THE MOMENTUM TRANSFER CROSS SECTION FOR ELECTRONS IN HELIUM

By R. W. Crompton,* M. T. Elford,* and R. L. Jory*†

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Summary

Measurements of the drift velocity, the ratio of diffusion coefficient to mobility, and the "magnetic drift velocity" for electrons in helium have been made at 293°K in the range $1\cdot8\times10^{-19} < E/N < 3\times10^{-17}$ V cm^2. From an analysis of the drift velocity data, an energy-dependent momentum transfer cross section has been derived for which an error of less than $\pm2\%$ is claimed over the central portion of the energy range. The cross section agrees with the theoretical cross section of La Bahn and Callaway to within 2% over the whole energy range. The agreement with the cross section derived by a number of procedures from the total elastic scattering cross section measured by Golden and Bandel is less satisfactory. The drift data are sufficiently accurate to enable a search to be made for the effects of fine structure in the cross section at low energy. The results do not support the existence of such structure.

I. Introduction

Despite the theoretical interest in the energy dependence of the cross section for elastic scattering between low energy electrons and helium atoms, there have been surprisingly few attempts to measure it directly. Following the earlier work in the 1920's (e.g. Ramsauer 1921; Brode 1925; Ramsauer and Kollath 1929; Normand 1930) there were no other direct experimental determinations until the work reported recently by Golden and Bandel (1965). The rather large differences that were reported both in the shape of the cross section curve and in the absolute magnitude of the cross section led these later workers to conclude that, at the time they initiated their own work, the cross section was certainly not known to better than 25%. In the light of recent detailed theoretical calculations of this cross section, its redetermination was therefore certainly called for.

Golden and Bandel describe in detail the attention paid to the accurate determination of all the experimental parameters, and an analysis of errors led them to the conclusion that the estimated probable error in their experiments was $\pm3\%$. In experiments of the type used by them there are, however, some inherent difficulties which are not easily overcome and whose effects are often difficult to

* Ion Diffusion Unit, Australian National University, Canberra.

† Present address: Department of Physics, The Florida State University, Tallahassee, Florida, U.S.A.

estimate. The first of these difficulties is associated with the low pressures (of the order of 10^{-2} torr and less) that must be used in the collision chamber to preserve single-scattering conditions. Several problems associated with the measurement of low pressures by McLeod gauges are reported in the literature (see, for example, Kieffer and Dunn 1966). Even though the present discussion refers to helium, for which the Gaede–Ishi effect is reported to be negligible, the measurement of pressure in this range is necessarily indirect and subject to greater error than the measurement of pressures above 1 torr. Furthermore, there may be some difficulty in inferring from the gas pressure the gas number density along the path of the electron beam within the collision chamber. In Golden and Bandel's apparatus, for example, differential pumping was employed to maintain a high vacuum in the energy selection region against the relatively high pressure in the scattering chamber. It is therefore possible that pressure and density gradients existed within the chamber, although the results obtained under static conditions suggest that errors from this source were not serious. Whether or not any of these effects are important in each of the experiments described in the literature is difficult to assess, but any error in the effective number density along the collision path will be directly reflected in the absolute magnitude of the measured cross sections. This seems a likely explanation of some of the discrepancies that exist between the results of investigations of this type.

A further complication arises in determining the absolute energy scale, particularly at low energies. Although momentum selection has usually been employed to produce a beam of good energy resolution, Golden and Bandel found that the energy could not be inferred reliably from the magnitude of the magnetic field and they resorted to retarding-potential measurements to establish the energy of their electron beam. While they took precautions to minimize contact potential differences within the apparatus, the complete elimination of such effects is extremely difficult, so that accurate calibration of the energy scale, at least below 0·5 eV, is always subject to some doubt.

These and other problems of determining low energy collision cross sections by direct methods, and the discrepancies that have resulted from them, have been responsible for attempts to obtain the cross sections by so-called "indirect" methods, in which the procedure is to measure one or more transport coefficients for an assembly of electrons in the gas and to find subsequently the energy-dependent cross section that is consistent with these results. In contrast with the methods just described, the experimental conditions are now chosen to ensure a very large number of collisions in the collision chamber rather than a single collision. As a result, collisions occur between the neutral particles and electrons having a relatively wide distribution of energies. While on the one hand this procedure is responsible for the higher accuracy obtainable from swarm experiments when studying very low energy collisions, it is also responsible for the complexity of the analytical procedures and for the fact that, until recently, the inherent accuracy had not been attained.

The application of computer techniques to the analysis of the results of swarm experiments is the factor chiefly responsible for the greatly increased accuracy that can now be claimed for the cross section obtained in this way. Although little use could be made of them, accurate formulae for the energy distribution (e.g. Chapman and Cowling 1939) and for the drift and diffusion (Barbiere 1951) of an electron swarm moving through a gas in the presence of a uniform electric field have been known for many years.* Identical formulae have been obtained more recently by free path methods (Davidson 1954; Huxley 1957; see also Huxley 1960), but before the availability of computers the only practical approach to the problem of extracting collision parameters from the transport data was through the use of somewhat drastic assumptions. In the earlier approach (e.g. Huxley and Zaazou 1949; Crompton and Sutton 1952) formulae for diffusion and drift were developed on the assumption that the energy-dependent momentum transfer cross section could be replaced by an effective mean value for any given energy distribution corresponding to a particular value of the ratio E/N of electric field strength to gas number density. Used in conjunction with assumed forms of the energy distribution function (usually Maxwellian or Druyvesteyn) these values enabled the mean value of the cross section to be calculated and plotted as a function of a parameter characterizing the energy distribution, for example, the mean energy $\bar{\epsilon} = \frac{1}{2}m\overline{c^2}$ or the mean electron speed $\bar{c}$ (see Appendix II). While this approach was the only one then possible, it clearly could not lead to an accurate determination of the energy-dependent cross section.

Recently, Frost and Phelps (1964) and Crompton and Jory (1965) have made use of the more rigorous formulae by applying iterative procedures to find an energy-dependent cross section consistent with the one or more of the transport coefficients. This procedure is described in Section V. One result of this new approach has been the demonstration of the sensitivity of the derived cross sections to the basic transport data (Crompton and Jory 1965). On the other hand it has been shown recently that the transport coefficients can be obtained with high accuracy (Crompton and Jory 1962; Crompton, Elford, and Gascoigne 1965; Elford 1966), and it now seems reasonable to expect that the energy dependence of the cross section at low energies might be obtained more accurately by this technique than has been hitherto possible by any other technique. The results for helium presented in the present paper support this view.

In addition, we have used the method to investigate the possibility of the existence of fine structure in the cross section curve at low energies. Such structure was reported by the early investigators (Ramsauer and Kollath 1929; Normand 1930) and has received further attention recently (Golden and Bandel 1965; Schulz 1965). Because the energy resolution of the "probe" in swarm experiments of this kind is relatively low, swarm experiments cannot usually compete with single-collision experiments when investigating resonance phenomena. Nevertheless, unlike beam experiments, swarm experiments are just as easy to perform, and are just as precise, at energies of 0·1–1 eV (the energy range where these phenomena have been reported) as they are at somewhat higher energies. It therefore seemed worth while to see what information could be gained from them. The results of this investigation are discussed in Section VI, where it is shown that the analysis of

* It should be noted that while the energy distribution function as discussed by Chapman and Cowling is applicable only to the case of elastic collisions, the formulae for drift and diffusion are not restricted in this way.

transport data of high relative accuracy leads to the conclusion that, if there is fine structure, it is far less significant that has been reported.

Finally, the results of our experiments are compared with recent theoretical calculations of the cross section.

II. Design of Experiments

At sufficiently low energies the energy distribution of an electron swarm in a monatomic gas is determined entirely by the ratio E/N of the electric field strength to the gas number density, the gas temperature T, and the cross section for momentum transfer between the electrons and atoms. It is possible in principle, therefore, to determine the energy dependence of the cross section simply by measuring the variation of one transport coefficient as the energy distribution of the swarm is changed by varying E/N, or T, or both. Of the transport coefficients which might be used, the drift velocity W and the ratio of the diffusion coefficient D to the mobility μ $(= W/E)$ are the two that seem capable of being measured with the highest accuracy. The measurement of both has been studied in considerable detail (Crompton and Jory 1962; Crompton, Elford, and Gascoigne 1965; Elford 1966) and it is necessary to decide which measurements are likely to lead to the more accurate cross section determination. Measurements of the other coefficient can then be used to cross-check the result. A further check is provided by the measurement of a third transport coefficient, the magnetic drift velocity W_{M} (Jory 1965 and references therein).

Over the energy range of importance in this investigation the momentum transfer cross section q_{m} in helium is known to be approximately constant. It is therefore possible to obtain information that is useful in planning the experiments through the use of simplified formulae for diffusion and drift, which result from the assumption that q_{m} is constant and that inverse collisions between the atoms and electrons are unimportant in determining the energy distribution. With these assumptions it can be shown (see Appendix I, equations (A6), (A7)) that

$$W \propto \{(E/N)q_{\mathrm{m}}^{-1}\}^{\frac{1}{2}} \qquad \text{and} \qquad D/\mu \propto \{(E/N)q_{\mathrm{m}}^{-1}\}.$$

It follows that, if q_{m} does not vary rapidly, the determination of the cross section is twice as precise when values of D/μ are used in preference to values of W, provided the measurements are of equal accuracy and apply to the non-thermal region (see also Frost and Phelps 1964). In contrast, it should be noted that it is impossible to obtain any information from measured values of D/μ in the thermal region, since the value of this ratio is then determined almost entirely by the gas temperature whereas the drift velocity W is directly proportional to q_{m}^{-1} in this region (equation (A1)). Moreover, in practice, it seems possible to measure the drift velocity with considerably higher accuracy than the corresponding value of D/μ (Crompton, Elford, and Gascoigne 1965; Elford 1966). On balance, therefore, these considerations suggest that the drift data should in general take preference as the primary data for the cross section determination. However, when the cross section is not approximately constant these conclusions no longer apply.

In designing the experiments, it is obviously desirable to make provision for studying electron swarms over the widest energy range possible. The low energy limit is set by the gas temperature, provided it is possible to make accurate measurements at very low values of E/N to ensure that the swarm is practically in thermal equilibrium with the gas. With the apparatus used in the present experiments it was not practicable to extend the measurements to low temperatures, as has been done previously by Pack and Phelps (1961). The lower limit for the mean energy of the swarm in our experiments is therefore about 0·045 eV, while the upper limit, set by the onset of electrical breakdown in the apparatus, is a little less than 2 eV. Because of the spread of the energy distributions, the transport coefficients at each end of the range of E/N are affected by electrons of considerably lower and higher energy than the mean values quoted. Significant information about the cross section can therefore be obtained over a somewhat larger energy range than the mean values quoted (Section VI).

III. Experimental Procedures

(a) General

In this section a number of techniques will be described that were common to the three separate experiments to determine W, D/μ, and W_{M}.

The experimental parameters which control the steady state energy distribution are the electric field strength E, the gas number density N, and the gas temperature T. The accurate determination of the transport coefficients therefore rests in the first instance on the precise measurement of each of these parameters, and the aim was to determine each to within 0·1%. The experimental techniques developed to achieve this aim are described in greater detail by Crompton, Elford, and Gascoigne (1965) and by Elford (1966). It is worth noting that the parameter which is difficult to measure accurately in single-collision experiments, namely, the gas number density N, presents few difficulties in our experiments. All the measurements are carried out under static conditions, so that the gas number density is accurately known throughout the apparatus provided the gas pressure and temperature are measured accurately. In both the drift tube and the D/μ apparatus the gas temperature was measured to within 0·2°C by one or more copper–constantan thermocouples either attached or adjacent to the electrode structure. For the drift velocity measurements, pressures were measured by a Texas Instruments quartz spiral manometer with a resolution of 2 parts in 100 000 over the pressure range 0–250 torr. For the measurements of D/μ and W_{M}, which were made before the quartz spiral manometer was available, precision capsule gauges (Crompton and Elford 1957) covering the pressure ranges 0–20, 0–40, and 0–500 torr were used. All gauges were calibrated against a CEC type 6201 primary pressure standard modified to give an accuracy of 0·015% consistent with the manufacturer's specifications. The pressures used in the drift velocity measurements are considered to be in error by less than 0·1%, while a $\pm 1\%$ error limit is placed on the measurement of pressure in the D/μ and W_{M} experiments. The results given in Section IV suggest that this latter value overestimates the error.

The vacuum techniques used in all the experiments are similar to those described by Crompton, Elford, and Gascoigne (1965) and by Elford (1966). Matheson Research Grade helium was used without further purification apart from that obtained by passing it through cold traps on admittance to the experimental tubes. Transfer was effected by means of a Matheson ultrapure transfer regulator and a UHV valve. The outgassing rates of the experimental tubes and associated vacuum systems were similar to those quoted previously and were such that the purity level claimed for the helium by its suppliers was not significantly degraded. The adequacy of these techniques was confirmed by the fact that no variation of the results occurred over the time required to take the measurement (several hours). Our claim that the accuracy of the measured transport coefficients is unaffected by the presence of impurity in the gas samples therefore rests on the significance of the impurity present in the gas supplied. With the exception of the work of Bowe (1960*a*, 1960*b*) all recent determinations of the elastic scattering cross section in helium have been made using research grade gas without attempting further purification. Bowe could not detect any change in his experimental results for helium after further purification of the reagent grade gas. His results show that traces of impurity in the gas supplied are of no significance, thus confirming the results of approximate calculations based on the manufacturer's analysis.

(*b*) *Drift Velocity Measurements*

The method used in these measurements was developed initially by Bradbury and Nielsen (1936). Lowke (1962) has studied the limitations to the accuracy attainable with experiments of this kind and his work determined the criteria that were followed in designing the apparatus used in our experiments. A diagram of our apparatus is shown in Figure 1. Although Lowke's work demonstrated the relative insensitivity of drift velocity measurements to field distortion in the drift space, the apparatus shown in the diagram was designed to be as free as possible from field distortion and was constructed with a high order of dimensional accuracy. It was thereby anticipated that the results obtained with this apparatus would act as a standard of reference to calibrate other apparatuses in which similar standards might be difficult or impossible to meet.

The electrode structure follows the design described by Crompton, Elford, and Gascoigne. This form of construction enables a highly uniform electric field to be generated, while at the same time it ensures screening of the drift space from stray electric fields exterior to the apparatus.

When the field produced by such an electrode structure is examined, it is found that the mid-planes of the electrodes, together with the planes passing through the centres of the gaps separating the electrodes, form a system of equipotential planes. The drift distance can therefore be terminated without disturbing the field configuration by arranging that each shutter coincide with one of the equipotential planes. In the apparatus shown in Figure 1 this arrangement is achieved by replacing two of the standard electrodes by composite electrodes with the shutters accurately positioned at their mid-planes. The thickness of the composite electrode is made equal to the thickness of the standard electrodes. A section through one of the shutter electrodes is shown in the lower part of Figure 1. The shutter itself consists of about 200 nichrome wires of 0·008 cm diameter with their centres accurately spaced 0·04 cm apart and mounted between two soda-glass annular rings of 8·8 cm

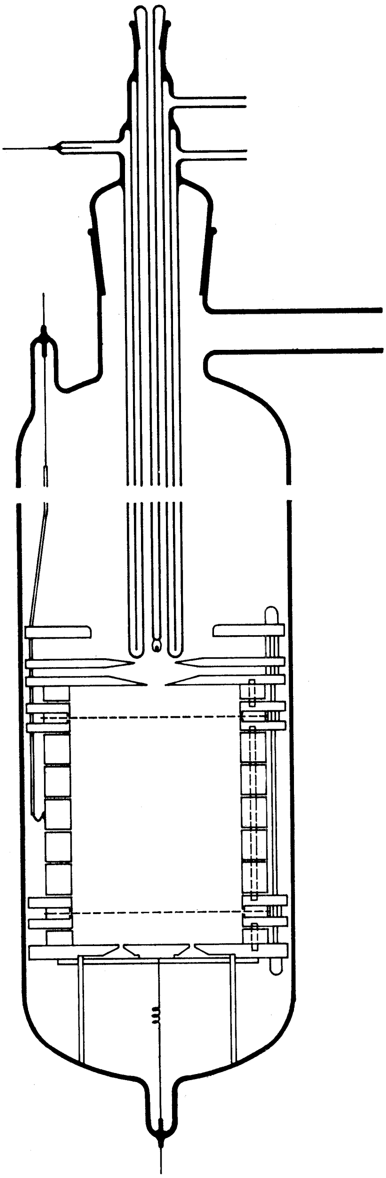
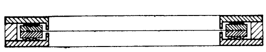


Fig. 1.—(*Above*) Diagram of the drift tube; (*below*) section through one of the electrical shutters.



internal diameter. In making the shutter the wires are stretched across a former and one of the annular rings is placed in contact with the wires. Corning Pyroceram glass frit is then applied to the annular area and the second ring placed in position to form a symmetrical ring structure with the wires at the mid-plane. The assembly is then heated in air to 400°C. This temperature is adequate for the sealing process but insufficiently high to cause appreciable oxidation of the nichrome wires. By following the procedure outlined above it is possible to construct shutters in which the wires remain in contact with accurately plane surfaces. In the completed apparatus, the drift space is thus terminated by shutters that are accurately plane and normal to the electric field.

The electrodes are made of copper and are separated by glass spacers 0·05 cm thick. The machining tolerances are similar to those for the apparatus described by Crompton, Elford, and Gascoigne. The principal dimensions of the present apparatus are:

Guard ring thickness	1·61 cm
Total chamber length	13·32 cm
Distance between shutters	9·991 ± 0·005 cm.

Before assembly all surfaces of the apparatus exposed to the electron stream were gold coated, the guard electrodes by electroplating and the shutter wires by vacuum deposition.

Although the vacuum envelope of the apparatus was designed for permanent sealing, the measurements in helium were made with the two halves of the envelope joined by a demountable Apiezon W 100 wax seal. Such an arrangement prevents the drift tube from being outgassed by baking; nevertheless the outgassing rates after prolonged pumping were found to be well within acceptable limits (see Section III(*a*)).

The experimental procedures used in the present investigation were similar to those described by Lowke and by Elford.

(*c*) *Measurements of the Ratio* D/μ

The Townsend–Huxley method was used for measuring D/μ. The experimental difficulties and their solution have been reviewed by Crompton and Jory (1962) and by Crompton, Elford, and Gascoigne (1965). The techniques used in the present experiments were similar to those described in these papers and require only one additional comment. As in the experiments in hydrogen reported by Crompton and Elford (1963), measurements made at the smallest values of E/p at the highest gas pressures showed a slight dependence on the magnitude of the total electron current. Typically, at $p = 500$ torr, $E/p_{293} = 0\cdot007$ V cm^{-1} torr^{-1}, an increase of the current from 10^{-12} to 2×10^{-12} A caused the apparent value of D/μ to change by 0·5%. In these experiments, and in a subsequent more detailed investigation into the cause of this phenomenon which will be described elsewhere, the apparent value of D/μ was found to be linearly dependent on the electron current, so that the true value of D/μ free of space charge effects could be found by simple linear extrapolation to zero current. In all cases the correction applied to the values measured at the smallest current was less than 1%; the effect was too small to observe for $E/p_{293} > 0\cdot02$ V cm^{-1} torr^{-1}.

The capsule gauges used to measure the gas pressures in this set of experiments showed noticeable hysteresis effects at the highest pressures ($\simeq$ 500 torr). The measurement of pressure in this range is therefore subject to larger errors than the measurements at lower pressures, particularly those made with the 0–40 and 0–20 torr gauges which did not exhibit this effect and for which an accuracy of $0 \cdot 25\%$ at full scale can be claimed. On the other hand, high gas pressures are only required for the measurements at very low values of E/N, where D/μ approaches kT/e and where the results are therefore relatively insensitive to the value of E/N. Small errors in pressure are thus of little importance as is confirmed by the results in Table 2.

(d) *Measurement of* W_M

Huxley's modification to Townsend's original method for measuring this transport coefficient was used in these experiments (Huxley and Zaazou 1949). The experimental apparatus and techniques used were identical with those described by Jory (1965).

Several factors contribute to a somewhat higher anticipated experimental error in these measurements. These data were taken before the quartz spiral manometer became available but, unlike the D/μ measurements, they are adversely affected by errors in the measurement of pressure at high as well as at low pressures. In addition, these measurements were taken without having the experimental tube either immersed in a water bath or surrounded by an insulating jacket, as was the case for the other experimental tubes. The temperature stability was therefore not as good.

IV. Experimental Results

(a) *Drift Velocity* W

The experimentally determined values of the electron drift velocity in helium are shown in Table 1. These values differ slightly (by less than 1%) from the values of Crompton and Jory (1965). The present results were obtained after better control of the gas temperature had been achieved and using the quartz spiral manometer rather than the less accurate capsule pressure gauges used previously.

The lower entries in Table 1 are the experimental results corrected for diffusion errors of the type described by Lowke (1962). In the first-order theory developed by Lowke, the apparent drift velocity $W' = 2f_0 d$ (where f_0 is the frequency of the first maximum in the current frequency curve and d is the drift distance) is related to the true drift velocity W by the equation

$$W' = W\left(1+\frac{C}{dW/D}\right). \tag{1}$$

The constant C depends on a number of factors, including the mode of operation of the shutters, the distribution of electron density at the first shutter (i.e. whether the electron groups are considered as originating from a point or a plane source), and the size of the collecting electrode. In Lowke's experiments the experimental results were generally consistent with equation (1) with $C = 3$. In the present experiments using an apparatus of different dimensions, equation (1) was found to overestimate the dependence of W' on pressure with this value of C, but results that agreed everywhere to within $0 \cdot 3\%$ over the range of pressures used could be obtained by using $C = 1 \cdot 5$. This value of C was therefore used to calculate the

Table 1

EXPERIMENTAL VALUES OF THE DRIFT VELOCITY W FOR ELECTRONS IN HELIUM (293°K)

The lower entries are the measured values corrected for diffusion errors

E/p_{293}	$10^{17}E/N$ (V cm^2)	$10^{-5}W$ (cm/sec) Pressure (torr) 500	200	100	50	20	Best Estimate $10^{-5}W$ (cm/sec)
0·006	0·01820	0·418(4)					
		0·418					0·418
0·007	0·02124	0·476(9)					
		0·477					0·477
0·008	0·0243	0·533(3)					
		0·533					0·533
0·009	0·0273	0·587(0)					
		0·586					0·586
0·010	0·0303	0·638(1)					
		0·637					0·637
0·012	0·0364	0·734					
		0·733					0·733
0·015	0·0455	0·864	0·866				
		0·863	0·863				0·863
0·018	0·0546	0·982	0·982				
		0·981	0·978				0·980
0·02	0·0607	1·054	1·054				
		1·053	1·050				1·052
0·025	0·0759	1·219	1·220				
		1·217	1·216				1·21(7)
0·03	0·0910	1·366	1·367	1·370			
		1·365	1·365	1·367			1·36(5)
0·04	0·1214	1·622	1·621	1·624			
		1·621	1·619	1·620			1·62(0)
0·05	0·1517	1·841	1·842	1·846			
		1·840	1·840	1·841			1·84(0)
0·06	0·1820		2·037	2·041	2·052		
			2·035	2·036	2·042		2·03(6)
0·07	0·2124		2·215	2·218	2·229		
			2·212	2·213	2·218		2·21(3)
0·08	0·243		2·377	2·380	2·390		
			2·374	2·374	2·379		2·37(4)

TABLE 1 (*Continued*)

E/p_{293}	$10^{17}E/N$ (V cm^2)	$10^{-5}W$ (cm/sec) Pressure (torr) 500	200	100	50	20	Best Estimate $10^{-5}W$ (cm/sec)
0·09	0·273		2·530	2·531	2·544		
			2·527	2·525	2·532		2·52(6)
0·10	0·303		2·671	2·674	2·686		
			2·668	2·668	2·674		2·66(8)
0·12	0·364		2·935	2·938	2·949		
			2·932	2·932	2·936		2·93(2)
0·15	0·455			3·287	3·305		
				3·280	3·291		3·28(0)
0·18	0·546			3·602	3·611	3·639	
				3·594	3·595	3·600	3·59(5)
0·20	0·607			3·799	3·804	3·827	
				3·791	3·788	3·787	3·78(9)
0·25	0·759			4·242	4·247	4·262	
				4·234	4·230	4·219	4·23(2)
0·30	0·910				4·649	4·672	
					4·630	4·625	4·63
0·40	1·214				5·359	5·373	
					5·338	5·320	5·33
0·50	1·517				5·994	6·035	
					5·971	5·977	5·97
0·60	1·820					6·61	
						6·55	6·55
0·70	2·124					7·13	
						7·07	7·07
0·80	2·43					7·65	
						7·57	7·57
0·90	2·73					8·15	
						8·07	8·07
1·00	3·03					8·65	
						8·57	8·57
1·20	3·64					9·56	
						9·47	9·47

corrected values shown in the table. However, the choice of the value of C does not critically affect the overall accuracy of the results. For example, if C differs from the chosen value by 50%, which would seem unlikely owing to the good agreement over the wide range of pressure used, the error in the corrected drift velocities is less than $0 \cdot 1\%$ for the results at 500 and 200 torr, less than $0 \cdot 3\%$ for the results at 100 and 50 torr, and less than $0 \cdot 5\%$ for the results at 20 torr.

TABLE 2

EXPERIMENTAL VALUES OF D/μ FOR ELECTRONS IN HELIUM (293°K)

E/p_{293}	$10^{17}E/N$ (V cm^2)	D/μ (volt) Pressure (torr) 40	80	100	200	400	500	Best Estimate D/μ (volt)
0·007	0·02124						0·0302	0·0302
0·008	0·0243					0·0315	0·0313	0·0313
0·009	0·0273					0·0327	0·0325	0·0325
0·010	0·0303					0·0338	0·0337	0·0337
0·012	0·0364					0·0364	0·0362	0·0362
0·015	0·0455				0·0404	0·0402	0·0400	0·0400
0·018	0·0546				0·0441	0·0442	0·0441	0·0441
0·02	0·0607				0·0468	0·0467	0·0468	0·0468
0·025	0·0759				0·0534	0·0536	0·0534	0·0534
0·03	0·0910			0·0607	0·0607	0·0604	0·0602	0·0604
0·04	0·1214		0·0743	0·0743	0·0743	0·0741	0·0738	0·0741
0·05	0·1517		0·0876	0·0876	0·0879	0·0876	0·0874	0·0874
0·06	0·1820		0·1007	0·1010	0·1015	0·1012		0·1012
0·07	0·2124		0·1141	0·1141	0·1151			0·1141
0·08	0·243	0·1273	0·1273	0·1271	0·1278			0·1271
0·09	0·273	0·1402	0·1405	0·1402	0·1412			0·1405
0·10	0·303	0·1533	0·1536	0·1536	0·1543			0·1536
0·12	0·364	0·1784	0·1786	0·1792	0·1802			0·1792
0·15	0·455	0·217(5)	0·216(8)	0·217(0)				0·217(0)
0·18	0·546	0·254(7)	0·253(7)	0·254(5)				0·254(5)
0·20	0·607	0·279(1)	0·278(6)	0·278(9)				0·278(9)
0·25	0·759	0·341	0·339	0·340				0·340
0·3	0·910	0·402	0·399					0·399
0·4	1·214	0·520						0·520
0·5	1·517	0·640						0·640
0·6	1·820	0·755						0·755
0·7	2·124	0·876						0·876
0·8	2·43	0·996						0·996
0·9	2·73	1·117						1·117
1·0	3·03	1·241						1·241

At a given value of E/p_{293} the best estimate of the value of W is found by weighting the results in favour of those taken at high pressures, since these results are the least subject to error. The "best estimate" values are considered to be in error by less than $0 \cdot 5\%$ over the range $0 \cdot 006 \leqslant E/p_{293} \leqslant 0 \cdot 6$ V cm^{-1} torr^{-1} and by less than 1% for $E/p_{293} > 0 \cdot 6$ V cm^{-1} torr^{-1}.

(b) *Ratio of Diffusion Coefficient to Mobility* D/μ

Table 2 shows the values of D/μ that were obtained from experiments in which pressures between 40 and 500 torr were used to cover the complete range of E/p_{293}. At some values of E/p_{293} it was possible to take results over the major part of the pressure range to test the self-consistency of the method. This test is somewhat

TABLE 3

EXPERIMENTAL VALUES OF W_M FOR ELECTRONS IN HELIUM (293°K)

Tabulated values are $10^{-5}W_M$ (cm/sec)

E/p_{293}	$10^{17}E/N$ (V cm^2)	Pressure (torr) 40		200		500		Best Estimate $10^{-5}W_M$ (cm/sec)
		Magnetic Flux Density (G) 40	100	40	100	40	100	
0·006	0·01820					0·487	0·480	0·480
0·007	0·02124					0·549	0·542	0·542
0·008	0·0243					0·605	0·602	0·602
0·009	0·0273					0·664	0·660	0·660
0·010	0·0303					0·716	0·715	0·715
0·012	0·0364					0·818	0·812	0·81
0·015	0·0455			0·95	0·94	0·956	0·954	0·95
0·018	0·0546			1·07	1·07	1·12	1·08	1·07
0·020	0·0607			1·15	1·14	1·15	1·16	1·15
0·025	0·0759			1·34	1·32	1·32	1·34	1·33
0·03	0·0910			1·49	1·47			1·48
0·04	0·1214			1·76	1·74			1·77
0·05	0·1517			1·99	1·97			1·98
0·06	0·1820			2·21	2·18			2·19
0·07	0·2124			2·38	2·37			2·37
0·08	0·243	2·52		2·54	2·53			2·53
0·09	0·273	2·68		2·71	2·71			2·71
0·10	0·303	2·82		2·86	2·84			2·84
0·12	0·364	3·11		3·14	3·11			3·11
0·15	0·455	3·44	3·46	3·51	3·49			3·49
0·18	0·546	3·85		3·83	3·82			3·82
0·20	0·607	4·00		4·08	4·00			4·00
0·25	0·759	4·45						4·45
0·3	0·910	4·86						4·86
0·4	1·214	5·60	5·62					5·61
0·5	1·517	6·24						6·24
0·6	1·820	6·80						6·80
0·7	2·124	7·32						7·32
0·8	2·43	7·89	7·92					7·92
0·9	2·73	8·31						8·31
1·0	3·03	8·76	8·91					8·91

more stringent than the corresponding test in the drift velocity measurements, since it involves not only a change in N and E but also a change in the distribution of current over the receiving electrode. For example, at $E/p_{293} = 0 \cdot 05$ V cm^{-1} torr^{-1} the current received by the central disk was about 25% of the total current when the pressure was 80 torr and about 85% when the pressure was 500 torr, yet the maximum discrepancy between any of the five values recorded in the table is considerably less than 1%.

Above $E/p_{293} = 0 \cdot 4$ V cm^{-1} torr^{-1} the results were taken at only one pressure, since electric field strengths greater than 16 V/cm could be used. Since random errors from such sources as contact potential differences become insignificant at such high field strengths, there is little point in checking the results at lower pressures. The agreement between the results at 40 torr and those at 80 and 100 torr for $E/p_{293} < 0 \cdot 25$ V cm^{-1} torr^{-1} reinforces the claim of reliability. Errors in pressure measurement (Section III(*a*)) may well be the major source of any residual error in these results.

The best estimate results were obtained by weighting the results somewhat in favour of those taken at the highest pressures, since these measurements were made using the highest field strengths. An examination of the tabulated results shows, however, that such a procedure is scarcely necessary or important. An analysis of the experimental errors, as described by Crompton and Jory (1962) and by Crompton, Elford, and Gascoigne (1965), suggests that these values are in error by less than 1%.

(*c*) *Magnetic Drift Velocity* W_M

The results for W_M are shown in Table 3.* A further check on the experimental data is possible when making these measurements in that more than one value of the magnetic flux density B can be used. At pressures of 200 and 500 torr flux densities of 40 and 100 G were used. In general the maximum discrepancy between results taken at different pressures or flux densities (or both) is less than 2%.

Because of a number of factors discussed by Jory (1965), the overall accuracy of the measurements is more difficult to assess than is the case for the measurements of W and D/μ. Jory's extensive set of data for nitrogen shows a maximum discrepancy of less than 3% between any two values of W_M measured at the same value of E/p with different values of p and B, and a further systematic error of less than 1% may be present owing to a possible error in the constant of proportionality between B and the current flowing through the Helmholtz coils. Since the measurements in helium were made without modifying the apparatus or experimental technique, a reasonable estimate of the maximum experimental error is about $\pm 2\%$.

V. Determination of the Momentum Transfer Cross Section

The motion of an electron swarm drifting and diffusing through a gas in the presence of an electric field has been discussed by many authors. The approach through the solution of the Boltzmann equation (e.g. Allis 1956) and an alternative

* These results differ somewhat at the lower values of E/p from those given by Crompton and Jory (1965). A redetermination of the results for $E/p_{293} < 0 \cdot 15$ V cm^{-1} torr^{-1} revealed a source of experimental error the removal of which enabled the range of measurement to be extended to $E/p_{293} = 0 \cdot 006$ V cm^{-1} torr^{-1}.

approach due to Huxley (1960) lead to the following formulae for diffusion and drift:

$$W = -(4\pi/3)(E/N)(e/m)\int_0^\infty \{c^2/q_{\mathrm{m}}(c)\}(\mathrm{d}f(c)/\mathrm{d}c)\,\mathrm{d}c\,, \tag{2}$$

$$D = (4\pi/3N)\int_0^\infty \{c^3/q_{\mathrm{m}}(c)\}f(c)\,\mathrm{d}c\,, \tag{3}$$

where e and m are the electronic charge and mass respectively and $f(c)$ is the function describing the distribution of the electron speeds c, as defined subsequently.

If, in addition, a magnetic field $\boldsymbol{B}$ is applied at right angles to $\boldsymbol{E}$, the longitudinal drift velocity $W_{\parallel}$ remains to a first order unchanged but a component of velocity $W_{\perp}$ perpendicular to both $\boldsymbol{E}$ and $\boldsymbol{B}$ is introduced whose magnitude is given by

$$W_{\perp} = -(4\pi/3)(E/N)(B/N)(e/m)^2\int_0^\infty \{c/q_{\mathrm{m}}^2(c)\}(\mathrm{d}f(c)/\mathrm{d}c)\,dc\,, \tag{4}$$

provided $Be/m \ll Nq_{\mathrm{m}}(c)\,c$ for the majority of electrons in the swarm (Huxley 1960).

For the reasons discussed in Section II, application of equation (2) forms the basis of the method for determining the energy dependence of q_{m} from the transport coefficients we have measured. The other transport coefficients are used simply to check the correctness of the cross section determined in this way.

Equation (2) shows that W depends explicitly on q_{m}; it also depends implicitly on the cross section through the distribution function $f(c)$. In general the energy distribution of the swarm is determined by the ratio E/N, the gas temperature T, and the elastic and inelastic collision processes between the electrons and the molecules of the gas. In the present case, however, the experiments were performed in a monatomic gas with swarms of electrons of insufficient energy to produce electronic excitation. The collision processes controlling the energy distribution were therefore limited to an exchange of kinetic energy between the electrons and helium atoms so that the momentum transfer cross section alone determined the distribution.

There have been many theoretical treatments of this situation (see, for example, Loeb 1955 and McDaniel 1964 for general references, and Huxley 1960). In several of these treatments the assumption is made that inverse collisions between the atoms and electrons produce a negligible modification to the distribution, i.e. that the value of E/N is sufficiently high that the gas temperature plays no part in determining the electron energy (e.g. Druyvesteyn 1930; Morse, Allis, and Lamar 1935). Such an assumption is not adequate in analysing the results of the present experiments, since, at the lowest values of E/N, the mean electron energy is less than twice the thermal energy. When the influence of inverse collisions is included it has been shown (e.g. Chapman and Cowling 1939) that the symmetric term in the velocity distribution function is given by

$$f(c) = A\exp\left\{-\int^c \frac{3mc\,\mathrm{d}c}{3kT+M(e/m)^2(E/N)^2\{cq_{\mathrm{m}}(c)\}^{-2}}\right\}, \tag{5}$$

where M is the atomic mass and k is Boltzmann's constant. $4\pi c^2 f(c)\,\mathrm{d}c$ is the fraction of electrons with speeds between c and $c+\mathrm{d}c$; the constant A can therefore be determined from the normalizing relation

$$4\pi\int_0^\infty f(c)c^2\,\mathrm{d}c = 1.$$

Equations (2) and (5) can be used to determine $q_m(c)$ in the following way. A trial set of 50 values of q_m as a function of c is used in conjunction with equation (5) to calculate $f(c)$ for a set of values of E/N corresponding to those at which the transport coefficients have been measured experimentally. At each value of E/N, $f(c)$ is calculated by a digital computer for 100 values of c distributed over a chosen range. To determine this range, the measured value of D/μ at each value of E/N is used to estimate the most probable electron speed for this value of E/N. A suitable range on either side of the most probable speed can then be chosen to include a sufficiently high proportion of the electrons in the swarm. Equation (2) is now used in conjunction with the calculated values of $f(c)$ to compute the value of W. Finally the set of values of W computed in this way is compared with the experimental values. Successive adjustments to the input data for $q_m(c)$ are made until the fit between calculated and experimental values shows discrepancies only of the same order as the scatter in the experimental data, in our experiments of the order of $0\cdot1\%$.

In order to commence the iterative process it is necessary to have a first approximation to the energy-dependent cross section. In this instance the cross section calculated by Frost and Phelps (1964) was used as the initial input data to the computer programme. Their cross section was obtained using a procedure similar to ours, and, since their drift velocity data were on the whole in good agreement with our data, no large modification to their cross section was expected. In the absence of such information, however, an adequate approximation to the cross section can be found by applying a straightforward analysis to the transport data. This analysis is similar to that used earlier to deduce mean values of collision parameters from the transport coefficients (Huxley and Crompton 1962) and is outlined in Appendix II.

Unlike the situation for molecular gases (e.g. Frost and Phelps 1962), data for only one transport coefficient are required to determine the cross section in the present case. The other experimental data for D/μ and W_M therefore enable a cross-check to be made on the validity of the cross section deduced from the drift velocity data.

Equations (2) and (3) can be combined to give the following formula for D/μ:

$$D/\mu = \frac{-(m/e)\int_0^\infty \{c^3/q_m(c)\}f(c)\,\mathrm{d}c}{\int_0^\infty \{c^2/q_m(c)\}(\mathrm{d}f(c)/\mathrm{d}c)\,\mathrm{d}c}. \tag{6}$$

Values of D/μ can be calculated for appropriate values of E/N using equation (6) together with the cross section determined from the drift velocity data. These values are then compared with the experimental values.

The coefficient $W_\perp$ (equation (4)), which describes the transverse motion of the electron swarm in a magnetic field, is not the most convenient coefficient to use when comparing calculated and experimental results. The quantity that is actually measured by the Townsend–Huxley method is the ratio $W_\perp/W_\parallel$ (Jory 1965). While a comparison of this ratio could be made with values calculated using equations (4) and (2) (since $W_\parallel \simeq W$), this is still not the most convenient way of comparing experimental and calculated values, since, at a given value of E/N, the ratio $W_\perp/W_\parallel$

is a function of B/N. A more convenient quantity to use is W_{M}, which is defined by the equation

$$W_{\mathrm{M}} = (E/B)(W_{\perp}/W_{\parallel}) \simeq \frac{(Ee/Nm)\int_0^\infty \{c/q_{\mathrm{m}}^2(c)\}(\mathrm{d}f(c)/\mathrm{d}c)\,\mathrm{d}c}{\int_0^\infty \{c^2/q_{\mathrm{m}}(c)\}(\mathrm{d}f(c)/\mathrm{d}c)\,\mathrm{d}c}. \tag{7}$$

Inspection of equation (7) shows firstly that, like the other transport coefficients W and D/μ, W_{M} is a function of E/N and T only (since $f(c)$ and hence the values of the integrals are determined by these parameters) and secondly that W_{M} has the dimensions of velocity. Furthermore, when the momentum transfer collision

TABLE 4

THE MOMENTUM TRANSFER CROSS SECTION $q_{\mathrm{m}}(\epsilon)$ FOR ELECTRONS IN HELIUM

ϵ (eV)	$10^{16}q_{\mathrm{m}}(\epsilon)$ (cm^2)	ϵ (eV)	$10^{16}q_{\mathrm{m}}(\epsilon)$ (cm^2)
$2\cdot0\times10^{-2}$	5·40	$4\cdot0\times10^{-1}$	6·50
2·5	5·45	5·0	6·60
3·0	5·50	6·0	6·67
4·0	5·58	7·0	6·73
5·0	5·65	8·0	6·78
6·0	5·71	9·0	6·83
7·0	5·77	$1\cdot0\times10^{0}$	6·87
8·0	5·82	1·2	6·94
9·0	5·86	1·5	6·99
$1\cdot0\times10^{-1}$	5·89	1·8	7·00
1·2	5·97	2·0	7·00
1·5	6·07	2·5	6·96
1·8	6·15	3·0	6·89
2·0	6·19	4·0	6·60
2·5	6·29	5·0	6·26
3·0	6·37	6·0	6·01

frequency $\nu_{\mathrm{m}} = Nq_{\mathrm{m}}(c)c$ is constant, equation (7) reduces to

$$W_{\mathrm{M}} = Ee/m\nu_{\mathrm{m}},$$

that is, W_{M} is equal to the drift velocity W in this special case. W_{M} is, in fact, the "drift velocity" measured by Townsend and his collaborators (Healey and Reed 1941). When ν_{m} is approximately constant, or when the energy distribution is narrow, W_{M} is a good estimate of W. In other circumstances the use of this technique to measure the drift velocity may be quite unsatisfactory. If, for example, measurements are made by this method in the thermal region in a gas for which q_{m} is proportional to c, the drift velocity would be overestimated by a factor of three (cf. Jory 1965).* In the present work the data for W_{M} serve only as an additional check on the validity of the cross section. The results of this test, and of the test using the data for D/μ, are given in the following section.

* It has been shown (Huxley 1940) that the method used to determine $W_{\perp}/W_{\parallel}$ in the earlier experiments may be subject to appreciable error; drift data determined by this method are therefore subject to a further small error in addition to the error discussed here.

VI. Results and Discussion

The procedures described in the previous section have been used to derive the cross section listed in Table 4. In Table 5 a comparison is made between the experimental values of W and the values calculated using this cross section. This comparison shows not only the success that has been achieved in finding a cross section that is consistent with the experimental data but also the smallness of the scatter in these data. Table 5 also shows a similar comparison for the D/μ and W_M results. While the fit between calculated and measured values of W can be

Table 5

A comparison of experimental and computed values of W, D/μ, and W_M for electrons in helium (293°K)

E/p_{293}	$10^{17}E/N$ (V cm^2)	$10^{-5}W$ Expt. (cm/sec)	$10^{-5}W$ Calc.* (cm/sec)	D/μ Expt. (V)	D/μ Calc.* (V)	$10^{-5}W_M$ Expt. (cm/sec)	$10^{-5}W_M$ Calc.* (cm/sec)
0·006	0·01820	0·418	0·418			0·480	0·486
0·007	0·02124	0·477	0·477	0·0302	0·0303	0·542	0·551
0·008	0·0243	0·533	0·533	0·0313	0·0314	0·602	0·613
0·009	0·0273	0·586	0·586	0·0325	0·0326	0·660	0·671
0·010	0·0303	0·637	0·637	0·0337	0·0339	0·715	0·727
0·012	0·0364	0·733	0·733	0·0362	0·0364	0·812	0·830
0·015	0·0455	0·863	0·864	0·0400	0·0403	0·947	0·970
0·018	0·0546	0·980	0·981	0·0441	0·0443	1·07	1·10
0·020	0·0607	1·052	1·053	0·0468	0·0469	1·15	1·17
0·025	0·0759	1·217	1·218	0·0534	0·0537	1·33	1·35
0·03	0·0910	1·365	1·364	0·0604	0·0605	1·48	1·50
0·04	0·1214	1·620	1·619	0·0741	0·0741	1·77	1·77
0·05	0·1517	1·840	1·839	0·0874	0·0876	1·98	2·01
0·06	0·1820	2·036	2·034	0·1012	0·1009	2·19	2·21
0·07	0·2124	2·213	2·211	0·1141	0·1142	2·37	2·40
0·08	0·243	2·374	2·373	0·1271	0·1273	2·53	2·58
0·09	0·273	2·526	2·525	0·1405	0·1404	2·71	2·74
0·10	0·303	2·668	2·667	0·1536	0·1533	2·84	2·89
0·12	0·364	2·932	2·929	0·1792	0·1790	3·11	3·17
0·15	0·455	3·280	3·280	0·217(0)	0·217(1)	3·49	3·54
0·18	0·546	3·595	3·595	0·254(5)	0·254(8)	3·82	3·88
0·20	0·607	3·789	3·788	0·278(9)	0·279(7)	4·00	4·08
0·25	0·759	4·232	4·232	0·340	0·341	4·45	4·55
0·3	0·910	4·630	4·630	0·399	0·403	4·86	4·98
0·4	1·214	5·33	5·33	0·520	0·523	5·61	5·73
0·5	1·517	5·97	5·96	0·640	0·643	6·24	6·38
0·6	1·820	6·55	6·53	0·755	0·762	6·80	6·98
0·7	2·124	7·07	7·07	0·876	0·881	7·32	7·53
0·8	2·43	7·57	7·58	0·996	1·000	7·92	8·05
0·9	2·73	8·07	8·06	1·117	1·121	8·31	8·55
1·0	3·03	8·57	8·54	1·241	1·244	8·91	9·03
1·2	3·64	9·47	9·47				

* The theoretical values of W, D/μ, and W_M were computed using the momentum transfer cross section listed in Table 4.

carried out to any degree warranted by the data, the fit between calculated and measured values of the other transport coefficients represents a sensitive test of the validity of the cross section required to fit the data for W. The agreement to within $\pm 1\%$ for the D/μ data is therefore particularly satisfactory, since it lies within the claimed experimental accuracy of these results. The agreement for the W_{M} data is equally satisfactory, bearing in mind the higher experimental error discussed in Section IV(c). The measured values of the transport coefficients in the range $1\cdot 8\times 10^{-19} < E/N < 3\times 10^{-17}$ V cm^2 are therefore seen to form a body of data which is consistent with the momentum transfer cross section of Table 4 and Figure 2.

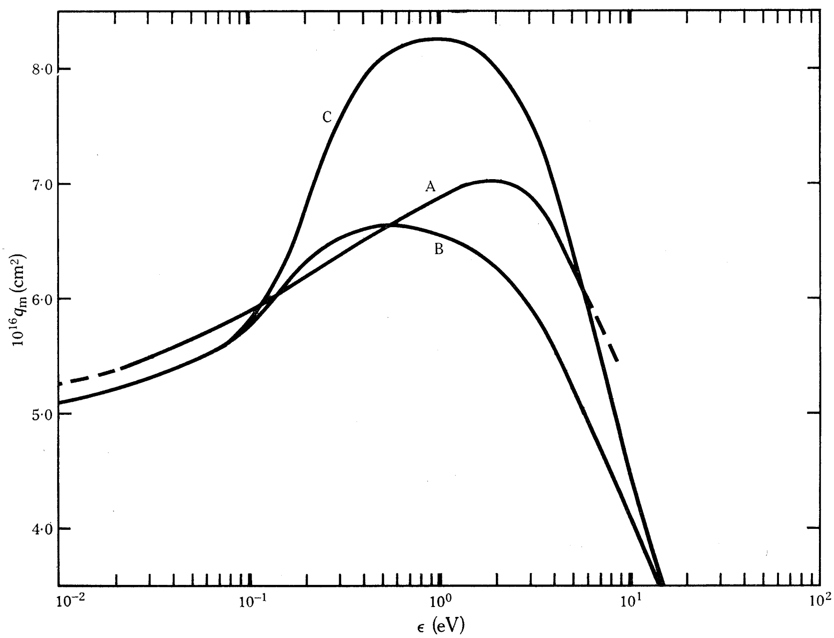


Fig. 2.—The energy dependence of the momentum transfer cross section determined from swarm experiments. Curve A, Crompton, Elford, and Jory; curve B, Frost and Phelps; curve C, Frost and Phelps (using Bowe's data).

The cross section of Table 4 is on the whole in good agreement with the cross section published earlier by Crompton and Jory (1965). Where there are discrepancies they arise from the use of the new drift velocity data, which are characterized by considerably reduced experimental scatter in addition to improved overall accuracy. The discrepancies between the two sets of drift velocity data amount to 1% or less, the experimental error claimed for the earlier data. For the reasons discussed in Appendix I, any discrepancy between two sets of drift velocity data leads to a discrepancy between the derived cross sections that is approximately twice as great. Moreover, an additional source of error arose in the earlier cross section determination from the fact that there was no justification for attempting to fit calculated and experimental values of W to better than 1%. It has now been possible to reduce considerably this source of error. Despite the somewhat higher error to be expected in the results of the earlier analysis, the agreement between the two cross sections is to within about 3% over the whole energy range.

(a) *A Comparison of Experimental Determinations of* q_m

There have been a number of determinations of q_m from analyses of drift velocity data or the data obtained from microwave experiments. Until the recent papers of Frost and Phelps (1964) and Crompton and Jory (1965) the approximations used were that either q_m is independent of electron energy or is related to it by a simple power law. Derivations based on these assumptions were surveyed by Frost and Phelps (1964). Since these earlier approximate methods are inferior to the more recent analyses in which these assumptions are not made, no data based on the use of such methods have been included in Figure 2.

The procedure used by Frost and Phelps to analyse the drift velocity data was shown to be identical with that used by Crompton and Jory and with that used in the present work by comparing the calculated transport coefficients when the same input cross sections were used in the respective computer programmes.* The cross section obtained by Frost and Phelps (1964) using the drift velocity data of Pack and Phelps (1961) is shown as curve B in Figure 2 while our cross section is shown as curve A. Curve C is the cross section calculated by Frost and Phelps from the drift velocity data of Bowe (1960*a*).

The energy range over which our cross section is considered to be reliable is indicated by the full curve. The lower limit to the energy range is set by there being an insufficient number of electrons in the swarm at the lowest values of E/N with energies less than about 2×10^{-2} eV to enable the curve-fitting procedure to be carried out. Frost and Phelps (1964) used drift velocity data taken at a gas temperature of 77°K and hence the electron swarms used in their measurements had a minimum mean energy of approximately 1×10^{-2} eV. The energy range over which their curve-fitting procedure is valid therefore extends to an energy of about 5×10^{-3} eV. A similar argument applies to the upper limit of the energy range, our cross section being considered unreliable for electron energies greater than about 6 eV.

The largest difference between the curves shown in Figure 2 is between curves B and C. At approximately 1 eV the difference is 20%. Our cross section is 15% higher than that of Frost and Phelps at 3 eV but the disagreement decreases as the energy decreases, the difference being less than 3% for all energies less than about 0·6 eV. Since the analysis of the experimental data is the same in each case, the disagreement between the three cross sections simply reflects the differences between the sets of drift velocity data from which they were derived.†

* This comparison was made possible through an exchange of input and output data with Dr Phelps. The authors are grateful to him for drawing attention to a numerical error in an earlier programme. Tests which were subsequently applied to check the adequacy of the programme are described in Appendix III.

† Bowe (personal communication) has recently revised his drift velocity data and they now agree with our data to within 2%. The cross section derived from his new data will therefore be in good agreement with our cross section.

In the remainder of this section we first compare experimental determinations of q_m with derivations of q_m from measurements of the total scattering cross section. Secondly, we discuss the existence of fine structure in the cross section at low energies in the light of our transport data. Finally, a comparison is made between the experimental results for the momentum transfer cross section and recent theoretical calculations.

(b) *A Comparison of the Momentum Transfer Cross Sections derived Experimentally with those calculated from Total Scattering Cross Sections*

The available experimental values of the total scattering cross section are shown in Figure 3. The most recent measurements are those of Golden and Bandel (1965). The other measurements shown were all made prior to 1931.

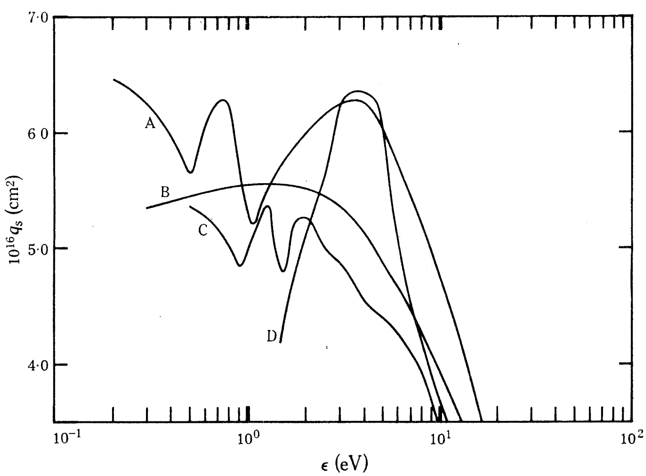


Fig. 3.—The energy dependence of the total elastic scattering cross section determined from single-scattering experiments. Curve A, Ramsauer and Kollath; curve B, Golden and Bandel; curve C, Normand; curve D, Brode.

In order to compare the total scattering cross section q_s obtained from single-collision experiments with the momentum transfer cross section q_m derived from an analysis of drift velocity data, a knowledge of the differential scattering cross section is required. The only experimental measurements of the angular scattering cross section for low energy electrons in helium are those of Ramsauer and Kollath (1932). Using these data, Barbiere (1951) has calculated values of q_m from the measured values of q_s of Ramsauer and Kollath (1929) (curve F, Fig. 4). We have applied a similar procedure to the data of Golden and Bandel (1965) to obtain curve E; in both cases the lower limit of approximately 1·5 eV is set by the range of angular scattering data available. In order to obtain values of q_m from Golden and Bandel's data at energies lower than 1·5 eV two procedures have been used. The first is to use modified effective range theory (O'Malley 1963); the momentum transfer cross section obtained by Golden (1966) using this technique is shown as curve C. The second method is to use the theoretical angular scattering data of La Bahn and Callaway (1966), the cross section so obtained being curve D. Of all the curves shown, the closest agreement between any of those derived from measurements of q_s (curves C, D, E, and F), and those determined directly from swarm experiments (curves A and B) is that between curves D and A. The agreement between the form of these two curves is gratifying, since both the measurements of Golden and Bandel and the calculations of La Bahn and Callaway are probably the most reliable of their kind available. The almost constant difference of 10% between curves D and A suggests, however, the existence of a systematic error. The reliability of the calculations of La Bahn and Callaway is discussed in Section VI(d).

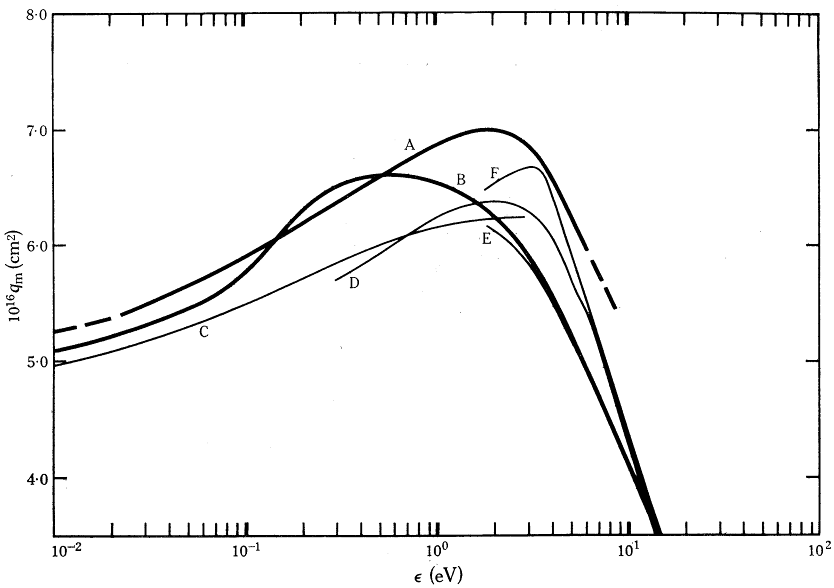


Fig. 4.—A comparison of the momentum transfer cross section found from swarm experiments with the cross section deduced from single-scattering experiments. Curve A, Crompton, Elford, and Jory; curve B, Frost and Phelps; curve C, Golden and Bandel–O'Malley; curve D, Golden and Bandel–La Bahn and Callaway; curve E, Golden and Bandel–Ramsauer and Kollath, Barbiere; curve F, Ramsauer and Kollath–Barbiere.

The differences between the curves of Figure 4 may arise from one or more of the following sources:

(1) errors in the measurements of q_s

(2) errors in the measurement of the differential scattering cross section

(3) an inadequacy in the theoretical derivation of the differential scattering cross section

(4) an inadequacy in the modified effective range theory

(5) errors in the experimental determination of q_m from drift velocity measurements.

Three facts support the view that the last possibility is the least likely. Firstly, the absolute value of the cross section determined by the analysis of drift data depends on the measurement of comparatively high pressures under static conditions. Such a measurement is a straightforward procedure. Secondly, the accuracy of the energy scale at low energies does not depend on the reliable determination of very small accelerating potentials. For even the lowest energy swarms, the potential difference between the shutter planes in the drift tube was never less than 30 V. Finally, the measured values of all three transport coefficients form a set which is consistent with our proposed cross section.

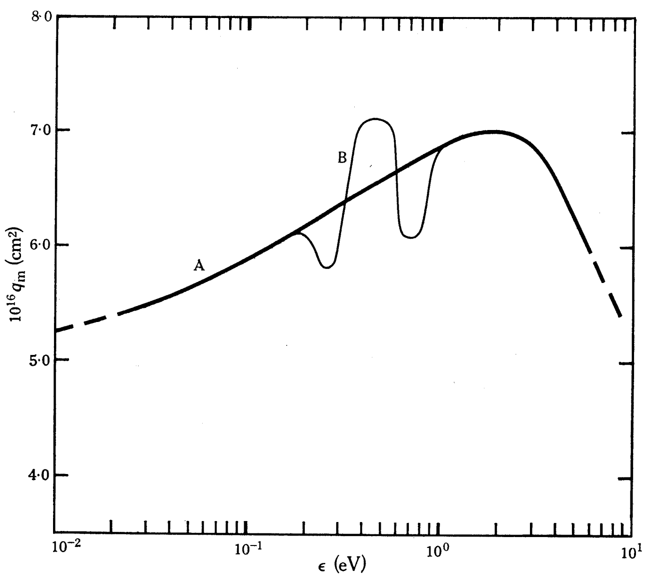


Fig. 5.—The cross sections used to investigate the effect of fine structure. Curve A ———, derived from transport coefficients (Crompton, Elford, and Jory); curve B ———, curve A incorporating resonance.

(c) *Fine Structure in the Momentum Transfer Cross Section*

From Figure 3 it can be seen that Normand (1930) and Ramsauer and Kollath (1929) observed fine structure in the total cross section for electrons in helium in the energy region 0·5–2 eV. More recently Schulz (1965) reported the presence of a "resonance" in the total cross section for electrons in helium at 0·45 eV, although no details were given of either the shape or magnitude of the resonance. On the other hand, Golden and Bandel (1965), using the same technique as that used by Ramsauer and Kollath (1929), found no evidence of fine structure.

If fine structure exists in the total cross section, similar structure should occur in the momentum transfer cross section and should therefore be found in the cross section obtained by the procedure given in Section V. No evidence for an oscillation was found in the cross section derived from our drift velocity data but, since in swarm measurements the electrons have a wide distribution of energies, it might be thought that a resonance which occurs over a narrow range of energies would produce little effect on the transport coefficients.

To check whether the effect on the transport coefficients would be observable, the following procedure was adopted. A hypothetical resonance similar to that of Ramsauer and Kollath was placed on our momentum transfer cross section (see Fig. 5). The position of the resonance was taken from Schulz's observation, while its shape and amplitude were based on Ramsauer and Kollath's data. This cross section was then used as input data and the computed values of W were compared with those obtained experimentally (Fig. 6). It can be seen that the computed

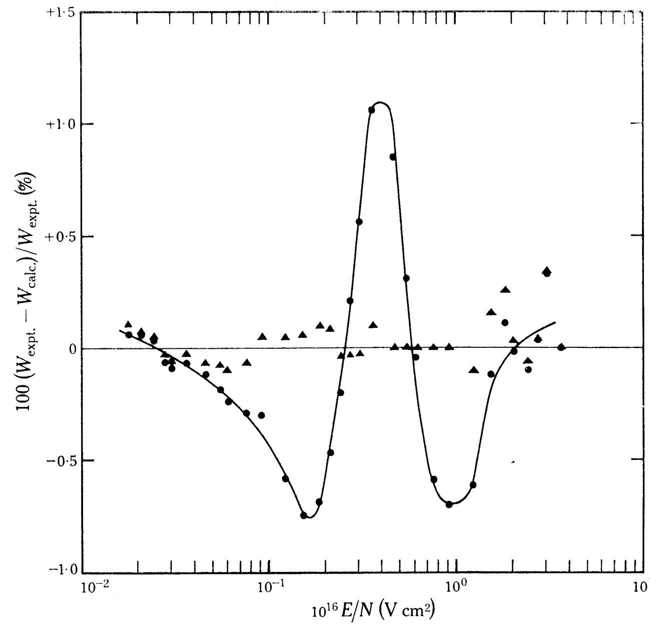


Fig. 6.—The differences between experimental and calculated drift velocities using the cross sections of Figure 5; ▲ using cross section A, ● using cross section B.

values of W differ from the experimental values by much more than the experimental scatter. It should be noted that, although we have not claimed an absolute accuracy of better than 0·5–1% for our drift data, the high *relative* accuracy obtained, as evidenced by the small experimental scatter shown in Figure 6, enables a perturbation in the drift velocity caused by an oscillation of the type shown in Figure 5 to be easily detected. Furthermore, it can be seen that if a resonance of the form shown in Figure 5 did exist, then its magnitude would have to be less than a quarter of that used in the present calculation.

An error curve of the same type as that shown in Figure 6 was also obtained for the values of D/μ. The magnitude of the maximum deviation in this case is, however, only approximately twice the experimental scatter and hence no firm conclusions could be made on the basis of the D/μ data alone.

O'Malley (1963) has discussed the theoretical basis for the existence of an oscillation in this cross section at an electron energy of approximately 0·5 eV. Both theory and experiment indicate that the higher order phase shifts are almost negligibly small below 2 eV so that the structure would have to originate from the behaviour of the S-wave phase shift. No mechanism has been suggested that would produce such behaviour. Calculations which have been made suggest that the S-wave phase shift should vary smoothly with electron energy. O'Malley has suggested that the oscillation observed by Normand and by Ramsauer and Kollath could have been caused by impurities in the gas used in their experiments and has calculated that the oscillation which they observed could result from contamination of their helium by 3% of oxygen and nitrogen.

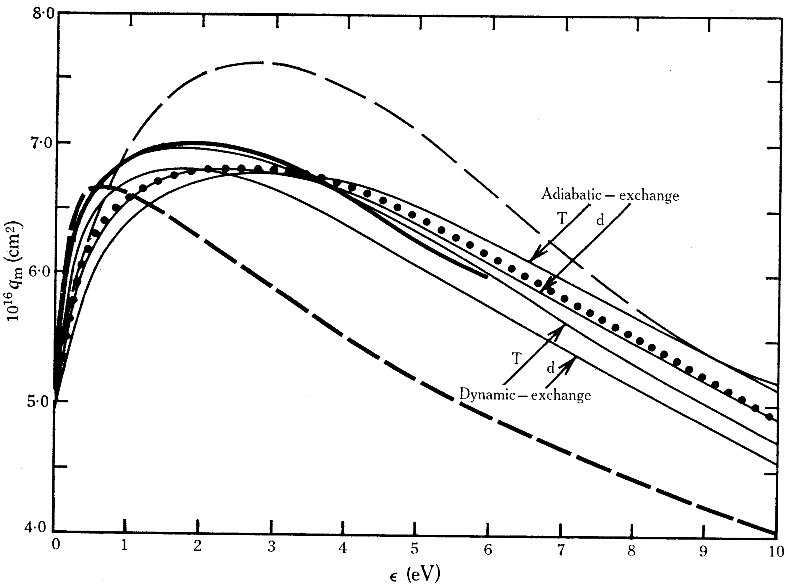


Fig. 7.—A comparison between theoretical and experimental momentum transfer cross sections. ——— Crompton, Elford, and Jory; — — Frost and Phelps; — — Bauer and Browne; ——— La Bahn and Callaway; ● ● ● Williamson and McDowell.

(*d*) *Comparison between Experimental and Theoretical Momentum Transfer Cross Sections*

Since the first extensive calculations of the total scattering cross section by Morse and Allis (1933) there have been a number of theoretical treatments of this problem. The most recent are those of La Bahn and Callaway (1964), Bauer and Browne (1964), Williamson and McDowell (1965), and La Bahn and Callaway (1966). The results of these calculations are shown in Figure 7 together with our cross section and that of Frost and Phelps.

Four curves are shown as being due to La Bahn and Callaway (1966). In order to discuss these curves, each of which is based on a particular set of assumptions, it is necessary to discuss the interaction forces between a scattering electron and a helium atom. There are two types of forces. The first is the exchange force which occurs as a result of the Pauli exclusion principle between the incident electron and the atomic electrons. The importance of this force was shown by the work of Morse and Allis (1933). The second type of interaction results from the distortion of the atomic system by the presence of the scattering electron. These distortion effects are of several kinds. The first distortion effect is direct polarization. In dealing with the polarization interaction most theoretical treatments have been based on the adiabatic approximation, in which it is assumed that the velocity of the incident electron always remains substantially smaller than that of the atomic electrons; usually only the dipole term of the polarization interaction is retained. The curve marked adiabatic-exchange-d was obtained by La Bahn and Callaway (1964) by making these approximations and including the exchange interaction. When higher order terms of the polarization interaction are retained the curve marked adiabatic-exchange-T was obtained. In 1966 La Bahn and Callaway suggested that the adiabatic approximation is no longer valid at higher electron energies since a velocity-dependent interaction term in the distortion effects should be present. This velocity-dependent term results from the failure of the atomic system to readjust instantaneously as the position of the incident electron changes and was shown by La Bahn and Callaway to give rise to an additional repulsive interaction which has a long range behaviour. The curve obtained when this velocity-dependent interaction term is taken into account and when only the dipole term of the polarization interaction term is retained is marked dynamic-exchange-d. When the higher order terms of the polarization force are included, the curve marked dynamic-exchange-T is obtained. One point to note is that, although the velocity-dependent interaction term was introduced to improve the theory for higher energy electron scattering, there is a significant change in the cross section in the lower energy region as well.

The cross sections of Williamson and McDowell (1965) and of Bauer and Browne (1964) were derived on the adiabatic assumption. There is good agreement between the results of Williamson and McDowell and those of La Bahn and Callaway (adiabatic-exchange-d), both derivations being made using similar assumptions.

Our experimentally determined cross section is in excellent agreement with the curve marked dynamic-exchange-T. Over the energy range for which our cross section is considered reliable (2×10^{-2} to 6 eV) the maximum difference is less than 2%. This suggests that it is necessary to include the velocity-dependent interaction in a theoretical discussion of this problem and also that the higher order polarization terms are significant.

The cross section of Frost and Phelps is approximately 3% lower than the dynamic-exchange-T curve at 1 eV and 20% lower at 6 eV.

Our results do not extend to sufficiently low energies to enable the scattering length to be determined by extrapolation to zero energy. However, the agreement between our results and those of La Bahn and Callaway (dynamic-exchange-T) suggests that the scattering length calculated by them for this cross section, that is, $a = 1 \cdot 186 a_0$ (where a_0 is the Bohr radius), is not greatly in error.

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Appendix I

In this appendix formulae for the transport coefficients are developed on the basis of one or more simplifying assumptions. There are two reasons for doing so. Firstly, the simple formulae derived enable the dependence of W and D/μ on q_m and the experimental parameters to be easily seen, thus enabling a decision to be made as to which experimental measurements of the transport coefficients are likely to lead to the most accurate determination of the cross section. Equations (2) and (3) do not lend themselves to an examination of this kind. Secondly, a recent determination of the cross section from drift velocity data (Bowe 1966) has been made using arguments similar to those developed here, and it is necessary to examine the validity of the assumptions upon which this approach rests.

With the assumption that q_m is constant, equations (2) and (3) lead to the well-known formulae:

$$W = \tfrac{2}{3}(E/N)(e/m)q_m^{-1}\overline{c^{-1}}, \tag{A1}$$

$$D = (1/3N)q_m^{-1}\bar{c}. \tag{A2}$$

A third equation relates the balance of the energy supplied to the electrons by the electric field to the loss of energy as a result of collisions (in this case, elastic). Expressed in terms of a coefficient η, which is a measure of the average energy loss per electron per collision (e.g. Huxley and Crompton 1962), this equation reads

$$\begin{aligned} eEW &= \eta\bar{\epsilon}\bar{\nu}_m(c) \\ &= \eta.\tfrac{1}{2}m\overline{c^2}.Nq_m\bar{c}, \end{aligned} \tag{A3}$$

($\nu_m(c)$ being the momentum transfer collision frequency) with the assumption of constant q_m. When only elastic collisions need be considered, the value of η approaches an asymptotic value as E/N is increased. This situation is attained when the mean energy of the electrons in the swarm is considerably in excess of the thermal value (Huxley and Crompton 1962).*

With these two assumptions, therefore, namely that q_m is constant and that inverse collisions between the atoms and electrons are unimportant in determining the energy distribution, equations (A1), (A2), and (A3) can be solved to give the following formulae for W and D/μ:

$$W = \{(4\eta/27)[(\overline{c^{-1}})^3\overline{c^2}\bar{c}]\}^{\frac{1}{4}}(e/m)^{\frac{1}{2}}\{(E/N)q_m^{-1}\}^{\frac{1}{2}}, \tag{A4}$$

$$D/\mu = \{(1/3\eta)[\bar{c}/\overline{c^{-1}}\,\overline{c^2}]\}^{\frac{1}{2}}\{(E/N)q_m^{-1}\}. \tag{A5}$$

In equations (A4) and (A5) the terms $[(\overline{c^{-1}})^3\overline{c^2}\bar{c}]$ and $[\bar{c}/\overline{c^{-1}}\,\overline{c^2}]$ are dimensionless factors whose values depend on the distribution of electron speeds. With the assumptions already made, the distribution function represented by equation (5) reduces to the form derived by Druyvesteyn and the evaluation of these terms is then straightforward. Substitution of numerical values for the dimensionless factors, for η, and for the physical constants leads to the following formulae in the case of helium:

* With the assumption of Druyvesteyn's distribution, to which the distribution will closely approximate when $\bar{\epsilon} > \frac{3}{2}kT$, equation (53) in this paper becomes $\eta = 2\cdot40(m/M)\times(1-3kT/2\bar{\epsilon})$, so that $\eta = 2\cdot40\ m/M$ for $\bar{\epsilon} \gg \frac{3}{2}kT$.

$$W = 4\cdot07\times10^{6}\{(E/N)q_{\mathrm{m}}^{-1}\}^{\frac{1}{2}} \text{ cm/sec,} \tag{A6}$$

$$D/\mu = 27\cdot8\{(E/N)q_{\mathrm{m}}^{-1}\} \text{ volts,} \tag{A7}$$

where E is expressed in V/cm.

The significance of the inverse collisions, whose influence was ignored in deriving these formulae, can be found by comparing the values of W and D/μ calculated from these formulae with the corresponding values calculated for the same constant cross section using the computer programme described in Section V. Using a value of $q_{\mathrm{m}} = 7\cdot0\times10^{-16}$ cm^2, such a comparison showed agreement to within 1% for W and just outside 2% for D/μ for $E/p_{293} > 0\cdot5$ V cm^{-1} torr^{-1}. As would be expected, the magnitude of the discrepancies steadily increases as the mean energy of the swarm approaches the thermal value. Thus, provided that the cross section does not vary rapidly and that $D/\mu \gg kT/e$, equations (A6) and (A7) represent with reasonable accuracy the dependence of the transport coefficients on the cross section. Their use in designing the experiments is discussed in Section II.

Recently, Bowe (1966) has pointed out that for both his results and for our own the quotient $W(E/p)^{-\frac{1}{2}}$ is almost constant for the range $0\cdot1 < E/p_{293} < 1\cdot0$ V cm^{-1} torr^{-1}, from which he has inferred that q_{m} is constant, at least over a restricted range of energy. The derivation of equation (A4) shows that the validity of this conclusion depends on whether or not the effect of the molecular motion is negligible over the whole of the range of E/p in question. As a test for the significance of the molecular motion, the constant value of $W(E/p)^{-\frac{1}{2}}$ calculated from equation (A6) was compared with values of the same factor calculated from the results of the computer programme, the same constant cross section being used in each case. The computer-calculated results showed a decreasing value for this ratio as E/p decreased, the values falling away with increasing rapidity.

From equation (A4) it follows that

$$W(E/p)^{-\frac{1}{2}} \propto \{\eta[(\overline{c^{-1}})^{3}\,\overline{c^{2}}\,\bar{c}]\}^{\frac{1}{2}}q_{\mathrm{m}}^{-\frac{1}{2}}, \tag{A8}$$

from which it can be seen that the decrease in $W(E/p)^{-\frac{1}{2}}$ is a result of a decrease in the term $\{\eta[(\overline{c^{-1}})^{3}\,\overline{c^{2}}\,\bar{c}]\}$ as the mean energy of the swarm approaches the mean molecular energy.

As Bowe observed, the factor $W(E/p)^{-\frac{1}{2}}$ found from the experimental results is almost constant for $0\cdot1 < E/p_{293} < 1\cdot0$ V cm^{-1} torr^{-1}. Although there is a slight decrease in $W(E/p)^{-\frac{1}{2}}$ as $E/p \to 0\cdot1$, this trend is considerably less than the trend in the computer-calculated values. Equation (A8) shows that a cross section which decreases towards lower energies would provide the compensation required to keep $W(E/p)^{-\frac{1}{2}}$ constant despite the fact that $\eta[(\overline{c^{-1}})^{3}\,\overline{c^{2}}\,\bar{c}]$ decreases as E/p decreases. At the highest values of E/p in this range the influence of the molecular motion is found to be negligible, that is, the computer-calculated results agree with the value calculated from equation (A6). On the other hand the experimental values of $W(E/p)^{-\frac{1}{2}}$ increase slightly as E/p increases, and this trend can only be accounted for by assuming that the cross section decreases towards higher energies. A cross section that varies in the way shown in Figure 2 is therefore seen to be more reasonable than a constant cross section in explaining the behaviour of the factor $W(E/p)^{-\frac{1}{2}}$, the approximate constancy of which is seen to be fortuitous.

Appendix II

In the absence of any information on the energy dependence of q_{m}, the following procedure can be used to obtain a first approximation for use as the initial input data for the computer calculation (Section V). It will be shown that, at least in the case of helium where the cross section does not vary rapidly, the cross section calculated by the procedure described below is a good approximation to the cross section finally determined.

The procedure is to calculate an effective mean value of q_{m}, denoted by q_{m}^{*}, which is consistent with the measured transport coefficients at each value of E/N, together with an estimate of the mean speed $\bar{c}$ in each case. Pairs of values of q_{m}^{*} and $\bar{c}$ then serve as the first approximation to pairs of values of q_{m} and c. At first sight, equation (A4) appears suitable for calculating q_{m}^{*}, but it has to be remembered that this relation can be conveniently used only when the coefficient η is constant, in which case it leads to equation (A6). The assumption of a constant value for η implies that no energy is fed back from the atoms to the electrons in inverse collisions, so that it is clearly unrealistic to use equation (A6) near thermal equilibrium. An alternative approach is possible, however, when experimental data for both W and D/μ are available. By combining equations (A1) and (A2) the following formula for q_{m} is obtained

$$q_{\mathrm{m}} = \tfrac{1}{3}(2e/m)^{\frac{1}{2}}[\bar{c}\,\overline{c^{-1}}]^{\frac{1}{2}}(E/N)/W(D/\mu)^{\frac{1}{2}}. \tag{A9}$$

This formula is, of course, only strictly applicable when q_{m} is constant. Nevertheless, when q_{m} is a slowly varying function of the electron energy, equation (A9) can be used to calculate an effective mean value of q_{m}, that is, q_{m}^{*}.

In calculating the cross section from the measured transport coefficients in this way, the value of $\overline{c^{-1}}$ which appears in equation (A1) is calculated from the mean energy of the swarm, which is in turn estimated from the measured value of D/μ. This is in contrast to any method using the power balance equation (e.g. the use of equation (A4)), which relies on a knowledge of the energy loss at collisions, in this discussion expressed through the coefficient η. When equation (A9) is used, detailed knowledge of the collision processes is required only to determine the *form* of the energy distribution in order to calculate the dimensionless speed average term $[\bar{c}\,\overline{c^{-1}}]$. In practice, as shown below, the exact form of the distribution is, in fact, unimportant.

Equations (A1) and (A2) can be used to estimate the mean speed $\bar{c}$ of the electrons in the swarm. It can be readily shown that $\bar{c}$ is given by

$$\bar{c} = (2e/m)^{\frac{1}{2}}[\bar{c}\,\overline{c^{-1}}]^{\frac{1}{2}}(D/\mu)^{\frac{1}{2}}. \tag{A10}$$

The Druyvesteyn distribution represents the energy distribution reasonably well over a significant portion of the range of E/N, and with this distribution the dimensionless average factors can be evaluated to give

$$q_{\mathrm{m}} = 2\cdot 15\times 10^{7}\{(E/N)/W(D/\mu)^{\frac{1}{2}}\}\ \mathrm{cm}^{2}, \tag{A11}$$

$$\bar{c} = 5\cdot 93\times 10^{7}(D/\mu)^{\frac{1}{2}}\ \mathrm{cm/sec}, \tag{A12}$$

where E/N is expressed in V cm^2, W in cm/sec, and D/μ in volts.

As $E/N \to 0$ it would of course be more appropriate to use a Maxwellian distribution. However, with this distribution, the coefficients in the formula for q_{m} and $\bar{c}$ are changed by less than 5%. Errors arising from an incorrect assumption as to the form of the energy distribution are therefore trivial in this application and equations (A11) and (A12) may be used for all E/N in determining q^*_{m} as a function of $\bar{c}$. It was this procedure that was used in many earlier investigations before the procedure described in Section V became practicable.

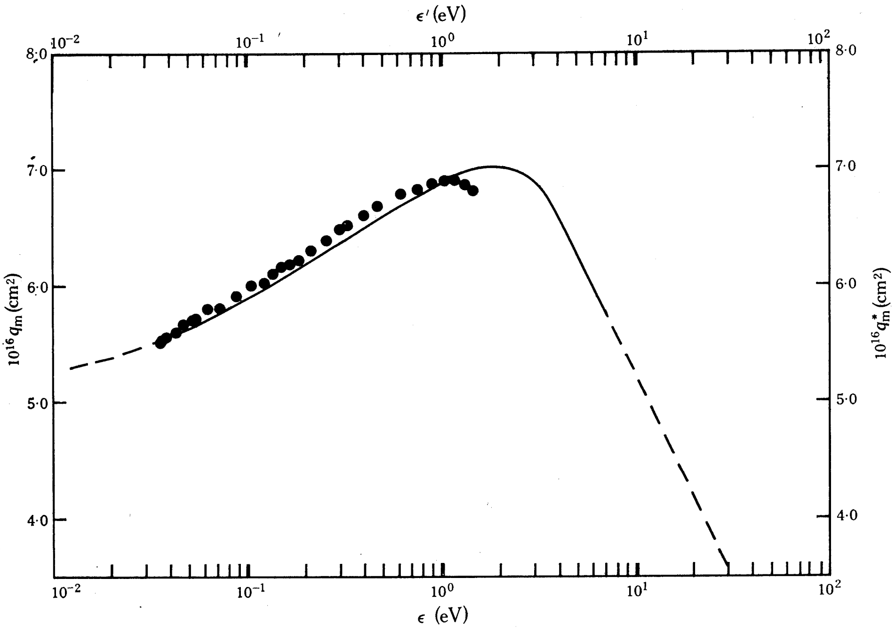

Fig. 8.—A comparison between the cross section obtained from the accurate analysis with that obtained using approximate formulae.

• $q^*_{\mathrm{m}} = 2\cdot 15\times 10^7\ (E/N)/W(D/\mu)^{\frac{1}{2}}$.

——— Final computed cross section.

The results of this procedure are shown in Figure 8, in which q^*_{m} is plotted against $\epsilon' = \frac{1}{2}m(\bar{c})^2$. On the same diagram curve A of Figure 2 is plotted for comparison. Where the cross section varies slowly, the comparison shows that $q^*_{\mathrm{m}}(\epsilon')$ is a good approximation to $q_{\mathrm{m}}(\epsilon)$ and that it is, in fact, quite adequate everywhere to serve as the initial input in the iterative process for determining q_{m}.

Two other points should be made. Firstly, the approximate treatment described above is useful in deciding the range over which the final cross section is likely to be reasonably accurate. The $q^*_{\mathrm{m}}(\epsilon')$ curve can be drawn for values of ϵ' covering the range from the value corresponding to the value of $\bar{c}$ at the lowest value of E/N to the corresponding value at the highest value of E/N for which measurements were made. It is therefore to be expected that q_{m} could be accurately calculated from the transport data, at least over this energy range. In practice it can be calculated with reasonable accuracy from these data over a somewhat wider energy range (Section VI), since there are significant numbers of electrons in the swarm with speeds much less than $\bar{c}$ at the lowest value of E/N and much greater than $\bar{c}$ at the highest value of E/N.

Secondly, this procedure is useful in serving as a guide when adjusting the input values of q_{m} during the iterative process. The curve for $q_{\mathrm{m}}^{*}(\epsilon')$ shown in Figure 8 is derived from the experimentally determined values of W and D/μ and is therefore invariant. A similar curve can be obtained from the computer-calculated values of W and D/μ at any stage during the iterative process and this curve can be compared with the curve in Figure 8. The points of disagreement between the two curves can be used as a guide when correcting the input data; when the final cross section curve is obtained the two curves of $q_{\mathrm{m}}^{*}(\epsilon')$ calculated from the experimental and computed data should be coincident.

While this procedure is useful in arriving at the final values for the cross section, it should be stressed again that the final curve depends only on the drift velocity data, since adjustments are made until the W versus E/N curve is fitted to the accuracy justified by the experimental results. In this way the maximum accuracy is achieved, since the determination rests only on the more accurately determined transport coefficient W.

Appendix III

The computer programme described in Section V was checked in the following way.

At sufficiently small values of E/N, the electron swarm is in thermal equilibrium with the gas molecules so that the energy distribution is completely specified once the gas temperature is specified. If the momentum transfer cross section is assumed constant, the integral in equation (2) (Section V) can be evaluated to give the following expression for the drift velocity of thermal electrons in any gas

$$W = \tfrac{2}{3}(2e^2/\pi mkT)^{\frac{1}{2}}\,(E/N)q_{\mathrm{m}}^{-1}\,. \tag{A13}$$

The value of W calculated from equation (A13) can be used to check the computer-calculated value obtained for a value of E/N chosen to ensure that $\bar{\epsilon} = \frac{3}{2}kT$.

It is also possible to check the programme when E/N is made so large that $M(e/m)^2[cq_{\mathrm{m}}(c)]^{-2}(E/N)^2 \gg 3kT$ (equation (5)). The distribution function again assumes a simple form, provided $q_{\mathrm{m}}(c)$ is of the form $q_{\mathrm{m}}(c) = q_0\,c^n$, so that equation (2) once more reduces to a form that can be calculated simply without numerical integration. For example, for constant q_{m} the following expression for the drift velocity is obtained

$$W = \tfrac{2}{3}\{\Gamma(\tfrac{1}{2})/\Gamma(\tfrac{3}{4})\}(e/m)^{\frac{1}{2}}\{3m/4M\}^{\frac{1}{4}}\{(E/N)q_{\mathrm{m}}^{-1}\}^{\frac{1}{2}}\,. \tag{A14}$$

As is to be expected, equation (A4) reduces to equation (A14) after making the appropriate substitution for η.

When these two tests were applied to our programme, agreement between the computer-calculated values and the values obtained from equations (A13) and (A14) was better than $0\cdot1\%$.