SHORT COMMUNICATIONS

INTERFERENCE BETWEEN LEVELS IN NUCLEAR REACTIONS*

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In this note we consider the cross section for a nuclear reaction in which one of the product nuclei is unstable, with two or more levels contributing to its decay. Previously a formula had been derived from R-matrix theory for the case where contributions come from only a single level of the nucleus with a given spin and parity. The many-level generalization of this formula is based on analogy with the form of the cross section for a simple two-stage reaction. Applications to some ^{8}Be levels are considered.

A general formula for the cross section of a two-stage reaction

$$A+a \rightarrow C \rightarrow B+b \tag{1}$$

proceeding through any number (say N) of levels of the compound nucleus C with the same spin and parity J^π has been given in the R-matrix formalism by Lane and Thomas (1958). For the case where only one channel spin s and relative orbital angular momentum l occur for each of the initial system $A+a$ and the final system $B+b$ (assumed different from $A+a$), and for the particular case $\mathbf{R}^0 = 0$, equation (IX, 1.14) of Lane and Thomas (1958) leads to

$$\sigma_{ab} = \frac{\pi g_J}{k_a^2}\left|2P_a^{\frac{1}{2}}P_b^{\frac{1}{2}} \sum_{\lambda,\mu=1}^{N} \gamma_{\lambda a}\gamma_{\mu b}A_{\lambda\mu}\right|^2. \tag{2}$$

The $A_{\lambda\mu}$ are elements of a level matrix that depend on the eigenenergies E_λ and on the level widths $\Gamma_{\lambda\mu}$ and level shifts $\Delta_{\lambda\mu}$ defined by

$$\left.\begin{aligned} \Gamma_{\lambda\mu} &= \sum_c \Gamma_{\lambda\mu c}, \qquad & \Gamma_{\lambda\mu c} &= 2P_c\gamma_{\lambda c}\gamma_{\mu c}, \\ \Delta_{\lambda\mu} &= \sum_c \Delta_{\lambda\mu c}, \qquad & \Delta_{\lambda\mu c} &= -(S_c-B_c)\gamma_{\lambda c}\gamma_{\mu c}, \end{aligned}\right\} \tag{3}$$

where the summations are over all channels c, including the formation and decay channels a and b. The $\gamma_{\lambda c}$ are reduced width amplitudes, and P_c, S_c, and B_c are the penetration factor, shift factor, and boundary condition parameter respectively

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(Lane and Thomas 1958). In the one-level approximation, (2) becomes

$$\sigma_{ab} = \frac{\pi g_J}{k_a^2} \frac{\Gamma_{1a}\Gamma_{1b}}{(E_1+\Delta_1-E)^2+(\frac{1}{2}\Gamma_1)^2}, \tag{4}$$

with the usual abbreviations

$$\Gamma_\lambda = \Gamma_{\lambda\lambda}, \qquad \Gamma_{\lambda c} = \Gamma_{\lambda\lambda c}, \qquad \Delta_\lambda = \Delta_{\lambda\lambda}. \tag{5}$$

and E the excitation energy of the nucleus C. The cross section in the two-level approximation has also been given explicitly (equation (XII, 4.2) of Lane and Thomas 1958) and is quite complicated in form.

In the case of a reaction

$$D+d \rightarrow C+c, \qquad C \rightarrow B+b, \tag{6}$$

where the first stage may proceed via compound nucleus formation or by direct transition, the cross section has been obtained by Lane and Thomas (1958) from R-matrix theory for the case where only one level of C with given J^π is contributing. Here we are concerned only with the dependence on E, so this cross section may be written

$$\sigma_{dcb} \propto \frac{G_{1dc}\Gamma_{1b}}{(E_1+\Delta_1-E)^2+(\frac{1}{2}\Gamma_1)^2}, \tag{7}$$

where G_{1dc} is a real positive feeding factor that is generally a slowly varying function of E. The cross section (7) is similar to (4) with G_{1dc} replacing Γ_{1a}, but whereas the feeding channel contributes to the total width Γ_1 and level shift Δ_1 in (4) it does not do so in (7).

The form of the cross section for (6) has not been given for the case when there are contributions from more than one level of C with the same J^π. The different levels are expected to contribute coherently and an obvious generalization of (7) is

$$\sigma_{dcb} \propto \left| \sum_{\lambda=1}^{N} \frac{G_{\lambda dc}^{\frac{1}{2}}\Gamma_{\lambda b}^{\frac{1}{2}}}{E_\lambda+\Delta_\lambda-E-\frac{1}{2}\mathrm{i}\Gamma_\lambda} \right|^2. \tag{8}$$

This form has been used by Browne, Callender, and Erskine (1966) to interpret the observed cross section for the $^{10}B(d,\alpha)^8Be(\alpha)^4He$ reaction, with contributions from the two 2^+ levels of ^8Be at 16·6 and 16·9 MeV. With the Δ_λ neglected and the Γ_λ independent of energy, a good fit to experiment was obtained.

However, it seems that a more reasonable R-matrix generalization of (7) for the many-level case can be obtained from the many-level cross section σ_{ab} given by (2) by using the same procedure that leads from the formula (4) to (7) in the one-level case, i.e. by replacing $(2P_a)^{\frac{1}{2}}\gamma_{\lambda a} \equiv \Gamma_{\lambda a}^{\frac{1}{2}}$ by $G_{\lambda dc}^{\frac{1}{2}}$ (and changing the factor $\pi g_J/k_a^2$). The $A_{\lambda\mu}$ are unchanged, the contributions to $\Gamma_{\lambda\mu}$ and $\Delta_{\lambda\mu}$ coming only from channels to which the nucleus C can decay. The cross section so obtained is quite complicated in general, but becomes simple in form in some particular cases. We consider two cases that are specially relevant to certain levels of ^8Be.

(i) If only the one channel b is contributing to the decay of C and therefore to Γ_λ and Δ_λ, the cross section for the reaction (6) may be obtained from (2) as suggested above, or more simply one may start from the form of the cross section for (1) in terms of the channel matrix $\mathbf{R}$ without transforming to the level matrix form. Thus

$$\sigma_{ab} \propto |[\mathbf{P}^{\frac{1}{2}}(1-\mathbf{RL}^0)^{-1}\mathbf{R}\,\mathbf{P}^{\frac{1}{2}}\,\mathbf{w}]_{ab}|^2 \tag{9}$$

(from equation (VII, 1.6a) of Lane and Thomas 1958) leads to

$$\sigma_{dcb} \propto \left| \frac{\sum_{\lambda=1}^{N} G^{\frac{1}{2}}_{\lambda dc}\,\Gamma^{\frac{1}{2}}_{\lambda b}/(E_\lambda - E)}{1+\sum_{\lambda=1}^{N} (\Delta_{\lambda b}-\frac{1}{2}\mathrm{i}\Gamma_{\lambda b})/(E_\lambda - E)} \right|^2 . \tag{10}$$

In terms of the nuclear phase shift δ_b and the hard-sphere phase shift $(-\phi_b)$ for the $B+b$ elastic scattering, this can be written

$$\sigma_{dcb} \propto \sin^2(\delta_b+\phi_b) \left[\frac{\sum_{\lambda=1}^{N} G^{\frac{1}{2}}_{\lambda dc}\,\Gamma^{\frac{1}{2}}_{\lambda b}/(E_\lambda - E)}{\sum_{\lambda=1}^{N} \Gamma_{\lambda b}/(E_\lambda - E)} \right]^2 . \tag{11}$$

The formulae (10) and (11) should be applicable to reactions proceeding through the low-lying levels of ^{8}Be, which decay only through the $\alpha+\alpha$ channel.

(ii) For reactions involving the 16·6 and 16·9 MeV levels of ^{8}Be, the formulae (10) and (11) are not applicable even though only the $\alpha+\alpha$ channel is open, as the ^{7}Li+p and ^{7}Be+n channels can contribute appreciably to the level shifts. If the 16·6 and 16·9 MeV levels are denoted by 1 and 2 and the channels $\alpha+\alpha$, $^7\mathrm{Li}_{\mathrm{g.s.}}$+p, and $^7\mathrm{Be}_{\mathrm{g.s.}}$+n by α, p, and n respectively, then it is reasonable to take $\gamma_{1\mathrm{n}} = \gamma_{2\mathrm{p}} = 0$ (Barker 1966). Since S_α does not change appreciably in the region of the levels, one can take $B_\alpha = S_\alpha$ and then $\Delta_{12} = 0$. Also $P_\mathrm{n} = P_\mathrm{p} = 0$ so that $\Gamma_{12} = \Gamma_{12\alpha} = \Gamma^{\frac{1}{2}}_{1\alpha}\Gamma^{\frac{1}{2}}_{2\alpha}$. Then from equation (2), or from equation (XII, 4.2) of Lane and Thomas (1958), one obtains

$$\sigma_{dc\alpha} \propto \left| \frac{\sum_{\lambda=1}^{2} G^{\frac{1}{2}}_{\lambda dc}\,\Gamma^{\frac{1}{2}}_{\lambda\alpha}/(E_\lambda+\Delta_\lambda - E)}{1-\frac{1}{2}\mathrm{i}\sum_{\lambda=1}^{2} \Gamma_{\lambda\alpha}/(E_\lambda+\Delta_\lambda - E)} \right|^2 . \tag{12}$$

This may be further simplified by using the approximation (Lane and Thomas 1958) that the Δ_λ are linear functions of E.

One significant difference between the various forms of σ_{dcb} is that, if a particular level λ is not fed in the reaction so that $G_{\lambda dc} = 0$, then that level has no effect on σ_{dcb} given by (8) but does affect σ_{dcb} given by (10)–(12) (a similar situation exists for the cross section σ_{ab} given by (9), where a level with $\gamma^2_{\lambda a} = 0$ still modifies σ_{ab} due to the non-diagonal factor $(1-\mathbf{R}\,\mathbf{L}^0)^{-1}$). As an example we consider the 0^+ states of ^{8}Be. In α–α scattering the ground state appears to interfere with other broad 0^+ states (Barker, Hay, and Treacy, in preparation), but in reactions such as ^{9}Be(p, d)^{8}Be(α)^{4}He one may assume that the higher 0^+ states are not fed significantly. The ground state is observed in both scattering and reactions at the same energy within experimental errors (Benn *et al.* 1966; Reichart *et al.* 1966). This is what is expected if (10) or (11) is used, but if (8) is used it follows only if $B_\alpha = S_\alpha(E_1)$, which is not a reasonable requirement for a many-level approximation.

The formula (12) can give fits to the $^{10}B(d,\alpha)^{8}Be(\alpha)^{4}He$ experimental data of comparable quality to those obtained by Browne, Callender, and Erskine (1966) using (8).

We started by assuming that only a single set of *sl* values occurred for each of the initial and final systems in the reaction (1). More generally contributions to σ_{ab} from different sets of *sl* values add incoherently. Similarly one expects incoherent contributions to the reaction (6). For example, for reactions involving the 16·6 and 16·9 MeV levels of ^{8}Be, (12) may be generalized to

$$\sigma_{dc\alpha} \propto \sum_{x} \left| \frac{\sum_{\lambda=1}^{2} G^{\frac{1}{2}}_{\lambda dc,x}\, \Gamma^{\frac{1}{2}}_{\lambda\alpha}/(E_\lambda+\Delta_\lambda-E)}{1-\frac{1}{2}\mathrm{i}\sum_{\lambda=1}^{2} \Gamma_{\lambda\alpha}/(E_\lambda+\Delta_\lambda-E)} \right|^2, \tag{13}$$

where x labels the quantum numbers for the formation process that give incoherent contributions. For any number of x values, only three combinations of feeding factors occur; these are H_{11}, H_{22}, and H_{12}, where

$$H_{\lambda\mu} = \sum_{x} G^{\frac{1}{2}}_{\lambda dc,x} G^{\frac{1}{2}}_{\mu dc,x}, \qquad \text{so that} \qquad H_{11}H_{22} \geqslant H_{12}^2$$

for admissible cross sections. In the approximation that widths are independent of energy and shifts are linear functions of energy, the $\sigma_{dc\alpha}$ given by (13) and by the similar generalization of (8) can be made identical functions of E by suitably relating the parameters. However, the form (13) is the less restrictive as some cross sections that are admissible from (13) are inadmissible from the generalization of (8), but all cross sections admissible from the generalization of (8) are also admissible from (13).

For the $^{10}B(d,\alpha)^{8}Be$ reaction, if only the $T = 0$ parts of the ^{8}Be states are fed, then $G_{1dc,x}/G_{2dc,x}$ is independent of x and $H_{11}H_{22} = H_{12}^2$, so that (13) effectively reduces to (12). If, however, the $T = 1$ parts of the ^{8}Be states are also fed, as suggested by Browne, Callender, and Erskine (1966), then (13) can be more general than (12). Another interesting reaction is $^{8}B(\beta^{+})^{8}Be$ involving the 16·6 and 16·9 MeV levels of ^{8}Be. The Gamow–Teller and Fermi contributions add incoherently. Because $|G|^2$ for the $\Delta T = 1$ transition is approximately equal to $|F|^2$ while $|G|^2$ for $\Delta T = 0$ is much smaller (Barker 1966), the contribution from the H_{12} term is expected to be very small. Experimental investigation of this is difficult because the cross section is dominated by the Fermi function (Matt *et al.* 1964).

More details of applications of these formulae to ^{8}Be levels will be given in a paper with H. J. Hay and P. B. Treacy, to whom the author is grateful for discussions.

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