STRUCTURE OF POLYTROPES WITH PURELY TOROIDAL MAGNETIC FIELDS

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Summary

In this paper the influence of a purely toroidal magnetic field upon the structure of a polytrope is investigated and the first-order perturbations in the density distribution, the geometry of the boundary, and the mass–radius relation are obtained.

I. Introduction

In a recent paper, Van der Borght (1967) has investigated the structure of a polytrope in the presence of a magnetic field having both poloidal and toroidal components. The structure of a polytrope in the case of only a toroidal field does not, however, follow from his investigations. Roxburgh (1966) investigated the structure of a toroidal magnetic field in a polytrope, and in a following paper (Roxburgh 1967) indicated the form of the radial perturbation of the polytrope $n = 3$ in the presence of a toroidal magnetic field but did not give any detailed calculations.

The existence of a toroidal magnetic field in a polytrope is not unrealistic. In the present work, we study the effect of a toroidal field on the structure of polytropes employing a method previously used by Chandrasekhar (1933) to study rotating polytropes. The geometry of the boundary, the oblateness, and the mass variation are obtained for polytropes $n = 1\cdot5(0\cdot5)3\cdot5$.

II. General Equations

The general Lundquist equations for a self-gravitating fluid are

$$\frac{\mathrm{D}\mathbf{V}}{\mathrm{D}t} = -\frac{\nabla p}{\rho} - \nabla\Phi + \frac{1}{\bar{\mu}\rho}(\nabla\times\mathbf{B})\times\mathbf{B}\,, \tag{1}$$

$$\partial\rho/\partial t = -\nabla\,.\,(\rho\mathbf{V})\,, \tag{2}$$

$$\partial\mathbf{B}/\partial t = \nabla\times(\mathbf{V}\times\mathbf{B})\,, \tag{3}$$

$$\nabla\,.\,\mathbf{B} = 0\,, \qquad \nabla\times\mathbf{B} = \bar{\mu}\mathbf{J}\,, \tag{4}$$

and

$$\mathrm{D}S/\mathrm{D}t = 0\,, \tag{5}$$

where $\mathbf{V}$ is the velocity of the fluid, $\mathbf{B}$ is the magnetic field, p is the pressure, ρ is the

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density, S is the entropy, $\mathbf{J}$ is the current density, $\bar{\mu}$ is the permeability of the medium, and $\mathrm{D}/\mathrm{D}t \equiv \partial/\partial t + \mathbf{V}.\nabla$. Further, the gravitational potential Φ satisfies the Poisson equation

$$\nabla^2\Phi = 4\pi G\rho. \tag{6}$$

In spherical coordinates (r, ϑ, ϕ), the field may be expressed as

$$\mathbf{B} \equiv (0,0,B).$$

In the absence of any azimuthal velocity component, equations (2)–(5) may be solved to obtain

$$B/r\sin\vartheta\,\rho = f(S),$$

giving a general expression for B.

III. Equilibrium Configuration

In the steady state, which we take as the equilibrium configuration, the energy equation (5) can be replaced by the polytropic equation

$$p = K\rho^{1+n^{-1}}, \tag{7}$$

where K is a constant and n is the polytropic index. B now may be given by

$$B = Lr\sin\vartheta\,\rho, \tag{8}$$

where L is a constant. The equation (8) gives the same expression for B as that obtained by Roxburgh (1966) in a different way for a polytrope in equilibrium.

In the equilibrium state, equation (1) gives rise to the equations

$$\frac{\partial p}{\partial r} + \rho\frac{\partial\Phi}{\partial r} + \frac{B}{\bar{\mu}}\frac{\partial B}{\partial r} + \frac{B^2}{\bar{\mu}r} = 0,$$

$$\frac{\partial p}{\partial\vartheta} + \rho\frac{\partial\Phi}{\partial\vartheta} + \frac{B}{\bar{\mu}}\frac{\partial B}{\partial\vartheta} + \frac{B^2}{\bar{\mu}}\cot\vartheta = 0.$$

Using equations (7) and (8), the above equations can be solved to give the integral

$$(n+1)K\rho^{n^{-1}} + (L^2/\bar{\mu})r^2\sin^2\vartheta\,\rho + \Phi = \text{constant}. \tag{9}$$

On substituting equation (9) into equation (6) we obtain

$$\nabla^2\{(n+1)K\rho^{n^{-1}} + (L^2/\bar{\mu})r^2\sin^2\vartheta\,\rho\} + 4\pi G\rho = 0. \tag{9a}$$

We write

$$\rho = \lambda\Theta^n, \qquad \mu = \cos\vartheta, \qquad \alpha^2 = \{(n+1)/4\pi G\}K\lambda^{n^{-1}-1},$$

$$r = \alpha\xi, \qquad \beta^2 = L^2/4\pi G\bar{\mu},$$

where λ is the central density, ξ is a dimensionless distance, and β^2, the square of a characteristic Alfvén Mach number of the medium, is a measure of the strength of the magnetic field. The equation (9a) becomes

$$\frac{1}{\xi^2}\left[\frac{\partial}{\partial\xi}\left(\xi^2\frac{\partial}{\partial\xi}\right)+\frac{\partial}{\partial\mu}\left\{\left(1-\mu^2\right)\frac{\partial}{\partial\mu}\right\}\right]\left(\Theta+\beta^2\xi^2(1-\mu^2)\Theta^n\right)=-\Theta^n, \tag{10}$$

and it is this equation that has to be solved to find the density distribution in a polytrope in equilibrium with a toroidal magnetic field.

In the absence of a magnetic field ($\beta=0$) the above equation reduces to Emden's equation

$$\frac{1}{\xi^2}\frac{\mathrm{d}}{\mathrm{d}\xi}\left(\xi^2\frac{\mathrm{d}\theta}{\mathrm{d}\xi}\right)+\theta^n=0, \tag{11}$$

of index n, where the spherically symmetric function θ is introduced by $\rho_\mathrm{u}=\lambda\theta^n$, the subscript u denoting the solution for the unperturbed polytrope.

Assuming the magnetic field to be small ($\beta^2 \ll 1$), a solution of equation (7) will be sought in terms of θ, up to first order in β^2, by considering a solution of the type (cf. Chandrasekhar 1933)

$$\Theta=\theta+\beta^2\Psi(\xi,\mu). \tag{12}$$

Substituting equation (12) into equation (10) and using equation (11), it is found that Ψ satisfies the equation

$$\frac{1}{\xi^2}\frac{\partial}{\partial\xi}\left(\xi^2\frac{\partial\Psi}{\partial\xi}\right)+\frac{1}{\xi^2}\frac{\partial}{\partial\mu}\left\{\left(1-\mu^2\right)\frac{\partial\Psi}{\partial\mu}\right\}+n\theta^{n-1}\Psi$$

$$=\left(\mu^2-1\right)\frac{1}{\xi^2}\frac{\mathrm{d}}{\mathrm{d}\xi}\left\{\xi^2\frac{\mathrm{d}}{\mathrm{d}\xi}\left(\xi^2\theta^n\right)\right\}-(6\mu^2-2)\theta^n. \tag{13}$$

From equation (12), it follows that

$$\Psi=\partial\Psi/\partial\xi=0, \qquad \text{at} \qquad \xi=0.$$

Ψ is expanded as

$$\Psi=\psi_0(\xi)+\sum_{j=1}^{\infty}\psi_j(\xi)\,P_j(\mu), \tag{14}$$

and we find that

$$\psi_j(0)=\psi_j'(0)=0.$$

Substituting the expansion (14) into equation (13) and equating coefficients of $P_j(\mu)$, we get the equations

$$\frac{1}{\xi^2}\frac{\mathrm{d}}{\mathrm{d}\xi}\left(\xi^2\frac{\mathrm{d}\psi_j}{\mathrm{d}\xi}\right)=\left(\frac{j(j+1)}{\xi^2}-n\theta^{n-1}\right)\psi_j, \qquad j\neq 0,2, \tag{15}$$

$$\frac{1}{\xi^2}\frac{\mathrm{d}}{\mathrm{d}\xi}\left(\xi^2\frac{\mathrm{d}\psi_2}{\mathrm{d}\xi}\right) = \left(\frac{6}{\xi^2} - n\theta^{n-1}\right)\psi_2 + \frac{2}{3}\frac{1}{\xi^2}\frac{\mathrm{d}}{\mathrm{d}\xi}\left\{\xi^2\frac{\mathrm{d}}{\mathrm{d}\xi}\left(\xi^2\theta^n\right)\right\} - 4\theta^n \tag{16}$$

and

$$\frac{1}{\xi^2}\frac{\mathrm{d}}{\mathrm{d}\xi}\left(\xi^2\frac{\mathrm{d}\psi_0}{\mathrm{d}\xi}\right) = -n\theta^{n-1}\psi_0 - \frac{2}{3}\frac{1}{\xi^2}\frac{\mathrm{d}}{\mathrm{d}\xi}\left\{\xi^2\frac{\mathrm{d}}{\mathrm{d}\xi}\left(\xi^2\theta^n\right)\right\} \tag{17}$$

to determine the ψ_j.

The equation (15) is different in form from equations (16) and (17), in as much as if ψ_j is a solution of (15) so is $A_j\psi_j$, where A_j is an arbitrary constant. A proper expansion of Ψ would then be

$$\Psi = \psi_0(\xi) + \psi_2(\xi)P_2(\mu) + \sum_{j=1}^{\infty}{}' A_j\psi_j(\xi)P_j(\mu), \tag{18}$$

where the prime denotes exclusion of the term with $j = 2$ from the summation. Equation (10) does not contain Φ explicitly and remains the same whatever be the external gravitational field. This indeterminacy may be resolved by calculating the potential from the solutions found and then making it satisfy the basic equation (9). This will also lead to the determination of the A_j.

Poisson's equation (6) may be rewritten as

$$\frac{1}{\xi^2}\frac{\partial}{\partial\xi}\left(\xi^2\frac{\partial\Phi}{\partial\xi}\right) + \frac{1}{\xi^2}\frac{\partial}{\partial\mu}\left((1-\mu^2)\frac{\partial\Phi}{\partial\mu}\right)$$

$$= -D[\theta^n + n\theta^{n-1}\beta^2\{\psi_0 + \psi_2P_2(\mu) + \Sigma' A_j\psi_jP_j(\mu)\}], \tag{19}$$

where $D = -(n+1)K\lambda^{n-1}$. To the first order in β^2, Φ may be developed as

$$\Phi = \bar{\Phi} + \beta^2\left(\Phi_0(\xi) + \sum_{j=1}^{\infty}\Phi_j(\xi)P_j(\mu)\right),$$

where $\bar{\Phi}$ is the potential of the polytrope without a magnetic field. Substituting in equation (19) and equating the coefficients of $P_j(\mu)$, we get the equations

$$\frac{1}{\xi^2}\frac{\mathrm{d}}{\mathrm{d}\xi}\left(\xi^2\frac{\mathrm{d}\Phi_j}{\mathrm{d}\xi}\right) = \frac{j(j+1)}{\xi^2}\Phi_j - Dn\theta^{n-1}A_j\psi_j, \qquad j \neq 0, 2, \tag{20}$$

$$\frac{1}{\xi^2}\frac{\mathrm{d}}{\mathrm{d}\xi}\left(\xi^2\frac{\mathrm{d}\Phi_2}{\mathrm{d}\xi}\right) = \frac{6}{\xi^2}\Phi_2 - Dn\theta^{n-1}\psi_2, \tag{21}$$

$$\frac{1}{\xi^2}\frac{\mathrm{d}}{\mathrm{d}\xi}\left(\xi^2\frac{\mathrm{d}\Phi_0}{\mathrm{d}\xi}\right) = -Dn\theta^{n-1}\psi_0, \tag{22}$$

$$\frac{1}{\xi^2}\frac{\mathrm{d}}{\mathrm{d}\xi}\left(\xi^2\frac{\mathrm{d}\bar{\Phi}}{\mathrm{d}\xi}\right) = -D\theta^n. \tag{23}$$

Equation (20) becomes, using equation (15),

$$\frac{1}{\xi^2}\frac{\mathrm{d}}{\mathrm{d}\xi}\left(\xi^2\frac{\mathrm{d}\Phi_j}{\mathrm{d}\xi}\right)-\frac{j(j+1)}{\xi^2}\Phi_j = DA_j\left\{\frac{1}{\xi^2}\frac{\mathrm{d}}{\mathrm{d}\xi}\left(\xi^2\frac{\mathrm{d}\psi_j}{\mathrm{d}\xi}\right)-\frac{j(j+1)}{\xi^2}\psi_j\right\}.$$

Since there is no singularity at the centre, the admissible complementary function is $DB_j\,\xi^j$, where B_j is any constant. A particular solution is given by $\Phi_j = DA_j\psi_j$. Hence the general solution is

$$\Phi_j = D(A_j\psi_j+B_j\,\xi^j), \qquad j\neq 0,2.$$

Equation (21) treated with equation (16) yields

$$\frac{1}{\xi^2}\frac{\mathrm{d}}{\mathrm{d}\xi}\left(\xi^2\frac{\mathrm{d}\Phi_2}{\mathrm{d}\xi}\right)-\frac{6}{\xi^2}\Phi_2$$
$$= D\left[\frac{1}{\xi^2}\frac{\mathrm{d}}{\mathrm{d}\xi}\left\{\xi^2\frac{\mathrm{d}}{\mathrm{d}\xi}\left(\psi_2-\tfrac{2}{3}\xi^2\theta^n\right)\right\}-\frac{6}{\xi^2}\left(\psi_2-\tfrac{2}{3}\xi^2\theta^n\right)\right],$$

and, as above, its solution is

$$\Phi_2 = D(\psi_2-\tfrac{2}{3}\xi^2\theta^n+B_2\,\xi^2),$$

where B_2 is constant. Substituting for ψ_0 from equation (17) into equation (22), we get

$$\frac{1}{\xi^2}\frac{\mathrm{d}}{\mathrm{d}\xi}\left(\xi^2\frac{\mathrm{d}\Phi_0}{\mathrm{d}\xi}\right) = D\frac{1}{\xi^2}\frac{\mathrm{d}}{\mathrm{d}\xi}\left\{\xi^2\frac{\mathrm{d}}{\mathrm{d}\xi}\left(\psi_0+\tfrac{2}{3}\xi^2\theta^n\right)\right\},$$

which has the solution

$$\Phi_0 = D(\psi_0+\tfrac{2}{3}\xi^2\theta^n)+\text{constant}.$$

Finally, comparing equation (23) with Emden's equation (11), we get

$$\overline{\Phi} = D\theta+\text{constant}.$$

Substituting these values of $\overline{\Phi}$ and Φ_j in equation (14), the expression for Φ becomes after a readjustment of terms

$$\Phi = D\left\{\Theta+\beta^2\left(\sum_{j=1}^{\infty} B_j\,\xi^j P_j(\mu)+\tfrac{2}{3}\xi^2\theta^n\{1-P_2(\mu)\}\right)\right\}+\text{constant}.$$

Φ so obtained must satisfy equation (9) identically, and this gives

$$\Theta+\tfrac{2}{3}\beta^2\xi^2\{1-P_2(\mu)\}\theta^n = \Theta+\beta^2\left(\sum_{j=1}^{\infty} B_j\,\xi^j P_j(\mu)+\tfrac{2}{3}\xi^2\theta^n\{1-P_2(\mu)\}\right).$$

It follows that $B_j = 0$ for all j. Hence

$$\Phi = D[\Theta+\tfrac{2}{3}\beta^2\xi^2\theta^n\{1-P_2(\mu)\}]+\text{constant}. \qquad (24)$$

Furthermore, the potential arising from the mass must be continuous with the potential in the free space outside on the boundary. The density, being of higher order than the first, in the space between $\xi = \xi_1$, the first zero of Emden's function with index n, and the new boundary $\xi = \xi_0$, is negligible here. Thus on $\xi = \xi_1$, Φ and its normal derivative should be continuous to the first order with those of Φ_{ext}, which may be expressed as

$$\Phi_{\mathrm{ext}} = D\left(\frac{c_0}{\xi}+\beta^2\sum_{j=1}^{\infty}\frac{c_j}{\xi^{j+1}}P_j(\mu)\right)+\text{constant.}$$

Bearing in mind the requirement $n > 1$, arising from the fact that ρ and $\nabla\times\mathbf{B}$ vanish at the boundary, we find that the above conditions imply that

$$\text{(i) } A_j = 0\,, \qquad j \neq 2, \qquad \text{and}$$

$$\text{(ii) } \xi_1\psi_2'(\xi_1)+3\psi_2(\xi_1) = 0\,. \tag{25}$$

Hence the solution is given as

$$\Theta = \theta(\xi)+\beta^2\{\psi_0(\xi)+\psi_2(\xi)P_2(\mu)\},$$

where ψ_2 and ψ_0 are the solutions of equations (16) and (17) respectively. This specifies the structure of a polytrope in equilibrium with a toroidal magnetic field.

IV. Numerical Integration

It is not possible to solve equations (16) and (17) exactly. Assuming the expansions

$$\psi_0 = -\tfrac{2}{3}\xi^2+\ldots,$$

$$\psi_2 = \bar{\psi}_2\,\xi^2+\ldots,$$

and

$$\theta = 1-\tfrac{1}{6}\xi^2+\ldots$$

at the centre, these equations were integrated numerically using a Runge–Kutta method together with Emden's equation (11) for the cases $n = 1\cdot5(0\cdot5)3\cdot5$. A suitable value for $\bar{\psi}_2$ was determined so that the boundary condition (25) is satisfied. Values of ψ_0 and ψ_2 obtained for polytropes of different indices are given in Table 1. The value of $\bar{\psi}_2$ in each case is also given there.

V. New Boundary

On the surface $\Theta(\xi_0) = 0$ and this gives the expression

$$\xi_0 = \xi_1-\frac{\beta^2}{(\mathrm{d}\theta/\mathrm{d}\xi)_{\xi=\xi_1}}\Big(\psi_0(\xi_1)+\psi_2(\xi_1)\,P_2(\mu)\Big),$$

for the new boundary. The second term gives the mean expansion of the polytrope,

TABLE 1

PERTURBATIONS IN DENSITY FOR $n = 1{\cdot}5(0{\cdot}5)3{\cdot}5$

Parameter	Perturbations										$\bar{\psi}_2$
$n = 1{\cdot}5$											
ξ/ξ_1	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1·0	
$10\psi_0$	−0·852	−2·984	−5·339	−6·732	−6·382	−4·182	−0·654	3·323	6·812	8·824	1·149
$10\psi_2$	1·489	5·384	10·230	14·321	16·379	15·942	13·380	9·609	5·751	3·087	
$n = 2{\cdot}0$											
ξ/ξ_1	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1·0	
$10\psi_0$	−1·163	−3·627	−5·311	−4·884	−2·495	0·792	3·891	6·161	7·417	7·746	1·153
$10\psi_2$	2·049	6·797	11·290	13·325	12·553	9·962	6·841	4·125	2·266	1·376	
$n = 2{\cdot}5$											
ξ/ξ_1	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1·0	
$10\psi_0$	−1·637	−4·113	−4·108	−1·589	1·618	4·179	5·728	6·429	6·582	6·458	1·157
$10\psi_2$	2·944	8·390	11·246	10·450	7·796	5·034	2·944	1·633	0·937	0·635	
$n = 3{\cdot}0$											
ξ/ξ_1	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1·0	
$10\psi_0$	−2·335	−3·764	−1·216	2·118	4·301	5·303	5·585	5·521	5·526	5·114	1·160
$10\psi_2$	4·374	9·433	9·078	6·232	3·653	1·991	1·066	0·596	0·374	0·268	
$n = 3{\cdot}5$											
ξ/ξ_1	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1·0	
$10\psi_0$	−3·106	−1·382	2·434	4·301	4·790	4·719	4·476	4·212	3·975	3·778	1·162
$10\psi_2$	6·461	8·287	5·036	2·516	1·216	0·606	0·327	0·200	0·141	0·109	

whereas the third term gives rise to an ellipticity. Since the value of $P_2(\mu)$ is $+1$ at the poles and $-\frac{1}{2}$ at the equator, the oblateness of the configuration is given by

$$\delta = \frac{3}{2}\frac{\psi_2(\xi_1)}{\xi_1(\mathrm{d}\theta/\mathrm{d}\xi)_{\xi=\xi_1}}\beta^2,$$

and depends upon the strength of the magnetic field. The values of the relevant quantities are given in Table 2, for different values of n.

TABLE 2

VALUES OF PARAMETERS DEFINING NEW CONFIGURATION

n	ξ_1	$-\psi_0(\xi_1)/\theta'(\xi_1)$	$-\psi_2(\xi_1)/\theta'(\xi_1)$	δ/β^2	$-\psi_0'(\xi_1)/\theta'(\xi_1)$
1·5	3·65375	4·340	1·389	0·570	−0·181
2·0	4·35287	6·087	1·081	0·373	0·142
2·5	5·35528	8·464	0·832	0·233	0·434
3·0	6·89685	12·045	0·631	0·137	0·681
3·5	9·53581	18·166	0·526	0·083	0·899

VI. Mass of New Configuration

To the first order in β^2, the mass of the polytrope may be given by

$$M = 4\pi\int r^2\rho\,\mathrm{d}r.$$

The ellipticity term does not contribute anything on the average. The mass is, therefore,

$$M = 4\pi\left(\frac{n+1}{4\pi G}K\lambda^{\frac{1}{2}(3-n)n^{-1}}\right)^{3/2}\int_0^{\xi_1}(\theta^n+\beta^2 n\theta^{n-1}\psi_0)\xi^2\,\mathrm{d}\xi$$

$$= -4\pi\left(\frac{n+1}{4\pi G}K\lambda^{\frac{1}{2}(3-n)n^{-1}}\right)^{3/2}\xi_1^2\left(\frac{\mathrm{d}\theta}{\mathrm{d}\xi}\right)_{\xi_1}\left\{1+\beta^2\left(\frac{\mathrm{d}\psi_0}{\mathrm{d}\xi}\div\frac{\mathrm{d}\theta}{\mathrm{d}\xi}\right)_{\xi_1}\right\},$$

obtained on using equations (11) and (17) and remembering that $n>1$.

If M_{u} is the mass of the polytrope with the same central density when there is no magnetic field, we get

$$M = M_{\mathrm{u}}\left\{1+\beta^2\left(\frac{\mathrm{d}\psi_0}{\mathrm{d}\xi}\div\frac{\mathrm{d}\theta}{\mathrm{d}\xi}\right)_{\xi_1}\right\},$$

which shows that a magnetic polytrope with a toroidal field has a greater mass only if $\mathrm{d}\psi_0/\mathrm{d}\xi<0$ at the boundary. Values of $(\psi_0'/\theta')_{\xi_1}$ are given in Table 2.

VII. Discussion

It is found that ψ_0 is negative from the centre up to a fraction ϵ of the radius, when it becomes positive and remains so to the surface, which shows that the inner core of the perturbed polytrope is less dense and the outer layers are more dense than the unperturbed one. The value of ϵ decreases with increasing value of n.

The structure of the polytrope depends largely upon the strength of the magnetic field, but the polytropic index n has also a remarkable influence on it. The relative mean expansion, $-\psi_0(\xi_1)/\{\xi_1\,\theta'(\xi_1)\}$, of the polytrope increases and its ellipticity decreases with increasing values of n. Thus perturbed polytropes with larger n are relatively larger in size and less elliptical in shape.

The mass M has been shown to be greater than, equal to, or less than M_{u} according as

$$\left(\frac{\mathrm{d}\psi_0}{\mathrm{d}\xi}\right)_{\xi=\xi_1} \lesseqqgtr 0\,.$$

It is found that $(\psi_0')_{\xi_1}$ is positive for small values and negative for large values of n. Equation (17) was integrated with n as an eigenvalue and the above critical value was found to correspond to $n = 1\cdot80$. It follows that a magnetic polytrope with the toroidal field (8) has a greater or smaller mass according as its polytropic index is greater than or less than $1\cdot80$, and further, that this ratio increases with n.

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