NUCLEAR LEVEL DENSITIES IN INTERMEDIATE AND HEAVY NUCLEI

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Summary

The level densities of intermediate and heavy nuclei have been fitted to the free gas model formula with some improvement over previous fits. The predictions of the model regarding cross section behaviour have been tested and found to lead to anomalous behaviour of the partial widths for incident neutron energies above 1·5 MeV.

I. Introduction

Knowledge of the average resonance parameters for a large number of nuclides is necessary for applications in astrophysics, reactor physics, and fission physics. In many instances, owing to the extremely short lifetimes of the nuclides involved, no data are available or else the data are too crude to give sufficiently reliable estimates. Of 80 major fission product nuclides causing reactor poisoning, 40% have no measured resonance parameters. To calculate their effect on the neutron flux, the capture cross section (averaged over energy groups) must be computed using estimated values for the s-wave strength function S_0, the average radiation width $\bar{\Gamma}_\gamma$, and the average level spacing $\bar{D}$. Of these, the first two appear to be reasonably systematic functions of mass number, and linear interpolation between experimentally determined values is expected to yield reliable estimates for the unknown parameters. However, the average level spacing sensitively depends on even–odd effects and on nuclear shell structure, making interpolation difficult and inaccurate. The level density for a gas of free nucleons was first derived by Bethe (1936, 1937) and this expression, after suitably correcting for interactions present in actual nuclei, has been widely used to predict nuclear level densities. The most recent and most accurate free gas formula is due to Gilbert and Cameron (1965) and involves the use of parameters derived from a semi-empirical mass law (Cameron 1957) to correct for the effects of nucleon pairing and shell structure. However, this formula still gives an average deviation factor between experimental and calculated level densities of $\sim 1\cdot 75$, which is well outside the experimental errors. Since the capture cross section depends quite sensitively on $\bar{D}$ (once S_0 and $\bar{\Gamma}_\gamma$ have been determined), the present treatment was devised to improve upon this fit, which is not exact enough for reactor physics applications.

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II. The Level Spacing Formula

At an energy E above the ground state, the density of states of spin J is given by Gilbert and Cameron (1965) as

$$\rho(E,J) = \frac{\pi^{\frac{1}{2}}}{12}\frac{\exp\{2(aU)^{\frac{1}{2}}\}}{a^{\frac{1}{4}}U^{5/4}}\frac{(2J+1)\exp\{-(J+\frac{1}{2})^2/2\sigma^2\}}{2(2\pi)^{\frac{1}{2}}\sigma^3}. \quad (1)$$

Since actual nuclei have important interaction energies which depress their ground states below those of the corresponding gases, an "effective" excitation energy U measured from the conceptual ground state of the gas is used in equation (1) in place of the nuclear excitation energy E. If the ground state of the gas is approximated by the reference mass of odd–odd nuclei, then U is given by

$$U = E - \Delta E\,, \quad (2)$$

where ΔE is the nucleon pairing energy, which may be subdivided into separate contributions from neutrons and protons:

$$\Delta E = P(N) + P(Z)\,. \quad (3)$$

The pairing correction has the practical justification of removing even–odd effects from the level density parameter a. The variable σ is the spin cutoff parameter and is related to the nuclear moment of inertia and the nuclear temperature. Gilbert and Cameron (1965) give

$$\sigma^2 = 0\cdot 0888\,(aU)^{\frac{1}{2}}A^{\frac{2}{3}}\,, \quad (4)$$

where A is the mass number of the compound nucleus. The level density parameter in Bethe's theory is directly proportional to A, but, although this general trend prevails, rather large deviations occur in the vicinity of magic number nuclei. These deviations are removed very well by use of the shell correction parameters of Cameron's semi-empirical mass law. The total shell correction S is again subdivided into proton and neutron contributions,

$$S = S(N) + S(Z)\,, \quad (5)$$

and it is found that a linear correlation between a/A and S exists for undeformed nuclei,

$$a/A = 0\cdot 00917S + 0\cdot 142, \quad (6)$$

while deformed nuclei can be represented by a parallel line,

$$a/A = 0\cdot 00917S + 0\cdot 120. \quad (7)$$

This gives a certain physical meaning to the shell correction since the level density parameter is directly related to the density of single-particle states at the Fermi energy. Thus the pairing corrections and shell corrections derived from the semi-empirical mass law remove the even–odd effects and shell effects from the level density parameter. Since the mass formula gives deviations of the order of 200 keV from the measured masses, we cannot expect the correction parameters to be known to better than this accuracy. Furthermore, the separation of the total mass correction into pairing and shell contributions is not unique; the pairing

TABLE 1

SHELL AND PAIRING CORRECTIONS

Z or N	P(Z) (MeV)		P(N) (MeV)		S(Z) (MeV)	S(N) (MeV)
	Present Work	Gilbert and Cameron	Present Work	Gilbert and Cameron		
28	1·28	1·20		1·30	−18·60	
29	0·26				−18·70	
30	0·88	1·06		1·27	−18·01	
31	0·19				−17·87	
32	1·35	1·35		1·29	−17·08	
33	−0·05		0·08		−16·60	15·52
34	1·52	1·43	1·41	1·41	−16·75	16·38
35	−0·09		−0·08		−16·50	17·16
36	1·17	1·17	1·50	1·50	−16·35	17·55
37	0·04		−0·05		−16·22	18·03
38	1·24	1·24	2·24	1·50	−16·41	17·59
39	0·29		−0·47		−16·89	19·03
40	1·09	1·20	1·43	1·43	−16·43	18·71
41	0·26		−0·15		−16·68	18·80
42	1·17	1·28	1·44	1·88	−16·73	18·99
43	0·23		0·06		−17·45	18·46
44	1·15	1·28	1·56	1·47	−17·29	18·25
45	−0·08		0·25		−17·44	17·76
46	1·35	1·35	1·57	1·57	−17·82	17·38
47	0·34		−0·16		−18·62	16·72
48	1·05	1·36	1·46	1·46	−18·27	15·62
49	0·28		0·00		−19·39	14·38
50	1·27	1·19	0·93	0·93	−19·91	12·88
51	0·00		0·01		−19·14	13·23
52	1·05	1·14	0·62	0·72	−18·26	13·81
53	0·00		−0·50		−17·40	14·90
54	1·00	1·12	1·42	1·12	−16·42	14·86
55	0·09		0·13		−15·77	15·76
56	1·20	1·58	1·52	1·29	−14·37	16·20
57	0·20		−0·65		−13·91	17·62
58	1·40	1·17	0·80	0·94	−13·10	17·73
59	0·93		−0·08		−13·11	18·16
60	1·00	1·18	1·29	1·24	−11·43	18·67
61	−0·20		−0·47		−10·89	19·69
62	1·19	1·22	1·25	1·25	−10·75	19·51
63	0·09		−0·44		−10·62	20·17
64	0·97	0·97	0·97	1·57	−10·41	19·48
65	0·00		0·08		−10·21	19·98
66	0·92	0·92	1·65	1·32	−9·85	19·83
67	0·11		−0·11		−9·47	20·20
68	0·68	0·62	1·26	1·15	−9·03	19·72
69	0·05		−0·46		−8·61	19·87
70	0·68	0·68	1·06	1·24	−8·13	19·24
71	−0·22		0·22		−7·46	18·44
72	0·79	0·64	1·55	1·43	−7·48	17·61
73	0·09		−0·07		−7·20	17·10

TABLE 1 (*Continued*)

Z or N	P(Z) (MeV)		P(N) (MeV)		S(Z) (MeV)	S(N) (MeV)
	Present Work	Gilbert and Cameron	Present Work	Gilbert and Cameron		
74	0·69	0·72	1·37	1·09	−7·13	16·16
75	0·01		0·10		−7·06	15·90
76	0·72	0·75	1·20	1·20	−6·78	15·33
77	0·00		−0·27		−6·64	14·76
78	0·40	0·71	0·92	1·04	−6·64	13·54
79	0·16		−0·35		−7·68	12·63
80	0·73	0·87	1·19	0·70	−7·89	10·65
81	0·00		0·00		−8·41	10·10
82	0·46	0·83	1·05	0·85	−8·49	8·89
83	0·17		−0·25		−7·88	10·25
84	0·89	0·89	1·61	0·76	−6·30	9·79
85	0·00		−0·21		−5·47	11·39
86	0·79	0·79	0·90	0·92	−4·78	11·72
87	0·00		−0·21		−4·37	12·43
88	0·89	0·89	0·74	0·99	−4·17	12·96
89	0·00		−0·38		−4·13	13·43
90	0·81	0·78	0·72	1·10	−4·32	13·37
91	−0·06		−0·34		−4·55	12·96
92	0·69	0·69	0·92	0·92	−5·04	12·11
93	−0·20		−0·26		−5·28	11·92
94	0·71	0·61	0·94	0·73	−6·06	11·00
95	−0·12		0·01		−6·28	10·80
96		0·72	0·65	0·70		10·42
97			−0·36			10·39
98		0·77	0·83	0·87		9·69
99			0·11			9·27
100			0·67	0·61		8·93
101			0·05			8·57
102			1·00	0·69		8·02
103			0·51			7·59
104			1·04	0·55		7·33
105			0·33			7·23
106			0·68	0·40		7·05
107			−0·27			7·42
108			0·81	0·73		6·75
109			0·09			6·60
110			0·75	0·58		6·38
111			0·17			6·36
112			0·86	0·86		6·49
113			0·14			6·25
114			1·10	1·13		5·85
115			−0·22			5·48
116			0·84	0·84		4·53
117			−0·47			4·30
118			0·48	0·79		3·39
119			0·02			2·35
120			0·88	0·82		1·66

TABLE 1 (*Continued*)

Z or *N*	*P*(*Z*) (MeV)		*P*(*N*) (MeV)		*S*(*Z*) (MeV)	*S*(*N*) (MeV)
	Present Work	Gilbert and Cameron	Present Work	Gilbert and Cameron		
121			0·24			0·81
122			0·52	0·71		0·46
123			0·27			−0·96
124			0·41	0·41		−1·69
125			−0·05			−2·53
126			0·38	0·38		−3·16
127			0·15			−1·87
128			0·67	0·67		−0·41
129			0·00			0·71
130			0·61	0·61		1·66
131			0·00			2·62
132			0·78	0·78		3·22
133			0·00			3·76
134			0·67	0·67		4·10
135			0·00			4·46
136			0·67	0·67		4·83
137			0·00			5·09
138			0·79	0·79		5·18
139			0·00			5·17
140			0·60	0·60		5·10
141			0·04			5·01
142			0·64	0·57		4·97
143			−0·06			5·09
144			0·45	0·49		5·03
145			0·05			4·93
146			0·26	0·43		5·28
147			−0·22			5·49
148			0·39	0·50		5·50
149			0·00			5·37
150			0·39	0·39		5·30

corrections for odd Z or N are zero and those for even Z or N are found by making the variation of $S(Z)$ and $S(N)$ as smooth as possible. To obtain an exact fit to the experimental level densities, the shell and pairing corrections were re-evaluated subject to the restriction imposed by the mass formula that $S(Z)+P(Z)$ and $S(N)+P(N)$ remain unaltered. To do this, it was found necessary to relax the assumption of zero pairing correction for odd Z and N values, which corresponds to correcting the characteristic levels of the Fermi gases.

The experimental *s*-wave level spacings were weighted according to the percentage error and the correction parameters were least-squares adjusted. Care must be taken in the region of mass number 100, where a peak in the *p*-wave strength function occurs, since in these nuclei *p*-wave resonances are to be found at thermal energies. In most cases other than ^{93}Nb, l values for particular resonances are not

TABLE 2

LEVEL SPACINGS OF NUCLEI

Values followed by k are in units of keV

Nuclide	J	Binding Energy (MeV)	Observed $\bar{D}$ (eV)	Calculated $\bar{D}$ (eV)				
			s wave	s wave		s wave at 1 MeV	p wave	p wave at 1 MeV
				Present Work	Gilbert and Cameron			
28 Ni 60	0	7·97	23±3·0 k	23 k	19 k	11 k	12 k	5·9 k
29 Cu 63	3/2	7·916	1·0±0·15k	1·1 k	840 k	490	840	390
65	3/2	7·061	2·0±0·4 k	1·2 k	900	510	950	400
30 Zn 64	0	8·00		3·6 k	5·1 k	1·6 k	1·9 k	840
66	0	7·046	4·7±1·0 k	4·8 k	6·5 k	2·0 k	2·5 k	1·0 k
67	5/2	10·20	750±200	590	270	250	460	190
68	0	6·41		3·2 k	8·1 k	1·2 k	1·7 k	640
31 Ga 69	3/2	7·71	300±100	120	180	49	94	38
71	3/2	7·10	160±50	260	250	100	210	80
32 Ge 70	0	7·415	1·0±0·2 k	1·1 k	2·1 k	390	550	200
72	0	6·785	3·0±1·0 k	2·6 k	3·4 k	930	1·4 k	480
73	9/2	10·20	77±9	77	160	28	58	21
74	0	6·43		4·8 k	4·5 k	1·6 k	2·5 k	840
33 As 75	3/2	7·327	87±8	88	86	32	69	25
34 Se 74	0	7·96		660	720	240	340	120
76	0	7·408	1·5±0·35k	1·5 k	1·2 k	500	760	260
77	1/2	10·50	140±20	140	110	52	110	41
78	0	6·959	3·3±0·5 k	3·4 k	2·0 k	1·1 k	1·8 k	590
80	0	6·897	3·3±0·8 k	3·2 k	3·5 k	1·1 k	1·6 k	570
82*	0	5·0		72 k	63 k	24 k	38 k	13 k
35 Br 79	3/2	7·871	55±10	53	42	20	42	16
81*	3/2	7·597	65±15	67	99	26	53	20
36 Kr 83*	9/2	10·54		68	68	26	51	19
84*	0	6·92		4·8 k	4·8 k	1·8 k	2·5 k	930
85*	9/2	9·92		350	350	150	260	110
86*	0	5·53		31 k	31 k	11 k	16 k	6 k
37 Rb 85*	5/2	8·82	1·0±1·0 k	51	48	21	40	16
87*	3/2	6·24	1·25±0·5 k	1·2 k	1·2 k	480	980	380
38 Sr 87	9/2	11·14		110	110	48	83	36
88*	0	5·46		33 k	33 k	12 k	17 k	6 k
89*	5/2	7·768		920	1·1 k	340	710	260
90*	0	5·8		5·4 k	12 k	1·8 k	2·8 k	930
39 Y 89*	1/2	6·866	2·0±0·5 k	2·0 k	1·3 k	820	1·6 k	650
90*	2	7·9		420	310	170	330	130
91*	1/2	6·59		520	730	200	410	150
40 Zr 90	0	7·201	4·5±0·8 k	4·4 k	5·1 k	1·7 k	2·3 k	870
91*	5/2	8·633	290±40	290	400	110	220	85
92*	0	6·752	1·3±0·3 k	1·4 k	3·8 k	500	730	260
93*	5/2	8·2		500	360	170	390	130
94*	0	6·47	2·5±0·4 k	2·2 k	2·1 k	700	1·1 k	360
96*	0	5·6	1·0±0·3 k	890	3·4 k	250	460	130
41 Nb 93	9/2	7·197	83±20	83	120	30	62	23
42 Mo 92	0	7·860	2·4±1·0 k	2·6 k	3·1 k	1·0 k	1·4 k	540
94	0	7·42	1·0±0·3 k	790	2·1 k	290	400	150
95*	5/2	9·158	200±30	200	140	69	150	53
96	0	6·86	1·0±0·3 k	1·6 k	1·5 k	510	810	260
97*	5/2	8·29	160±25	240	200	74	190	57
98*	0	4·9		2·6 k	10 k	670	1·3 k	350
100*	0	5·6		1·5 k	2·1 k	380	770	190
43 Tc 99*	9/2	6·58	24±3	24	51	7·5	18	5·7
44 Ru 101*	5/2	9·218	14±3	14	23	4·5	11	3·4
102*	0	6·30		630	920	170	320	88
104*	0	5·2		740	2·4 k	170	380	89

* Nuclides that are major fission products.

TABLE 2 (*Continued*)

Nuclide	J	Binding Energy (MeV)	Observed $\bar{D}$ (eV)	Calculated $\bar{D}$ (eV)				
			s wave	s wave		s wave at 1 MeV	p wave	p wave at 1 MeV
				Present Work	Gilbert and Cameron			
45 Rh 103*	1/2	7·064	26±3	26	35	8·2	20	6·3
105*	7/2	6·47		4·1	12	1·2	3·1	0·9
46 Pd 105*	5/2	9·45	13·3±1·7	15	13	4·4	12	3·4
106*	0	6·38		220	490	57	110	29
107*	5/2	8·970		14	13	3·8	11	2·9
108*	0	6·24		190	410	47	98	24
47 Ag 107	1/2	7·271	12±6	14	15	4·1	11	3·2
109*	1/2	6·821	16±3	16	16	4·5	12	3·5
48 Cd 110	0	7·25		31	130	8·4	16	4·3
111	1/2	9·405	19·5±1·0	18	14	5	15	3·9
112	0	6·38	125±35	200	320	49	100	25
113*	1/2	9·046	22·7±4	22	21	5·9	17	4·6
114	0	6·16	140±40	170	380	39	85	20
49 In 113	9/2	7·346	6·0±1·5	5·7	2·9	1·6	4·3	1·2
115*	9/2	6·761	6·5±1·0	7·2	5·1	1·9	5·4	1·4
50 Sn 112	0	8·043	25±7	33	65	10	17	5·1
114	0	7·525	150±50	120	89	34	61	17
115	1/2	9·572	50±20	34	15	9·7	26	7·5
116	0	6·948	180±50	150	160	41	79	21
117	1/2	9·322	25±5	26	18	7·6	20	5·8
118	0	6·491	180±50	190	390	50	97	25
119	1/2	9·107	30±8	31	38	9·1	24	7·1
120	0	6·29		1·3 k	700	330	640	170
51 Sb 121	5/2	6·804	14±3	14	9·1	4	11	3
123	7/2	6·459	28±10	28	32	8·2	21	6·3
125*	7/2	6·21		94	77	28	71	22
52 Te 123	1/2	9·414	22±8	22	21	6·3	17	4·9
124	0	6·52		300	410	80	150	41
125	1/2	9·107	60±15	60	42	18	46	14
126	0	6·30		930	910	260	480	130
128*	0	5·09		5·1 k	10 k	1·4 k	2·6 k	710
130*	0	5·95	5·5±0·8 k	5·6 k	12 k	1·9 k	2·9 k	970
53 I 127*	5/2	6·801	13·5±0·7	13·0	11	3·9	10	3·0
129*	7/2	6·580	21±6	19	32	5·8	14	4·4
131*	7/2	6·310		110	210	36	81	27
135*	7/2	3·0		27 k	44 k	8 k	20 k	6 k
54 Xe 129	1/2	9·34		13	16	3·7	9·9	2·9
131*	3/2	8·932	25±10	25	41	7·9	20	6·1
132*	0	6·76		510	1·2 k	160	260	80
133*	3/2	6·14		7 k	3·5 k	2·1 k	5·5 k	1·6 k
134*	0	6·55		5·6 k	6·9 k	2·0 k	2·9 k	1·0 k
135*	3/2	7·94		2·7 k	2·4 k	1·1 k	2·1 k	830
136*	0	3·9		79 k	160 k	23 k	41 k	12 k
55 Cs 133*	7/2	6·702	18·5±0·5	19	31	5·7	14	4·3
134*	4	8·69		51	16	18	38	13
135*	7/2	6·99		140	110	48	100	37
137*	7/2	4·93		860	1·2 k	260	660	200
56 Ba 135	3/2	9·21	50±8	47	38	15	36	12
136	0	6·94		900	1·9 k	280	460	140
137	3/2	8·59		290	410	99	230	77
138*	0	4·70	8·6±4·0 k	10 k	36 k	2·6 k	5·3 k	1·3 k
57 La 138	5	8·78	32±5	32	14	11	24	8·2
139*	7/2	4·53		430	480	110	330	84
58 Ce 140*	0	5·38	3·0±1·0 k	2·3 k	2·4 k	540	1·2 k	280
142*	0	5·39	1·0±0·2 k	1·1 k	1·1 k	240	580	120
59 Pr 141*	5/2	5·86	115±10	110	27	29	86	22

* Nuclides that are major fission products.

TABLE 2 (*Continued*)

Nuclide	J	Binding Energy (MeV)	Observed $\bar{D}$ (eV)	Calculated $\bar{D}$ (eV)				
			s wave	*s* wave		*s* wave at 1 MeV	*p* wave	*p* wave at 1 MeV
				Present Work	Gilbert and Cameron			
60 Nd 143*	7/2	7·814	40±10	38	9	8·8	29	6·7
144*	0	5·93		86	200	18	44	9·2
145*	7/2	7·565	22±4	23	36	5·8	18	4·4
146*	0	5·27		500	1·2 k	110	260	54
148*	0	5·1		250	910	50	130	25
150*	0	3·9		2·7 k	9·4 k	450	1·4 k	230
61 Pm 147*	7/2	5·95	3·7±3	3·1	7·7	0·75	2·4	0·57
148*	1	7·25		2·5	6·7	0·61	1·9	0·47
62 Sm 147	7/2	8·13	8±13	7·5	8·4	1·8	5·7	1·4
148*	0	5·874		150	260	32	79	16
149*	7/2	7·986	2·8±0·3	2·7	5·1	0·6	2·1	0·5
150*	0	5·534		100	260	19	52	10
151*	5/2	8·30	1·3±0·25	1·3	3·2	0·28	0·98	0·22
152*	0	5·870	45±15	73	170	25	37	7·4
154*	0	5·814		160	300	33	80	17
63 Eu 151	5/2	6·36	0·71±0·09	0·75	1·4	0·17	0·57	0·13
153*	5/2	6·34	1·03±0·01	1·00	1·8	0·23	0·77	0·17
154*	3	8·10		0·76	0·62	0·19	0·58	0·14
155*	5/2	6·334		2·1	3·1	0·51	1·6	0·39
64 Gd 155*	3/2	8·528	1·8±0·15	1·8	1·8	0·42	1·4	0·32
156*	0	6·36	33±6	33	59	7·2	17	3·7
157*	3/2	7·932	5·5±1·2	9	5·7	2·1	6·9	1·6
65 Tb 159	3/2	6·389	4·7±0·6	4·7	4·6	1·1	3·6	0·87
66 Dy 161	5/2	8·175	2·2±0·15	2·2	2·5	0·53	1·7	0·41
162	0	6·280	42±6	42	94	9·4	21	4·8
163	5/2	7·630	10±1	10	11	2·4	7·7	1·8
164	0	5·714		420	330	91	210	46
67 Ho 165	7/2	6·27	6·1±0·4	6·1	3·7	1·4	4·7	1·1
68 Er 167	7/2	7·760	3·0±0·5	3·0	2·3	0·71	2·3	0·54
69 Tm 169	1/2	6·610	6·9±1·0	6·9	5·5	1·7	5·3	1·3
70 Yb 168	0	6·9		11	8·2	2·3	5·3	1·2
171	1/2	7·99	8·7±0·8	8·0	4·3	2·1	6·7	1·6
173	5/2	7·46	12±2	12	3·9	2·8	9·3	2·1
71 Lu 176	7/2	6·25	3·3±0·3	3·2	2·4	0·72	2·4	0·54
176	7	6·89	2·1±0·15	2·2	1·9	0·5	1·6	0·4
72 Hf 174	0	7·14	25±10	25	5·4	5·6	13	2·8
177	7/2	7·59	3·8±0·4	3·4	1·3	0·78	2·6	0·59
178	0	6·17	32±8	32	42	6·8	16	3·4
179	9/2	7·367	5·6±0·5	6·3	3·7	1·4	4·8	1·0
180	0	5·96	125±40	130	76	28	68	14
73 Th 180	8	7·632	1·5±0·5	1·3	0·84	0·3	0·93	0·2
181	7/2	6·059	4·35±0·20	4·7	3·0	1·0	3·5	0·8
74 W 182	0	6·29	50±12	47	41	10	24	5
183	1/2	7·42	12·5±0·8	13	9·3	2·9	9·9	2·2
184	0	5·77	130±30	140	97	27	69	14
75 Re 185	5/2	6·23	3·8±0·8	3·8	2·5	0·84	2·9	0·64
187	5/2	5·95	5·5±1·0	5·5	3·9	1·2	4·2	0·92
76 Os 189	3/2	7·89	5·1±1·2	5·1	5·8	1·1	3·9	0·87
77 Ir 191	3/2	6·145	3·3±0·3	3·3	5·6	0·77	2·5	0·6
193	3/2	5·960	7·7±0·6	8·0	24	2·0	6·2	1·5
78 Pt 192	0	6·29		19	67	4·3	9·6	2·2
195	1/2	7·92	16±1	16	68	4·5	12	3·5
198	0	5·2		9 k	10 k	2·6 k	4·6 k	1·3 k
79 Au 197	3/2	6·494	14·9±1·0	15	9·7	3·8	11	2·9

* Nuclides that are major fission products.

TABLE 2 (*Continued*)

Nuclide	J	Binding Energy (MeV)	Observed $\bar{D}$ (eV)	Calculated $\bar{D}$ (eV)				
			s wave	s wave		s wave at 1 MeV	p wave	p wave at 1 MeV
				Present Work	Gilbert and Cameron			
80 Hg 198	0	6·682	83±28	110	150	29	57	15
199	1/2	8·01	59±10	57	68	16	44	12
200	0	6·163	1·3±0·15k	1·2 k	990	340	630	170
201	3/2	7·760	90±25	85	180	26	66	20
202	0	6·06		8 k	6 k	2·6 k	4·1 k	1·3 k
81 Tl 203	1/2	6·53	2·0±0·8k	1·9 k	1·0 k	690	1·5 k	530
205	1/2	6·55	10±3	10 k	12 k	4·6 k	8 k	3·5 k
82 Pb 206	0	7·0	25±12 k	23 k	58 k	10 k	12 k	5·1 k
207	1/2	7·65		24 k	58 k	12 k	19 k	9·1 k
208	0	4·29	150±30 k	150 k	240 k	53 k	79 k	27 k
83 Bi 209	9/2	4·65	6·9±0·7k	6·9k	3·3 k	2·2 k	5·2 k	1·6 k
90 Th 232	0	5·07	17·5±0·7	18	19	2·4	9·7	1·2
91 Pa 231	3/2	5·58	0·45±0·07	0·46	0·49	0·07	0·35	0·06
233	3/2	5·03	0·86±0·12	0·82	1·2	0·12	0·63	0·09
92 U 232	0	5·9	7·6±1·5	5·9	5·2	0·9	3·0	0·46
233	5/2	6·76	0·66±0·05	0·58	0·54	0·09	0·44	0·07
234	0	5·25	13±0·8	13	18	1·8	6·6	0·93
235	7/2	6·39	0·65±0·03	0·65	0·73	0·10	0·50	0·08
236	0	5·42	14·5±1·5	14	12	2	7·1	1
238	0	4·76	17·7±0·7	19	36	2·4	9·6	1·2
93 Np 237	5/2	5·38	0·58±0·06	0·58	0·88	0·09	0·44	0·07
94 Pu 239	1/2	6·38	2·59±0·05	2·7	3·2	0·44	2·0	0·34
240	0	5·52	10±1	8·7	12	1·3	4·4	0·67
241	5/2	6·20	1·3±0·1	1·3	1·4	0·20	1·0	0·16
242	0	4·44	14·8±0·3	130	97	16	66	8·2
95 Am 241	5/2	5·47	0·43±0·06	0·43	1·1	0·07	0·33	0·06
243	5/2	5·15	1·25±0·15	1·2	1·7	0·19	0·92	0·15

* Nuclides that are major fission products.

known, and only such nuclei as ^{98}Mo and ^{100}Mo, with experimental level density parameters much greater than those given by equation (6), were not fitted. Our value of $\bar{D}$ for ^{93}Nb, calculated from s-wave levels only, is considerably greater than that quoted by Gilbert and Cameron (1965). The information on level spacings, nuclear binding energies, and ground state spins was taken mostly from Gilbert and Cameron (1965), where the original references are listed. Table 1 compares the new values for shell and pairing corrections with the previous values given by Gilbert and Cameron. Table 2 gives the values of s- and p-wave level spacings at the binding energy and at a bombarding energy of 1 MeV, calculated from the improved formula for nuclides of interest. Experimental level spacings are inserted where known and the calculated value from Gilbert and Cameron's formula is also given.

III. Cross Section Calculations

The task of predicting level spacings was initiated to assist in the estimation of unknown radiative capture and scattering cross sections for use in Australian Atomic Energy Commission nuclear data libraries. The property of the free gas model that the level spacing decreases with increasing neutron bombarding energy (evident from Table 2) leads to difficulties in the 1 MeV range. It is normally assumed that the scattering and radiative capture strength functions remain constant from thermal energies up to regions where direct reaction effects become important. Garrison and Roos (1962) obtained a good fit to the radiative capture cross section of ^{115}In up to 3 MeV by assuming that $\bar{D}$, the average radiation width $\bar{\Gamma}_\gamma$, and the

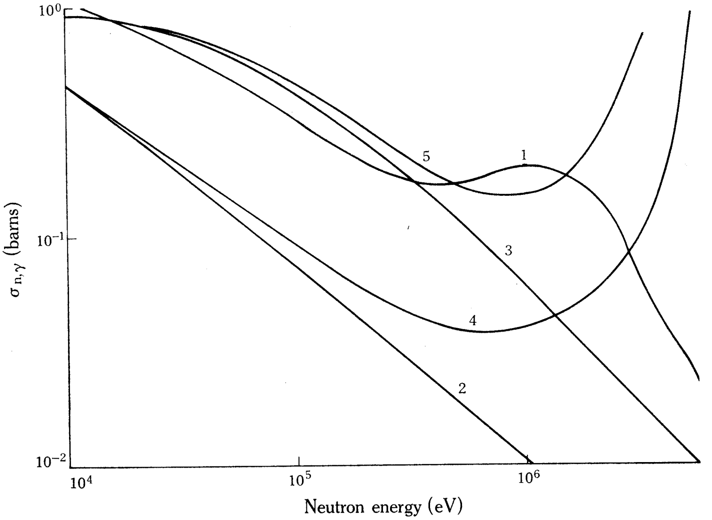

Fig. 1.—Cross section for the ^{115}In(n, γ) reaction: curve 1, experimental values and s, p, d waves from Garrison and Roos (1962); 2, s wave with $\bar{D}$ constant; 3, s, p wave with $\bar{D}$ constant; 4, s wave with $\bar{D}$ from the Bethe free gas model; 5, s, p wave with $\bar{D}$ from the Bethe free gas model.

average reduced widths for each orbital angular momentum state $\bar{\Gamma}_n^l$ were all constant. They assumed that the cross sections in the compound nucleus region are

$$\left.\begin{aligned} \sigma_t &\simeq 2\pi(\lambda/2\pi)^2 \sum_l g_J(\bar{\Gamma}_n^l/\bar{D})k\theta_l \simeq 2\pi(\lambda/2\pi)^2 \sum_l g_J S_l k\,\theta_l\,, \\ \sigma_{n,\gamma} &\simeq 2\pi(\lambda/2\pi)^2 \sum_l g_J(\bar{\Gamma}_n^l/\bar{D})k\theta_l(\bar{\Gamma}_\gamma/\bar{\Gamma}) \simeq 2\pi(\lambda/2\pi)^2 \sum_l g_J T\,\theta_l\,, \end{aligned}\right\} \tag{8}$$

where σ_t is the total cross section, θ_l is the penetration coefficient for each l state, g_J is the spin statistical weight factor, $\sigma_{n,\gamma}$ is the radiative capture cross section, $\lambda/2\pi$ is the neutron wavelength, k is the neutron momentum, $\bar{\Gamma}$ is the total width,

$$S_l = \bar{\Gamma}_n^l/\bar{D}, \qquad T = \bar{\Gamma}_\gamma/\bar{D}, \qquad \text{and} \quad \bar{\Gamma}_n^l k \gg \bar{\Gamma}_\gamma.$$

Since $\bar{\Gamma}_\gamma$ and $\bar{D}$ only appear in the ratio T, the radiative capture strength function, the results of Garrison and Roos are reproduced merely by assuming that T is constant. However, if $\bar{D}$ decreases according to the free gas formula, the experiments are reproduced only by assuming a roughly proportional decrease in each of $\bar{\Gamma}_n^l$ and $\bar{\Gamma}_\gamma$. Alternatively, should one choose these parameters to be constant, the predicted capture cross section undergoes an exponential increase above 1·5 MeV, which is not in accord with experiment, as illustrated in Figure 1.

The present strength function data were taken from Computor Index Neutron Data (1965). The experimental points quoted by Garrison and Roos were very dense, so that the curve 1 shown in Figure 1 is the line of best fit.

The most satisfactory procedure in the range from 1 to 10 MeV, as put forward by Zakharova and Malyshev (1965), is to calculate S_l from the optical model, $\bar{D}$ from the free gas model, and assume that $\bar{\Gamma}_{\mathrm{n}}^{l} = \bar{D}S_l$. The radiative widths can be estimated from the Weisskopf (1937) theory. Zakharova and Malyshev took a detailed account of photon cascades in their calculations for ^{127}I and obtained a reasonable fit to the radiative capture cross section. In their theory the decrease in $\bar{D}$ is compensated by decreases in the direct reaction contribution and in the widths for neutron radiative capture. It is clear from their results that the good agreement obtained by Garrison and Roos for ^{115}In must have been fortuitous.

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