)	-1.89(-7)
10.0	—	—	—	—	-3.79(-3)	-3.79(-5)	-3.79(-7)
	$J = 177$	92.0	24.4	3.05	0.313	0.0314	0.00314
$t = 400$							
0.001	-23.8	-0.439	-7.81(-3)	-9.17(-5)	-9.38(-7)	—	—
0.005	-119	-2.19	-3.90(-2)	-4.58(-4)	-4.67(-6)	—	—
0.01	-239	-4.39	-7.81(-2)	-9.17(-4)	-9.35(-6)	—	—
0.05	-1316	-22.0	-3.90(-1)	-4.58(-3)	-4.68(-5)	-4.68(-7)	—
0.10	—	-44.4	-7.81(-1)	-9.17(-3)	-9.35(-5)	-9.37(-7)	—
0.50	—	—	-3.93	-4.58(-2)	-4.68(-4)	-4.68(-6)	—
1.0	—	—	—	-9.17(-2)	-9.35(-4)	-9.37(-6)	—
5.0	—	—	—	—	-4.68(-3)	-4.68(-5)	-4.68(-7)
10.0	—	—	—	—	-9.35(-3)	-9.37(-5)	-9.37(-7)

* Each number in parentheses is the common logarithm of the multiplier; $-1.81(-3)$ means -1.81×10^{-3} .

TABLE 3 (Continued)

α	Values of K for						
	$\beta = 10^{-3}$	10^{-2}	10^{-1}	1	10	10^2	10^3
	$J = 125$	61.4	20.5	2.97	0.312	0.0314	0.00314
$t = 100$							
0.001	-35.5	-0.683	-1.33(-2)	-1.74(-4)	-1.80(-6)	—	—
0.005	-178	-3.42	-6.64(-2)	-8.68(-4)	-9.02(-6)	—	—
0.01	-358	-6.83	-1.33(-1)	-1.74(-3)	-1.80(-5)	-1.81(-7)	—
0.05	-2293	-3.44	-6.64(-1)	-8.68(-3)	-9.02(-5)	-9.06(-7)	—
0.10	—	-70.1	-1.33	-1.74(-2)	-1.80(-4)	-1.81(-6)	—
0.50	—	—	-6.79	-8.69(-2)	-9.02(-4)	-9.06(-6)	—
1.0	—	—	—	-0.174	-1.80(-3)	-1.81(-5)	-1.81(-7)
5.0	—	—	—	—	-9.02(-3)	-9.06(-5)	-9.06(-7)
10.0	—	—	—	—	-1.81(-2)	-1.81(-4)	-1.81(-6)
$J = 106$ 43.4 16.3 2.84 0.311 0.0314 0.00314							
$t = 25$							
0.001	-44.5	-1.01	-2.09(-2)	-3.12(-4)	-3.35(-6)	—	—
0.005	-223	-5.04	-1.05(-1)	-1.56(-3)	-1.67(-5)	-1.69(-7)	—
0.01	-450	-10.1	-2.09(-1)	-3.12(-3)	-3.34(-5)	-3.37(-7)	—
0.05	-3404	-51.1	-1.05	-1.56(-2)	-1.67(-4)	-1.68(-6)	—
0.10	—	-107	-2.10	-3.12(-2)	-3.34(-4)	-3.37(-6)	—
0.50	—	—	-11.1	-0.156	-1.67(-3)	-1.68(-5)	-1.69(-7)
1.0	—	—	—	-0.313	-3.34(-3)	-3.37(-5)	-3.37(-7)
5.0	—	—	—	—	-1.67(-2)	-1.69(-4)	-1.69(-6)
10.0	—	—	—	—	-3.36(-2)	-3.37(-4)	-3.37(-6)
$J = 100$ 33.9 12.0 2.60 0.307 0.0313 0.00314							
$t = 4$							
0.001	-48.7	-1.38	-3.32(-2)	-5.89(-4)	-6.75(-6)	—	—
0.005	-244	-6.92	-1.66(-1)	-2.94(-3)	-3.38(-5)	-3.43(-7)	—
0.01	-493	-13.85	-3.32(-1)	-5.89(-3)	-6.76(-5)	-6.87(-7)	—
0.05	-3940	-71.1	-1.66	-2.94(-2)	-3.38(-4)	-3.43(-6)	—
0.10	—	-156	-3.34	-5.89(-2)	-6.76(-4)	-6.87(-6)	—
0.50	—	—	-19.7	-2.96(-1)	-3.38(-3)	-3.43(-5)	-3.44(-7)
1.0	—	—	—	-5.99(-1)	-6.76(-3)	-6.87(-5)	-6.88(-7)
5.0	—	—	—	—	-3.40(-2)	-3.43(-4)	-3.44(-6)
10.0	—	—	—	—	-6.90(-2)	-6.87(-4)	-6.88(-6)

TABLE 3 (Continued)

α	Values of K for						
	$\beta = 10^{-3}$	10^{-2}	10^{-1}	1	10	10^2	10^3
	$J = 99.5$	31.9	10.4	2.42	0.304	0.0313	0.00314
	$t = 1$						
0.001	-49.4	-1.51	-4.10(-2)*	-8.20(-4)	-1.00(-5)	-1.03(-7)	—
0.005	-248	-7.54	-0.205	-4.10(-3)	-5.00(-5)	-5.13(-7)	—
0.01	-500	-15.1	-0.410	-8.20(-3)	-1.00(-4)	-1.03(-6)	—
0.05	-4018	-77.8	-2.06	-4.10(-2)	-5.00(-4)	-5.13(-6)	—
0.10	—	-173	-4.14	-8.20(-2)	-1.00(-3)	-1.03(-5)	-1.03(-7)
0.50	—	—	-27.6	-4.13(-1)	-5.00(-3)	-5.13(-5)	-5.15(-7)
1.0	—	—	—	-8.49(-1)	-1.00(-2)	-1.03(-4)	-1.03(-6)
5.0	—	—	—	—	-5.07(-2)	-5.13(-4)	-5.15(-6)
10.0	—	—	—	—	-1.05(-1)	-1.03(-3)	-1.03(-5)
	$J = 99.2$	31.4	9.73	2.30	0.302	0.0313	0.00314
	$t = 0.25$						
0.001	-49.6	-1.55	-4.52(-2)	-9.90(-4)	1.27(-5)	-1.32(-7)	—
0.005	-249	-7.74	-0.226	-4.95(-3)	-6.37(-5)	-6.59(-7)	—
0.01	-502	-15.5	-0.452	-9.90(-3)	-1.27(-4)	-1.32(-6)	—
0.05	-4046	-79.9	-2.27	-4.95(-2)	-6.37(-4)	-6.59(-6)	—
0.10	—	-178	-4.57	-9.91(-2)	-1.27(-3)	-1.32(-5)	-1.32(-7)
0.50	—	—	-33.1	-0.501	-6.37(-3)	-6.59(-5)	-6.61(-7)
1.0	—	—	—	-1.04	-1.27(-2)	-1.32(-4)	-1.32(-6)
5.0	—	—	—	—	-6.50(-2)	-6.59(-4)	-6.61(-6)
10.0	—	—	—	—	-0.139	-1.32(-3)	-1.32(-5)
	$J = 99.3$	31.2	9.51	2.24	0.300	0.0313	0.00314
	$t = 0.04$						
0.001	-49.7	-1.56	-4.71(-2)	-1.09(-3)	-1.45(-5)	-1.51(-7)	—
0.005	-249	-7.82	-2.36(-1)	-5.43(-3)	-7.24(-5)	-7.54(-7)	—
0.01	-503	-15.7	-4.71(-1)	-1.09(-2)	-1.45(-4)	-1.51(-6)	—
0.05	-4050	-80.7	-2.36	-5.43(-2)	-7.24(-4)	-7.54(-6)	—
0.10	—	-180	-4.76	-1.09(-1)	-1.45(-3)	-1.51(-5)	-1.52(-7)
0.50	—	—	-35.6	-5.52(-1)	-7.25(-3)	-7.54(-5)	-7.58(-7)
1.0	—	—	—	-1.16	-1.45(-2)	-1.51(-4)	-1.52(-6)
5.0	—	—	—	—	-7.44(-2)	-7.55(-4)	-7.58(-6)
10.0	—	—	—	—	-0.163	-1.51(-3)	-1.52(-5)

* Each number in parentheses is the common logarithm of the multiplier; $-4.10(-2)$ means -4.10×10^{-2} .

Clearly the absorption cross section asymmetries embodied in the K function decrease more rapidly with increasing β than does the J function. From Table 3 it is apparent that the contribution from K is negligible in homogeneous mixture assemblies where $\beta \gtrsim 500$ in practice.

It is quite apparent that the tables are interpolable in both a and t , and logarithmically interpolable in β . For large a the function K appears to be directly proportional to a for all t , though the relation (22) between J and K is not true for all t .

V. CONCLUSION

The effects of interference and resonance asymmetry in estimating the resonance escape probability have been investigated and found to be of less significance than errors in the resonance parameters and in the resonance theory approximation.

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