THE AKHIESER–POMERANCHUK–BLAIR MODEL FOR ELASTIC SCATTERING

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Summary

The semiclassical APB model is employed in analysing the angular distribution of protons elastically scattered from Fe, Cu, and Ag; deuterons from Ni, Zr, Ag, and Er; ^{3}He-particles from Cd and I; and α-particles from I. Effects of different parameters have been studied and the best-fit parameters obtained.

I. Introduction

In an attempt to explain the energy dependence of the elastic scattering of 13–43 MeV α-particles from a number of medium weight and heavy nuclei, as measured by Farwell and Wegner (1954), Blair (1954) extended the model proposed by Akhieser and Pomeranchuk (1945) for small-angle scattering of high energy charged particles. The model is, therefore, referred to as the Akhieser–Pomeranchuk–Blair (or simply the APB) model. If the incident beam is broken up into partial waves, then according to the model all partial waves up to a certain critical value are completely absorbed by the target nucleus, while the higher partial waves suffer only a Coulomb scattering. The consequence of the sharp cutoff assumption in orbital angular momentum is the appearance of a diffraction oscillation at higher energies or at larger angles, in contradiction to experimental results. The model was consequently modified by making the transition from complete absorption to Coulomb scattering gradual or smeared out, but the improvement was not significant (Wall, Rees, and Ford 1955; Ellis and Schecter 1956).

A further empirical modification was then suggested by McIntyre, Wang, and Becker (1960), where not only the amplitude $|A_l|$ but also the phase of the scattered waves δ_l were allowed to vary. $|A_l|$ was taken as

$$|A_l| = [1+\exp\{-(l-l_A)/\Delta l_A\}]^{-1},$$

where the transition $|A_l| = 0$ to $|A_l| = 1$ takes place over a range Δl_A in the neighbourhood of l_A. For δ_l was chosen the form

$$\delta_l = \delta[1+\exp\{(l-l_\delta)/\Delta l_\delta\}]^{-1},$$

with l_δ and Δl_δ bearing similar meanings to l_A and Δl_A respectively and δ being the real nuclear phase shift. The ratio of the differential cross section $\sigma(\theta)$ to the

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Coulomb cross section $\sigma_C(\theta)$, in this smoothed APB model, is given by

$$\sigma(\theta)/\sigma_C(\theta) = \Big|-\mathrm{i}\exp\{-\mathrm{i}\eta\ln(\sin^2\tfrac{1}{2}\theta)\}-\eta^{-1}\sin^2\tfrac{1}{2}\theta \times \sum_{l=0}^{\infty}(2l+1)(1-|A_l|\exp 2\mathrm{i}\delta_l)\exp\{2\mathrm{i}(\sigma_l-\sigma_0)\}\,\mathrm{P}_l(\cos\theta)\Big|^2, \qquad (1)$$

where η is the Coulomb parameter and σ_l is the Coulomb phase shift for the lth partial wave.

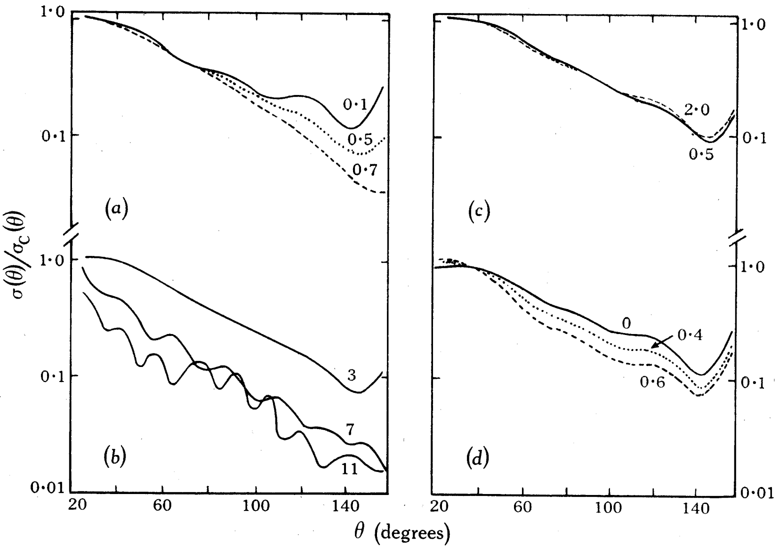


Fig. 1.—Effects of different parameters on angular distribution (θ is the c.m.s. angle).
(a) $l_A = 3\cdot0$, $\delta = 0\cdot2$, $\Delta l_\delta = 2\cdot0$, $\Delta l_A = 0\cdot1$, $0\cdot5$, $0\cdot7$;
(b) $\Delta l_A = 0\cdot5$, $\delta = 0\cdot1$, $\Delta l_\delta = 2\cdot0$, $l_A = 3$, 7, 11;
(c) $l_A = 3\cdot0$, $\Delta l_A = 0\cdot4$, $\delta = 0\cdot1$, $\Delta l_\delta = 2\cdot0$, $0\cdot5$;
(d) $l_A = 3\cdot0$, $\Delta l_A = 0\cdot3$, $\Delta l_\delta = 1\cdot5$, $\delta = 0$, $0\cdot4$, $0\cdot6$.

McIntyre, Wang, and Becker (1960) thus obtained a much better agreement with experimental results than had been previously obtained. (A review of all such works on α-particles has been made by Eisberg and Porter (1961).) The model has the advantage of simplicity and is expected to be valid in the regions of heavy and medium weight nuclei. It was therefore decided to see if this model could be applied to other projectiles such as protons, deuterons, and ^{3}He-particles. The present work was undertaken with this in view and we quote here analyses of the experimental elastic scattering data on protons from Fe (Benveniste *et al.* 1964), Cu (Hintz 1957), and Ag (Glassgold and Kellogg 1957); deuterons from Ni (Budzanowski *et al.* 1963), Zr (Igo, Lorenz, and Schmidt-Rohr 1961), Ag (Yntema 1959), and Er (Jolly, Lin, and Cohen 1963); ^{3}He-particles from Cd (Greenless and Rowe 1960) and I (Sen Gupta *et al.* 1964); and α-particles from I (Van Heerden and Prowse 1960).

II. Theoretical Analysis

The adjustable parameters in the APB model are l_A, Δl_A, l_δ, Δl_δ, and δ. It is usually assumed that l_δ is the same as l_A; the assumption is not arbitrary. Alster, Shreve, and Peterson (1966) in their analyses of the elastic scattering of 42 MeV α-particles from ^{88}Sr and ^{89}Y used l_δ as a free parameter but in the best-fit result the difference between the values of l_δ and l_A was found to be insignificant. Hence in all calculations we took l_δ to be the same as l_A.

The effect of the different parameters on angular distribution given by (1), was first studied by varying one while keeping the others fixed; the results and parameter values chosen are shown in Figure 1. It will be seen that increasing l_A decreases cross section steadily and, in addition, increases the number and amplitude of oscillations; Δl_A controls the ratio of backward to forward scattering without

Table 1

Parameters of the APB Model

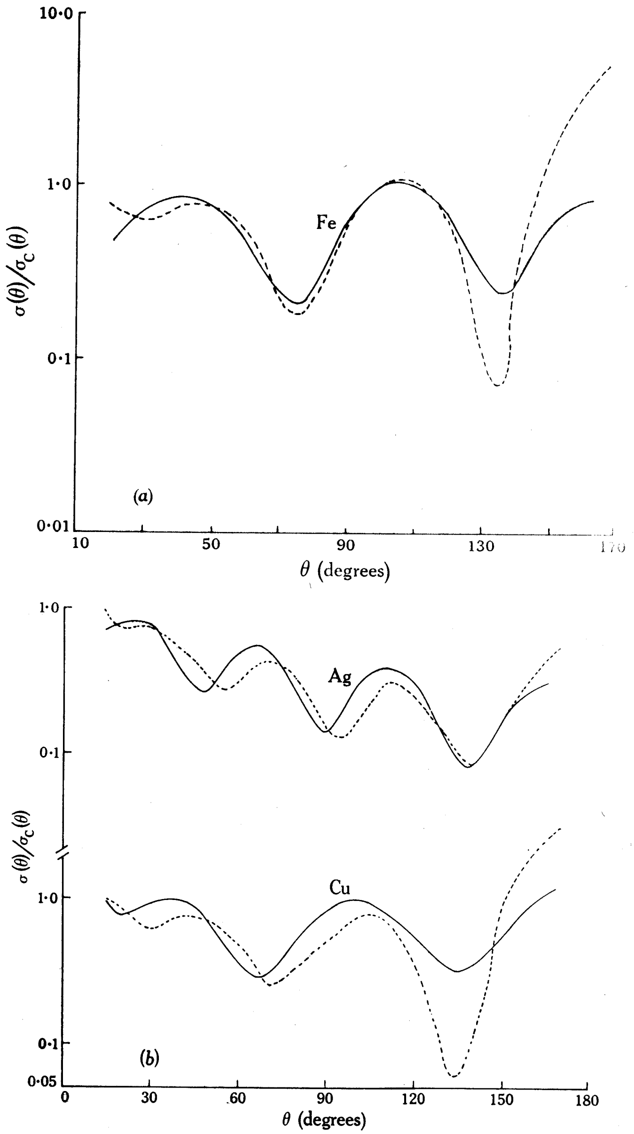
Beam	Beam Energy (MeV)	Target Nucleus	Parameter l_A	Δl_A	δ	Δl_δ
Proton	11·66	Fe	2·5	0·1	0·5	0·1
	9·8	Cu	2·7	0·1	0·5	0·1
	17·0	Ag	3·9	0·2	0·1	0·4
Deuteron	12·8	Ni	6·0	0·4	0·1	2·0
	11·8	Zr	4·75	0·3	0·1	1·0
	21·6	Ag	9·3	0·6	0·1	1·0
	15·0	Er	7·0	0·5	0	
^{3}He-particle	29·1	Cd	13·0	0·65	0	
	29·1	I	12·3	0·5	0·15	0·4
α-particle	38·09	I	16·25	1·7	0·27	0·35

significantly altering forward cross section and, in addition, damps out the oscillation as it is increased. The effect of an increase in δ is to depress the pattern throughout except at extreme forward angles, whereas variation of Δl_δ is relatively unimportant except at large angles.

For comparison with experimental results, δ was first put equal to zero and Δl_A and Δl_δ were given arbitrary values while l_A was varied so as to minimize χ^2, which is defined by

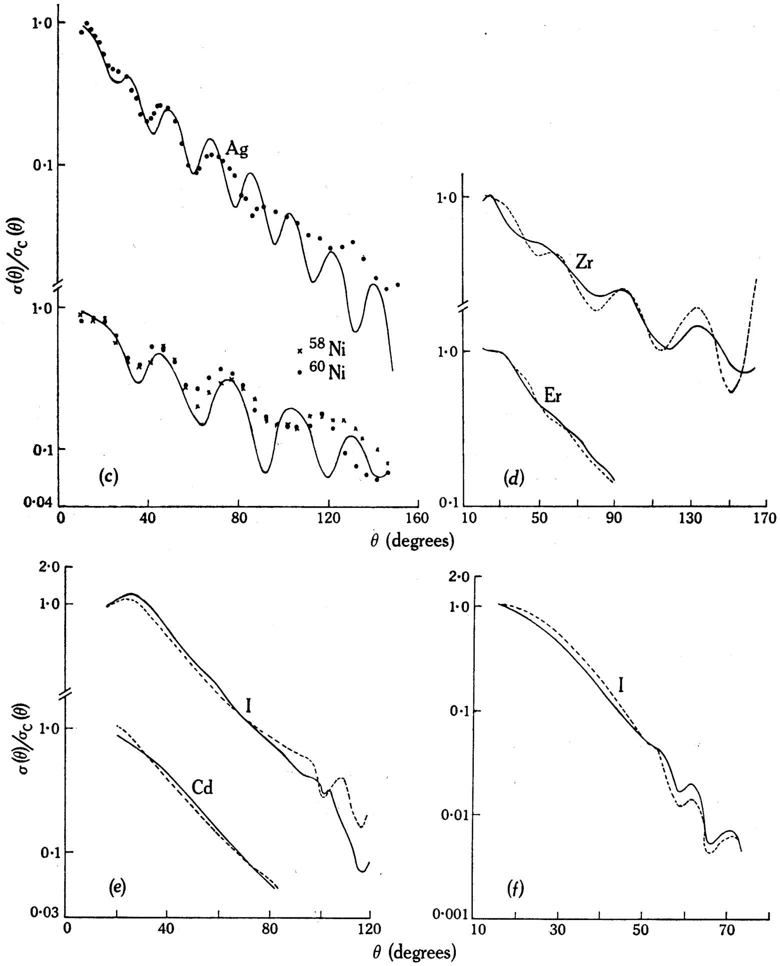
$$\chi^2 = \sum \left(\frac{\sigma(\theta)/\sigma_{\mathrm{C}}(\theta) - \sigma_{\mathrm{th}}(\theta)/\sigma_{\mathrm{C}}(\theta)}{\delta\{\sigma(\theta)/\sigma_{\mathrm{C}}(\theta)\}} \right)^2,$$

where the first term in the numerator is the ratio of the experimental to the Coulomb cross section, $\sigma_{\mathrm{th}}(\theta)$ in the second term is the theoretical cross section predicted by the APB model, and the denominator is the error in experimental measurement. The l_A thus obtained was kept constant and the remaining parameters were adjusted systematically one after another till a minimum in χ^2 was obtained.



Figs. 2(*a*)–2(*f*).—Analyses of the APB model for elastic scattering of (*a*) protons from Fe, (*b*) protons from Ag and Cu, (*c*) deuterons from Ag and Ni, (*d*) deuterons from Zr and Er, (*e*) ^{3}He-particles from I and Cd, and (*f*) α-particles from I. In each case except (*c*) the solid curve is the experimental result and the dashed curve is the theoretical distribution for the parameters listed in Table 1. In (*c*) the continuous curve is the theoretical distribution.

All calculations were carried out with the IBM 1620 computer of the Atomic Energy Centre, Dacca. Included with the parameter values in the input of the program were the scattering angles, the corresponding experimental cross sections, and errors in experimental values. The output gave the theoretical values of cross section with χ^2 for each set of parameters.



Figs. 2(a)–2(f) (*Continued*)

III. Results and Discussion

The best-fit parameters of the model for the individual cases are given in Table 1 and the corresponding results in Figures 2(a)–2(f).

It will be seen that the agreement in most cases is satisfactory and that the model reproduces the angular distributions fairly well. In case of d–Zr and d–Ni scattering, the agreement is not so good, the predicted minima being deeper than those observed. The model predicts a sharp oscillation at extreme backward angles, where experimental data show only a moderate oscillation, e.g. p–Fe, d–Zr, and d–Ni scattering. The data in d–Er scattering are better fitted than those in other deuteron scatterings, which at first sight appears to be surprising, since the APB model assumes both the interacting particles to be spherical while the Er nucleus is known to be non-spherical. However, it is to be noted that the model is expected to be particularly applicable to scattering from heavy and medium weight nuclei, as the validity criteria are better satisfied in these cases. Our calculations using the APB model for heavier nuclei have confirmed this point (Khan, Rahman, and Sen Gupta 1966), and of all the nuclei studied in the present case Er is the heaviest.

It is interesting to note that in heavy nuclei the cross section rises above the Coulomb value at extreme forward angles before an almost exponential falloff begins, and this pattern is well predicted by the model. In the case of ^{3}He–I elastic scattering (Sen Gupta *et al.* 1964) the optical model did not give any oscillation, which experimental data seemed to show, but this is again predicted by the APB model. Similar oscillation in cross section is also given in α–I elastic scattering.

It will be seen from Table 1 that the value of l_A, the critical orbital angular momentum, increases regularly when we pass from lighter to heavier projectiles, and a similar increase in Δl_A is also observed. The parameter δ is small in all cases, whereas Δl_δ does not show any systematic variation either from one projectile to another or from one target nucleus to another. However, it is to be noted that Δl_δ is not a very sensitive parameter (Fig. 1).

We conclude by saying that this semiclassical model can, in general, take a fair account of the elastic scattering process not only for α-particles but also for other projectiles. Whether or not the model is applicable to lighter nuclei and also if it is possible to get more than one set of parameters remain to be seen; this we hope to investigate in future work.

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