MEASUREMENT OF THE MAGNETIC DRIFT VELOCITY AND MAGNETIC DEFLECTION COEFFICIENT FOR SLOW ELECTRONS IN HYDROGEN AND DEUTERIUM AT 293°K

By R. P. Creaser*

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Summary

The magnetic drift velocity W_M relating to the drift of an electron swarm in crossed electric and magnetic fields has been measured in hydrogen and deuterium at 293°K. The measurements were made using Huxley's method of measuring W_x/W_z, the ratio of the transverse to the longitudinal drift velocity. Data are given for W_M and for the magnetic deflection coefficient Ψ for $6{\cdot}1\times10^{-19} < E/N < 2{\cdot}7\times10^{-16}$ V cm^2 for hydrogen and $6{\cdot}1\times10^{-19} < E/N < 2{\cdot}4\times10^{-16}$ V cm^2 for deuterium.

I. Introduction

The motions of electron swarms in crossed electric and magnetic fields have been investigated for many years, the first experiments by Townsend and Tizard (1913) being used as a method of measuring electron drift velocities. More recently, detailed examinations of the motions of electron swarms under these conditions (e.g. Huxley 1960) have shown that the electron drift velocity cannot be deduced in a simple way from the results of such measurements. On the other hand, it has been shown (see Section II) that under certain circumstances measurements in crossed fields can afford valuable information about the energy dependence of the momentum transfer cross section or the form of the energy distribution. As a consequence there has been renewed interest in making accurate measurements of this kind.

In experiments of the type initiated by Townsend and developed by Huxley, in which a uniform electric field E is applied along the z axis together with a uniform magnetic field B along the y axis, the quantity that is experimentally determined is the ratio of the transverse to longitudinal drift velocities W_x/W_z. However, W_x/W_z is a function not only of E/N, the ratio of electric field strength to gas number density N, but also of B/N (see equation (5)) and is therefore not the most convenient quantity to use when tabulating experimental data. Two quantities have therefore been introduced, namely, the magnetic drift velocity W_M and the magnetic deflection coefficient Ψ. These quantities are defined by the equations (see Jory 1965)

$$W_M = (E/B)(W_x/W_z), \tag{1}$$

$$\Psi = W_M/W, \tag{2}$$

where W is the electron drift velocity.

* Ion Diffusion Unit, Australian National University, Canberra.

Recent measurements have been made of W_M and Ψ in nitrogen (Jory 1965) and helium (Crompton, Elford, and Jory 1967). The present paper extends the measurements to hydrogen and deuterium.

II. Theory

In the presence of an electric field E directed along the z axis the drift velocity is given by (Allis 1956; Huxley 1960)

$$W = W_z = -\frac{4\pi}{3}\frac{E}{N}\frac{e}{m}\int_0^\infty \frac{c^2}{q_m}\frac{df(c)}{dc}dc, \tag{3}$$

where e and m are the electronic charge and mass respectively, q_m is the momentum transfer cross section, and $f(c)$ is the function describing the distribution of electron speeds c.

When a constant and uniform magnetic field B is applied along the y axis it has been shown that (e.g. Huxley 1960) the velocity of the centre of mass of the stream is then given by

$$\begin{aligned} W &= W_z + iW_x \\ &= -\frac{4\pi}{3}\frac{E}{N}\frac{e}{m}\int_0^\infty \frac{c^2}{q_m(1-i\omega/\nu_m)}\frac{df(c)}{dc}dc, \end{aligned} \tag{4}$$

where $\omega = eB/m$ and $\nu_m = Nq_m c$ is the momentum transfer collision frequency.

It follows from equation (4) that, if $\omega \ll \nu_m$,

$$W_x \simeq -\frac{4\pi}{3}\frac{Ee}{Nm}\frac{Be}{Nm}\int_0^\infty \frac{c}{q_m^2}\frac{df(c)}{dc}dc,$$

$$W_z \simeq -\frac{4\pi}{3}\frac{Ee}{Nm}\int_0^\infty \frac{c^2}{q_m}\frac{df(c)}{dc}dc = W,$$

and therefore

$$\frac{W_x}{W_z} \simeq \frac{Be}{Nm}\left\{\int_0^\infty \frac{c}{q_m^2}\frac{df(c)}{dc}dc\right\}\left\{\int_0^\infty \frac{c^2}{q_m}\frac{df(c)}{dc}dc\right\}^{-1}. \tag{5}$$

It follows that the magnetic drift velocity W_M as defined by equation (1) is given by

$$W_M = \frac{eE}{Nm}\left\{\int_0^\infty \frac{c}{q_m^2}\frac{df(c)}{dc}dc\right\}\left\{\int_0^\infty \frac{c^2}{q_m}\frac{df(c)}{dc}dc\right\}^{-1}, \tag{6}$$

while the magnetic deflection coefficient (equation (2)) is given by

$$\begin{aligned} \Psi = \frac{W_M}{W} &= \left(\frac{E}{B}\right)\left(\frac{W_x}{W_z W}\right) \\ &\simeq \left(\frac{E}{B}\right)\left(\frac{W_x}{W^2}\right), \\ &\simeq -\frac{3}{4\pi}\left\{\int_0^\infty \frac{c}{q_m^2}\frac{df(c)}{dc}dc\right\}\left\{\int_0^\infty \frac{c^2}{q_m}\frac{df(c)}{dc}dc\right\}^{-2}. \end{aligned} \tag{7}$$

From equation (7) it can be seen that Ψ is a function of the variation of momentum transfer cross section with electron speed and the distribution function $f(c)$. When the momentum transfer collision frequency is constant, $W_M = W$, that is, $\Psi = 1$, but it should also be noted that Ψ is approximately equal to unity when the distribution of electron speeds $f(c)$ is narrow, regardless of the energy dependence of ν_m. Equation (6) shows, moreover, that $W_M \geqslant W$, that is, $\Psi \geqslant 1$.

III. Experimental Procedure

The diffusion apparatus used was the same as that described by Jory (1965) and shown schematically in Figure 2 of his paper. Electrons generated thermionically by a heated platinum filament enter the main diffusion chamber through a hole of 1 mm diameter in the centre of the cathode. A uniform electric field is maintained both in the diffusion chamber itself and in the region immediately above the cathode. The electrons therefore acquire the steady state energy distribution appropriate to the electric field in the diffusion chamber before entering the chamber through the source hole.

The anode, which in this apparatus is 8·33 cm from the cathode, consists of a central disk surrounded by an outer annulus, both being concentric with the axis of the drift chamber. The disk and annulus are bisected along a diameter by a transverse slit parallel to the direction of the magnetic field. When the apparatus is used to measure W_M the segments are connected together to form two halves that are electrically insulated from each other. The currents used in the measurements were of the order of 10^{-12} A and the ratio of the currents received by the two halves of the anode was measured using the techniques described by Crompton and Jory (1962).

The hydrogen and deuterium were admitted to the apparatus through a heated palladium osmosis tube and a liquid nitrogen trap. The deuterium was obtained from cylinders supplied by the Liquid Carbonic Company, the isotope ratio being 99·9%. Gas pressures used were 5, 10, 20, 50, 100, and 200 torr and were measured to within 0·1% by a Texas Instruments quartz spiral manometer. For the measurements made with gas pressures of 100 and 200 torr, electric field strengths of from 3 to 40 V/cm were used, the lower limit being determined by the onset of significant errors due to contact potential differences over the anode surface. At the remaining pressures the lower limit was somewhat higher in order to prevent a significant proportion of the electron stream from entering the region of insufficiently uniform field adjacent to the guard electrodes.

The magnetic field was generated by a pair of Helmholtz coils whose axis was aligned parallel to the north–south direction. The magnitude of the field was determined by measuring the current passing through the coils and making the appropriate conversion, the current being measured by potentiometrically determining the potential difference across a standard resistor of 0·1 ohm placed in series with the coils. Although the current was measured to within 0·1%, the value of B could not be determined to better than 1% because of uncertainty in the relation between current and flux density. Measurements were made with two flux densities, 40 and 60 G.

The axis of the apparatus, which is defined by the line parallel to the electric field and passing through the centre of the source hole, nominally passes through the centre of the transverse slit in the anode. In the absence of an applied magnetic field the two halves of the anode should therefore receive equal currents, but in practice the ratio of the currents is not usually unity due to one or more of the following factors:

(1) the horizontal component of the Earth's magnetic field in the vicinity of the tube,

(2) contact potential differences across the anode surface,

(3) the centre of the transverse slit being off the axis of the diffusion chamber, and

(4) the effective centre of the electron source not coinciding with the centre of the source hole.

No error in the value of W_M need arise from (1) since the horizontal component of the Earth's magnetic field can be added vectorially to the value of B.

When present, the effects of contact potential differences can be largely nullified by the application of a compensating potential difference to one of the anode halves, using a procedure similar to that described by Crompton, Elford, and Gascoigne (1965). In order to obtain the correct compensating potential difference it is necessary to distinguish between the asymmetry arising from contact potential differences and that arising from (1), (3), and (4). To make the adjustment, the value of the current ratio is first determined at the highest value of E to be used, the Earth's field having first been nullified using the Helmholtz coils. This ratio is little affected by the presence of contact potential differences. The electric field is then reduced to the smallest value and the compensating potential adjusted until the ratio of the currents with this electric field is approximately equal to that measured at the largest field. This adjustment ensures that the stream has the same small degree of asymmetry over the range of E to be used.

In most cases (3) is a small effect that can be accounted for by measuring the current ratio with the magnetic field in each direction and averaging the two values of W_M obtained.

The main cause of inequality was found to be due to (4) and was overcome by placing a small electrode above the source hole, and adjacent to it, in order to produce a small transverse electric field in this region. The electrode was positioned so that the application to it of a small negative potential difference with respect to the cathode moved the position of the effective centre of the source in the appropriate direction, the value of the potential difference being adjusted until, with $B = 0$, the currents received by each half of the anode were equal.

It is worth noting that effects such as those described above could neither be observed nor corrected for in Townsend's method (Townsend and Tizard 1913), which relied on the adjustment of the magnetic field until equal currents were received on either side of an off-axis slit in the anode. In this respect the present method is superior since any residual asymmetry is accounted for by the reversal of the magnetic field.

The ratio W_x/W_z and hence W_M was calculated directly from the experimental parameters and the current ratios by means of a computer programme rather than by the use of computed tables (cf. Jory 1965). In order to proceed with the calculation it is necessary to know the value of W/D (D being the diffusion coefficient) appropriate to the values of E/p and p at which the current ratios were determined.

TABLE 1

EXPERIMENTAL VALUES OF W_M AND Ψ IN HYDROGEN AT 293°K

E/p (V cm^{-1} torr^{-1})	W_M (10^5 cm/sec) for Values of B (gauss) of: 40	60	40	60	40	60	Av. W_M (10^5 cm/sec)	W (10^5 cm/sec)	Av. Ψ	E/N (10^{-17} V cm^2)
	p = 200 torr		p = 100 torr		p = 50 torr					
0·02	1·07	1·07					1·07	0·878	1·21	0·061
0·025	1·28	1·28	1·29	1·29			1·29	1·07	1·20	0·076
0·03	1·48	1·48	1·51	1·49			1·49	1·26	1·18	0·092
0·04	1·86	1·87	1·87	1·87			1·87	1·59	1·17	0·121
0·05	2·21	2·21	2·22	2·22			2·22	1·89	1·17	0·152
0·06	2·54	2·53	2·53	2·53			2·53	2·18	1·16	0·182
0·07	2·84	2·83	2·81	2·82			2·83	2·45	1·15	0·212
0·08	3·12	3·11	3·11	3·11			3·11	2·70	1·15	0·243
0·09	3·40	3·39	3·38	3·38			3·39	2·94	1·15	0·273
0·10	3·63	3·63	3·62	3·62	3·63	3·63	3·63	3·15	1·15	0·303
0·15	4·74	4·75	4·73	4·73	4·73	4·74	4·74	4·09	1·16	0·455
0·20	5·63	5·60	5·58	5·58	5·59	5·58	5·59	4·85	1·15	0·607
0·30			6·92	6·93	6·86	6·90	6·90	5·97	1·16	0·911
0·40			7·88	7·88	7·88	7·87	7·88	6·81	1·16	1·21
0·50	p = 20 torr				8·63	8·63	8·63	7·45	1·16	1·52
0·60					9·32	9·33	9·33	8·04	1·16	1·82
0·70	9·94	9·89			9·93	9·95	9·93	8·57	1·16	2·13
0·80	10·4	10·4			10·5	10·5	10·5	9·08	1·15	2·43
0·90	11·0	11·0			11·0	11·0	11·0	9·56	1·15	2·73
1·00	11·5	11·5	p = 10 torr				11·5	10·1	1·14	3·03
1·20	12·5	12·5					12·5	11·0	1·13	3·64
1·50	13·8	13·9	13·8	13·8			13·8	12·3	1·13	4·55
1·80	15·1	15·2	15·1	15·1	p = 5 torr		15·1	13·5	1·12	5·46
2·00	16·0	16·0	16·0	15·9			16·0	14·3	1·12	6·07
3·00			19·7	19·7	19·6		19·7	17·9	1·10	9·11
4·00			23·0	23·0	22·9	22·9	23·0	21·0	1·09	12·14
5·00					25·9	25·8	25·9	24·0	1·08	15·18
6·00					28·6	28·6	28·6	26·7	1·07	18·21
7·00					31·2	31·2	31·2	29·5	1·06	21·25
8·00					33·8	33·7	33·7	31·9	1·06	24·28
9·00					36·3	36·2	36·3	34·2	1·06	27·3

The values of W/D for hydrogen were taken from data of Crompton and Jory (1962) and Crompton, Elford, and McIntosh (in preparation). For deuterium the majority of the values of W/D were taken from data of McIntosh (1966), the remainder being measured during the present investigation. The same apparatus was used for these measurements as was used for the determination of W_M, but the

segments of the anode were re-wired to form the configuration of a central disk and an outer annulus (Jory 1965).

For hydrogen the values of W needed for the calculation of Ψ were taken from data published by Lowke (1963) and for deuterium from those published by McIntosh (1966).

TABLE 2

EXPERIMENTAL VALUES OF W_M AND Ψ IN DEUTERIUM AT 293°K

E/p (V cm^{-1} torr^{-1})	W_M (10^5 cm/sec) for Values of B (gauss) of: 40	60	40	60	40	60	Av. W_M (10^5 cm/sec)	W (10^5 cm/sec)	Av. Ψ	E/N (10^{-17} V cm^2)
	p = 200 torr		p = 100 torr		p = 50 torr					
0·02	1·09	1·09					1·09	0·882	1·23	0·0607
0·025	1·31	1·31	1·31	1·31			1·31	1·075	1·22	0·0758
0·03	1·52	1·52	1·52	1·52			1·52	1·257	1·21	0·0916
0·04	1·90	1·90	1·90	1·90			1·90	1·592	1·19	0·121
0·05	2·24	2·25	2·24	2·24			2·24	1·895	1·18	0·152
0·06	2·55	2·56	2·56	2·56			2·56	2·17	1·18	0·182
0·07	2·81	2·84	2·84	2·84			2·83	2·42	1·17	0·212
0·08	3·09	3·09	3·10	3·09			3·09	2·65	1·17	0·243
0·09	3·33	3·33	3·32	3·33			3·33	2·85	1·17	0·273
0·10	3·55	3·56	3·54	3·55	3·51	3·51	3·54	3·05	1·16	0·303
0·15	4·43	4·45	4·43	4·43	4·39	4·39	4·42	3·81	1·16	0·455
0·20	5·10	5·08	5·09	5·09	5·05	5·05	5·08	4·37	1·16	0·607
0·30			6·00	6·03	5·98	6·00	6·00	5·18	1·16	0·911
0·40			6·80	6·77	6·73	6·68	6·75	5·82	1·16	1·21
0·50	p = 20 torr				7·38	7·38	7·38	6·41	1·15	1·52
0·60					7·97	7·98	7·98	6·95	1·15	1·82
0·70	8·52	8·52			8·55	8·58	8·54	7·48	1·14	2·13
0·80	9·06	9·07			9·10	9·07	9·08	7·98	1·14	2·43
0·90	9·59	9·60			9·62	9·64	9·61	8·46	1·14	2·73
1·00	10·1	10·1	p = 10 torr				10·1	8·93	1·13	3·03
1·20	11·0	11·0					11·0	9·80	1·12	3·64
1·50	12·3	12·3	12·2	12·2			12·3	10·98	1·12	4·55
1·80	13·5	13·5	13·4	13·4	p = 5 torr		13·4	12·10	1·11	5·46
2·00	14·2	14·2	14·2	14·1			14·2	12·77	1·11	6·07
3·00			17·3	17·4	17·3		17·3	15·86	1·09	9·11
4·00			20·2	20·1	20·1	20·0	20·1	18·59	1·08	12·14
5·00					22·6	22·5	22·5	21·1	1·07	15·18
6·00					24·5	24·5	24·5			18·21
7·00					27·1	27·2	27·2			21·25
8·00					29·5	29·5	29·5			24·28

IV. RESULTS AND DISCUSSION

The values of W_M that were measured in hydrogen and deuterium together with the derived values of Ψ are shown in Tables 1 and 2, while the variation of Ψ with E/N is shown in Figure 1. For each value of E/N, W_M was determined at several pressures using values of B of 40 and 60 G; under these conditions the dependence of W_M on the magnitude of B is smaller than the experimental error. For a given

value of E/N the majority of the results show a maximum discrepancy of about 1%. Since W_x/W_z is directly proportional to B, which itself may be subject to a systematic error of 1%, and since values of W which are subject to a possible error of $\pm 1\%$ must be used in the calculation of Ψ, a reasonable estimate of the maximum error in the values of W_M and Ψ is 2–3% (Jory 1965).

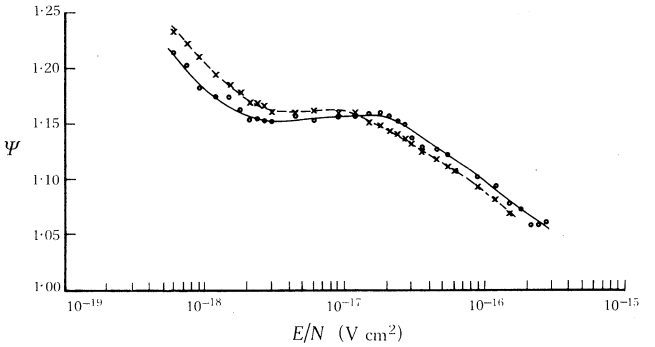

Fig. 1.—Variation of Ψ with E/N in hydrogen (solid curve) and deuterium (dashed curve) at 293°K.

In Figure 2 the values of Ψ for each gas have been plotted as a function of the characteristic energy D/μ rather than E/N. Figure 2 affords a more useful comparison than Figure 1 since (the energy dependence of the momentum transfer cross section being the same for both gases) any differences between the values of Ψ at the same value of D/μ arise from differences in the forms of the energy distribution functions. From Figure 2 it can be seen that the values of Ψ for the two gases

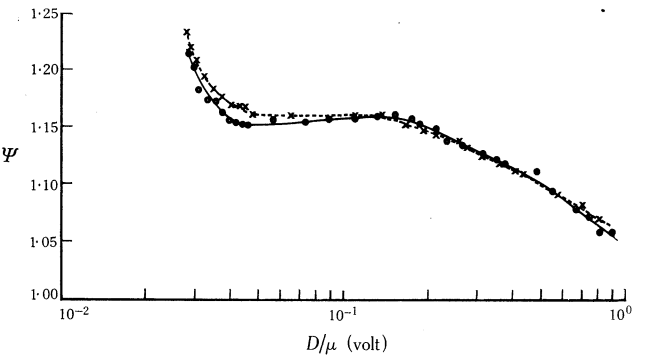

Fig. 2.—Variation of Ψ with D/μ in hydrogen (solid curve) and deuterium (dashed curve) at 293°K.

are in close agreement, with the exception of the values within the energy range $0{\cdot}03 < D/\mu < 0{\cdot}09$ V, where there is a maximum difference of almost 2% between the two curves. Although this difference is of the order of the overall experimental error claimed for Ψ, it nevertheless exceeds the random error in the data and may therefore be significant. Some differences between the energy distribution functions

for the two gases for similar values of D/μ are to be expected because of differences in the power losses to rotational and vibrational excitation in the two cases, but to answer the question as to whether these differences could account for the differences between the curves shown in Figure 2 would require detailed analysis.

In deuterium there are no other experimental or calculated values of W_M and Ψ with which the present results can be compared,* but in hydrogen a comparison can be made with Townsend's experimental data and with the calculated values of Engelhardt and Phelps (Engelhardt, personal communication). These comparisons are made in Figure 3. The values of Ψ attributed to Townsend were calculated from his data by dividing his tabulated "drift velocities" by the corresponding time-of-flight drift velocities published by Lowke (1963), while the values of Ψ attributed to Engelhardt and Phelps are the ratios of their calculated values of W_M and W.

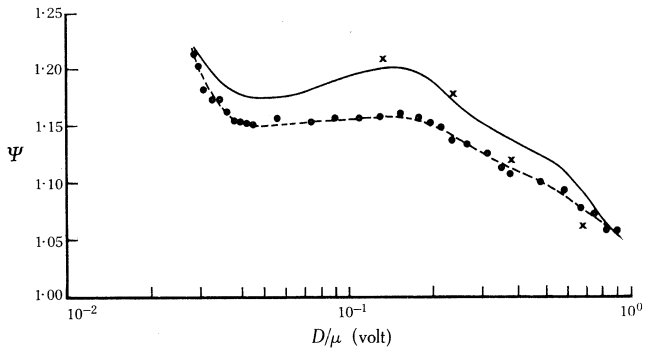

Fig. 3.—Comparison of present results in hydrogen at 293°K (dashed curve) with those of Engelhardt and Phelps at 300°K (full curve) and Townsend (**x**).

The agreement with Townsend's data is only fair, especially when it is remembered that the discrepancies represent differences in W_M rather than in Ψ, that is, any errors in W are common to the two sets of values of Ψ. In general the experimental results are somewhat lower than those calculated by Engelhardt and Phelps. An examination of the data for Ψ in nitrogen (Jory 1965) and in helium (Crompton, Elford, and Jory 1967), which have been obtained previously using this method, also reveals a general tendency for the data to be lower than the calculated data with which they are compared, although the discrepancies are rarely greater than the overall experimental scatter. The most significant comparison is that between calculated and experimental values in helium, where the experimental results are generally about 2% lower than those calculated using an energy-dependent momentum transfer cross section, which leads to excellent agreement between calculated and experimental values of W and D/μ (Crompton, Elford, and Jory 1967). These comparisons suggest the presence of a systematic error in the experimental results that could account for part, although probably not all, of the discrepancy

* Data are given in Engelhardt and Phelps (1963) from which these data could be calculated, but the scale of Figure 13 is too small to provide data with which significant comparisons can be made.

between the calculated and experimental results in the present instance. However, recalibration of the Helmholtz coil system, the pressure gauge, and the voltage divider supplying the electrode potentials failed to reveal a significant source of error. While more work is necessary to determine whether or not there is a further source of significant experimental error, the results presented here clearly demonstrate the expected similarity in the variation of Ψ with mean energy for the two gases and are of sufficient accuracy for making significant comparisons with calculated values of the transport coefficient.

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