DIFFRACTION BY A SLIT APERTURE FORMED BY TWO INCLINED SURFACES

By H. S. TAN*

[*Manuscript received April* 1, 1968]

Summary

The problem of diffraction by a slit aperture in a nonplanar screen formed by two inclined half-planes is formulated in terms of integral equations by means of the Lebedev transform. A convenient iterative solution of these equations is presented, based on a theorem by Karp and Zitron and its interpretations. The diffracted field is computed and compared with experimental measurements, and is found to contain a reflection interaction effect due to the nonplanar screen geometry.

I. INTRODUCTION

Studies in aperture diffraction theory have almost entirely been confined to apertures in plane screens. Out of the considerable literature on the subject, only a very few papers have been concerned with apertures in nonplanar screens. The reason for this may be that planar diffraction problems are far from being exhausted, and that a nonplanar screen introduces new difficulties in the mathematical analysis. However, a study of the problem of diffraction by an aperture in a nonplanar screen would be useful for comparison with diffraction by an identical aperture in a plane screen and for testing various approximate theories that have been used fairly successfully in planar problems. The planar problem possesses a plane of symmetry that results in certain simple properties of the diffracted field, e.g. the Babinet principle, and the property of an unperturbed tangential magnetic field and a normal electric field in the aperture plane. Without such a plane of symmetry, a nonplanar screen may have other characteristic properties not evident in the planar case.

The simplest nonplanar problem is that of a slit aperture formed by two inclined half-planes. Skalskaya (1963) has suggested an approximate solution of the integral equations for the induced currents on the screens for E-polarized plane wave incidence. His method was to assume that the slit width is large enough for the kernel to be suitably approximated so that an iterative solution can be computed. The purpose of the present paper is to formulate the more general problem of diffraction by a cylindrical wave, using an approach similar to Skalskaya. For the case of plane wave incidence, we show how a convenient iterative scheme may be used. Its interpretation in terms of interactions between the screens is discussed. The diffracted electric field intensity is also computed and compared with experiment.

* Department of Physics, University of Malaya, Kuala Lumpur, Malaysia.

II. Diffraction by an Incident Cylindrical Wave

For Dirichlet boundary conditions on the screens A and B with an incident cylindrical wave from a line source S, the induced screen currents j_A and j_B are given by the equations

$$\int_0^\infty j_A(\rho)\,H_0^{(2)}(k\,|r-\rho\,|)\,d\rho = -H_0^{(2)}(kS_A) - \int_0^\infty j_B(\rho)\,H_0^{(2)}(kR)\,d\rho\,, \tag{1}$$

$$\int_0^\infty j_B(\rho)\,H_0^{(2)}(k\,|r-\rho\,|)\,d\rho = -H_0^{(2)}(kS_B) - \int_0^\infty j_A(\rho)\,H_0^{(2)}(kR)\,d\rho\,, \tag{2}$$

where the notation is defined in Figure 1. The equation

$$\int_0^\infty j_{A0}(\rho)\,H_0^{(2)}(k\,|r-\rho\,|)\,d\rho = -H_0^{(2)}(kS_A)\,, \tag{3}$$

which describes the half-plane diffraction problem with cylindrical wave incidence, will be solved first, since the results will be used in the solution of (1) and (2).

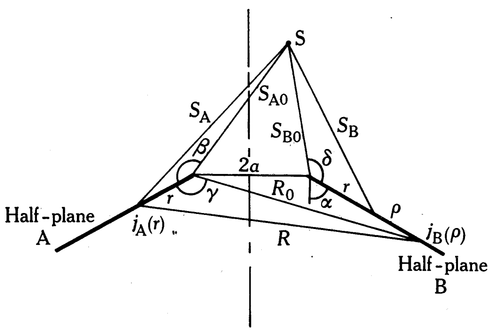

Fig. 1.—Diagram for diffraction by a cylindrical wave from S.

The Kontorovich–Lebedev transform and its inverse for a function $\phi(r)$ are defined as (Kontorovich and Lebedev 1938; Jones 1964)

$$\bar{\phi}(t) = \int_0^\infty r^{-1}\,\phi(r)\,e^{\frac{1}{2}\pi t}\,H_{it}^{(2)}(kr)\,dr\,, \tag{4}$$

$$\phi(r) = -\tfrac{1}{2}\int_0^\infty t\sinh(\pi t)\,\bar{\phi}(t)\,e^{\frac{1}{2}\pi t}\,H_{it}^{(2)}(kr)\,dt\,. \tag{5}$$

Applying this transform to (3), we have

$$\int_0^\infty j_{A0}(\rho)\,d\rho\int_0^\infty e^{\frac{1}{2}\pi t}\,H_{it}^{(2)}(kr)\,\frac{1}{r}\Big(H_0^{(2)}(k\,|r-\rho\,|) - e^{-ikr}\,H_0^{(2)}(k\rho)\Big)\,dr$$

$$= -\int_0^\infty e^{\frac{1}{2}\pi t}\,H_{it}^{(2)}(kr)\,\frac{1}{r}\Big(H_0^{(2)}(kS_A) - e^{-ikr}\,H_0^{(2)}(kS_{A0})\Big)\,dr\,. \tag{6}$$

The left-hand side of (6) becomes

$$-\frac{2}{it\sinh(\pi t)}\Big(H_0^{(2)}(kS_{A0}) - \cosh(\pi t)\,\overline{(\rho j_{A0})}\Big)\,,$$

where we have used the relation

$$\int_0^\infty j_{A0}(\rho)\, H_0^{(2)}(k\rho)\, d\rho = -H_0^{(2)}(kS_{A0}) \tag{7}$$

derived from (3) by putting $r = 0$.

The integral on the right-hand side of (6) can be evaluated to give (Oberhettinger and Higgins 1961)

$$\frac{2i}{t\sinh(\pi t)}\left(-e^{\frac{1}{2}\pi t}\, H_{it}^{(2)}(kS_{A0})\cosh(\pi-\beta)t + H_0^{(2)}(kS_{A0})\right),$$

so that equation (6) can now be written as

$$\overline{(\rho j_{A0})} = -e^{\frac{1}{2}\pi t}\, H_{it}^{(2)}(kS_{A0})\{\cosh(\pi-\beta)t\}/\cosh(\pi t). \tag{8}$$

On inverting the transform, we have

$$j_{A0}(r) = \frac{1}{2r}\int_0^\infty t\, e^{\pi t}\cosh(\pi-\beta)t\tanh(\pi t)\, H_{it}^{(2)}(kr)\, H_{it}^{(2)}(kS_{A0})\, dt. \tag{9}$$

To evaluate this integral, it is convenient to rewrite the integrand in terms of modified Bessel functions and to assume temporarily that the wave number k is purely imaginary. Using

$$K_\mu(z) = -\tfrac{1}{2}i\pi e^{-i\frac{1}{2}\pi\mu}\, H_\mu^{(2)}(ze^{-i\frac{1}{2}\pi}), \qquad k = -i\kappa,$$

equation (9) becomes

$$j_{A0}(r) = -\frac{2}{\pi^2 r}\int_0^\infty t\, K_{it}(\kappa r)\, K_{it}(\kappa S_{A0})\cosh(\pi-\beta)t\tanh(\pi t)\, dt, \tag{10}$$

which can be written as an integral along the entire real axis of t by use of the relation

$$K_\mu(z) = \tfrac{1}{2}\pi\{I_{-\mu}(z) - I_\mu(z)\}/\sin(\mu\pi). \tag{11}$$

Thus

$$j_{A0}(r) = \frac{i}{\pi r}\int_{-\infty}^\infty t\, K_{it}(\kappa S_{A0})\, I_{-it}(\kappa r)\,\frac{\cosh(\pi-\beta)t}{\cosh(\pi t)}\, dt, \qquad r < S_{A0}. \tag{12}$$

The path of integration is now closed by a semicircle in the upper half-plane, and Cauchy's theorem then yields

$$j_{A0}(r) = \frac{i}{2r}\sum_{n=0}^\infty (2n+1) H_{n+\frac{1}{2}}^{(2)}(kS_{A0})\, J_{n+\frac{1}{2}}(kr)\sin(n+\tfrac{1}{2})\beta, \qquad r < S_{A0}. \tag{13}$$

For $r > S_{A0}$, the arguments of $H_{n+\frac{1}{2}}^{(2)}$ and $J_{n+\frac{1}{2}}$ must be interchanged.

The solution of an equation of the type

$$\int_0^\infty j'_A(\rho)\, H_0^{(2)}(k|r-\rho|)\, d\rho = -\int_0^\infty j_B(\rho)\, H_0^{(2)}(kR)\, d\rho, \tag{14}$$

where only the second term on the right-hand side of (1) or (2) is present, is given by the same procedure from (6) to (13). Thus

$$j'_{\mathrm{A}}(r) = \int_0^\infty j_{\mathrm{B}}(\rho)\, j_0(r, R_0, \gamma)\, \mathrm{d}\rho\,,$$

where, from (13),

$$j_0(r, R_0, \gamma) = \frac{\mathrm{i}}{2r} \sum_{n=0}^{\infty} (2n+1) \mathrm{H}^{(2)}_{n+\frac{1}{2}}(kR_0)\, \mathrm{J}_{n+\frac{1}{2}}(kr) \sin(n+\tfrac{1}{2})\gamma\,, \qquad r < R_0\,, \tag{15}$$

is the current induced at r on half-plane A due to unit line current at ρ on half-plane B. Equations (1) and (2) have now been transformed into the Fredholm equations

$$j_{\mathrm{A}}(r) = j_0(r, S_{\mathrm{A}0}, \beta) + \int_0^\infty j_{\mathrm{B}}(\rho)\, j_0(r, R_0, \gamma)\, \mathrm{d}\rho\,, \tag{16a}$$

$$j_{\mathrm{B}}(r) = j_0(r, S_{\mathrm{B}0}, \delta) + \int_0^\infty j_{\mathrm{A}}(\rho)\, j_0(r, R_0, \gamma)\, \mathrm{d}\rho\,. \tag{16b}$$

III. Diffraction by an Incident Plane Wave

For an E-polarized plane wave incident on the screens in Figure 1, equation (1), for example, is modified to

$$\int_0^\infty g_{\mathrm{A}}(\rho)\, \mathrm{H}_0^{(2)}(k\,|\,r-\rho\,|)\, \mathrm{d}\rho = (c/\pi k)\exp(-\mathrm{i}kr\cos\theta) - \int_0^\infty g_{\mathrm{B}}(\rho)\, \mathrm{H}_0^{(2)}(kR)\, \mathrm{d}\rho\,, \tag{17}$$

where we have assumed that the incident wave makes an angle θ with the half-plane A. The solution of the equation

$$\int_0^\infty g_{\mathrm{A}0}(\rho)\, \mathrm{H}_0^{(2)}(k\,|\,r-\rho\,|)\, \mathrm{d}\rho = (c/\pi k)\exp(-\mathrm{i}kr\cos\theta) \tag{18}$$

is again obtained from the Kontorovich–Lebedev transform. The left-hand side of (18) becomes

$$\frac{2}{\mathrm{i}t\sinh(\pi t)}\left(\frac{c}{\pi k} - \cosh(\pi t)\,\overline{(\rho g_{\mathrm{A}0})}\right),$$

while the right-hand side gives

$$\frac{c}{\pi k}\frac{2\mathrm{i}}{t\sinh(\pi t)}\left(\cosh(\theta t) - 1\right).$$

Inverting the transform,

$$g_{\mathrm{A}0}(r) = -\frac{c}{\pi k}\frac{1}{2r}\int_0^\infty t\,\mathrm{e}^{\frac{1}{2}\pi t}\tanh(\pi t)\cosh(\theta t)\, \mathrm{H}_{\mathrm{i}t}^{(2)}(kr)\, \mathrm{d}t\,, \tag{19}$$

from which,

$$g_{\mathrm{A}0}(r) = -\frac{c}{\pi k}\frac{\mathrm{i}}{2r}\sum_{n=0}^{\infty}(2n+1)\mathrm{J}_{n+\frac{1}{2}}(kr)\,\mathrm{i}^{n+\frac{1}{2}}\sin(n+\tfrac{1}{2})\theta\,. \tag{20}$$

The solution of (17) with only the second term on the right-hand side present is given by (15), so that (17) now becomes

$$g_{\mathrm{A}}(r) = g_{\mathrm{A}0}(r,\theta) + \int_0^\infty g_{\mathrm{B}}(\rho)\, j_0(r, R_0, \gamma)\, \mathrm{d}\rho \tag{21}$$

with a similar equation for $g_{\mathrm{B}}(r)$.

IV. Solution of Integral Equation

An approximate iterative solution of the integral equations (16) and (21) can be obtained by the procedure used in Tan (1968), where the plane slit aperture was considered. Each of the line currents $g_{\mathrm{B}}(\rho)$ in (21), for example, is considered to be radiating a cylindrical wave that can be expanded in terms of plane waves and derivatives of plane waves with respect to their angles of incidence (Karp and Zitron 1964). The currents induced on half-plane A by $g_{\mathrm{B}}(\rho)$ are therefore equivalent to those induced by a series of plane waves and their derivatives. We thus write

$$j_0(r, R_0, \gamma) = \mathrm{H}_{\mathrm{op}}(\rho, \mathrm{d}/\mathrm{d}\theta)\, g_{\mathrm{A}0}(r,\theta)\,, \tag{22}$$

where the operator $\mathrm{H}_{\mathrm{op}}(\rho, \mathrm{d}/\mathrm{d}\theta)$ contains the series of plane waves and their derivatives. By using

$$\begin{aligned} \mathrm{H}^{(2)}_{n+\frac{1}{2}}(kR_0) &= (2/\pi kR_0)^{\frac{1}{2}}\exp\{-\mathrm{i}kR_0 + \mathrm{i}\tfrac{1}{4}\pi + \tfrac{1}{2}(n+\tfrac{1}{2})\pi\} \\ &\quad \times \sum_{m=0}^{\infty} \frac{\{1+4\,\mathrm{d}^2/\mathrm{d}\theta^2\}\{9+4\,\mathrm{d}^2/\mathrm{d}\theta^2\}\dots\{(2m-1)+4\,\mathrm{d}^2/\mathrm{d}\theta^2\}}{(-8\mathrm{i}kR_0)^m\, m!} \end{aligned} \tag{23}$$

in equation (15) and comparing with $g_{\mathrm{A}0}$ in (20), we have

$$\begin{aligned} \mathrm{H}_{\mathrm{op}}(\rho, \mathrm{d}/\mathrm{d}\theta) &= (2/\pi kR_0)^{\frac{1}{2}}\exp(-\mathrm{i}kR_0 + \mathrm{i}\tfrac{1}{4}\pi) \\ &\quad \times \left(1 + \mathrm{i}\,\frac{1+4\,\mathrm{d}^2/\mathrm{d}\theta^2}{8kR_0} - \frac{9+40\,\mathrm{d}^2/\mathrm{d}\theta^2 + 16\,\mathrm{d}^4/\mathrm{d}\theta^4}{128k^2R_0^2} + \dots\right). \end{aligned} \tag{24}$$

Equation (21) then becomes

$$g_{\mathrm{A}}(r) = g_{\mathrm{A}0}(r,\theta) + \left\{\int_0^\infty g_{\mathrm{B}}(\rho)\, \mathrm{H}_{\mathrm{op}}\left(\rho, \frac{\mathrm{d}}{\mathrm{d}\gamma}\right) \mathrm{d}\rho\right\} g_{\mathrm{A}0}(r,\gamma)\,, \tag{25}$$

where $g(r,\gamma)$ has been taken outside the integral because γ is assumed to be a slowly varying function of ρ. This approximation assumes that the main contribution of the interaction between the two screens comes from the induced currents near the edges, where $\gamma \simeq \frac{1}{2}\pi + \alpha$. The iteration procedure for solving (25) is now simple, since each iteration yields essentially the same integral. For the plane slit aperture γ is independent of ρ, and this procedure has been shown to give excellent results compared with the rigorous solution in terms of Mathieu functions. The iterative solution has the simple interpretation that it represents a series of successive interactions between the two screens.

V. Calculation of Diffracted Field

To calculate the diffracted field of screen A, for example, we write from equation (21) and Figure 2

$$E_{\mathrm{A}}(\mathrm{P}) = -\frac{\pi k}{c}\int_0^\infty g_{\mathrm{A}0}(r,\theta)\,\mathrm{H}_0^{(2)}(kr_{\mathrm{A}})\,\mathrm{d}r - \frac{\pi k}{c}\int_0^\infty g_{\mathrm{B}}(\rho)\,\mathrm{d}\rho\int_0^\infty j_0(r,R_0,\gamma)\,\mathrm{H}_0^{(2)}(kr_{\mathrm{A}})\,\mathrm{d}r\,. \tag{26}$$

The first term on the right-hand side of (26), E_1, is obviously the half-plane diffracted field of the incident plane wave, while the second term E_2 is the diffracted field

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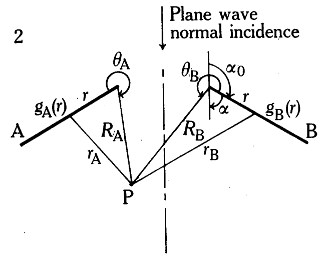

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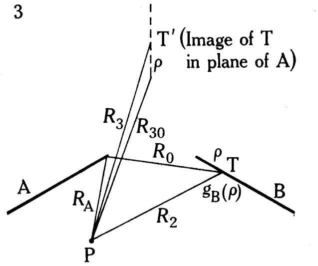

Fig. 2 (*above*).—Diagram for diffraction by a normally incident plane wave.

Fig. 3 (*above right*).—Diagram for field from source T and its image T′.

Fig. 4 (*right*).—Diagram for reflection interaction between screens.

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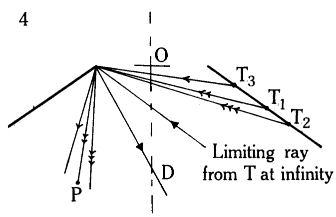

of the half-plane A due to a line source at ρ on the half-plane B. These fields are given by (Born and Wolf 1965)

$$E_1 = -\pi^{-\frac{1}{2}}\exp(\mathrm{i}\tfrac{1}{4}\pi)\big[\exp\{\mathrm{i}kR_{\mathrm{A}}\cos(\theta_{\mathrm{A}}-\alpha_0)\}F\{(2kR_{\mathrm{A}})^{\frac{1}{2}}\cos\tfrac{1}{2}(\theta_{\mathrm{A}}-\alpha_0)\} + \exp\{\mathrm{i}kR_{\mathrm{A}}\cos(\theta_{\mathrm{A}}+\alpha_0)\}F\{-(2kR_{\mathrm{A}})^{\frac{1}{2}}\cos\tfrac{1}{2}(\theta_{\mathrm{A}}+\alpha_0)\}\big]\,, \tag{27}$$

where, referring to Figure 3,

$$E_2 = \left.\begin{array}{ll} E_0+(\pi k/c)\mathrm{H}_0^{(2)}(kR_3)\,, & 0\leqslant 2\pi-\theta_{\mathrm{A}}\leqslant \pi-\gamma\,, \\ E_0\,, & \pi-\gamma < 2\pi-\theta_{\mathrm{A}} < \pi+\gamma\,, \end{array}\right\} \tag{28}$$

and

$$E_0 = \frac{2\mathrm{i}k}{c}\left(\exp(-\mathrm{i}kR_3)\int_{[k(R_1-R_3)]^{\frac{1}{2}}}^\infty \frac{\exp(-\mathrm{i}\mu^2)}{(\mu^2+2kR_3)^{\frac{1}{2}}}\,\mathrm{d}\mu + \exp(-\mathrm{i}kR_2)\int_{[k(R_1-R_2)]^{\frac{1}{2}}}^\infty \frac{\exp(-\mathrm{i}\mu^2)}{(\mu^2+2kR_2)^{\frac{1}{2}}}\,\mathrm{d}\mu\right). \tag{29}$$

The two integrals in E_0 can be approximated by Fresnel integrals to a degree of accuracy that is inadequate only if both the source T and the point of observation P are well within a wavelength of the diffracting edge of A. This approximation is done by replacing μ in the non-exponential part of the integrand by the lower limit of the integral (Clemmow 1950), giving

$$\left.\begin{aligned} E_0 &\simeq \frac{2\mathrm{i}k}{c}\left(\frac{\exp(-\mathrm{i}kR_3)}{\{k(R_1+R_3)\}^{\frac{1}{2}}}F(\{k(R_1-R_3)\}^{\frac{1}{2}})+\frac{\exp(-\mathrm{i}kR_2)}{\{k(R_1+R_2)\}^{\frac{1}{2}}}F(\{k(R_1-R_2)\}^{\frac{1}{2}})\right), \\ F(V) &= \int_V^\infty \exp(-\mathrm{i}\mu^2)\,\mathrm{d}\mu\,, \qquad R_1 = R_0+R_\mathrm{A}\,. \end{aligned}\right\} \tag{30}$$

If P lies in the region $0 \leqslant 2\pi-\theta_\mathrm{A} \leqslant \pi-\gamma$, the additional term $(\pi k/c)\mathrm{H}_0^{(2)}(kR_3)$ in (28) represents the reflected wave of the line source T, which appears to be radiated by an image source at T′. It is these reflected waves of the line sources between the two screens that account for the reflection interaction effect measured in certain parts of the diffracted field. This effect has been discussed in Tan (1967*a*, 1967*b*).

The point P does not necessarily receive the reflected waves of all the line sources on half-plane B. For example, in Figure 4, P will see the reflected waves of only those line sources on B for which $\rho > \rho_0$. A source T_2 will include P in its region of reflected waves, while a source T_3 will not. In general, the diffracted field from half-plane A will be, from (26),

$$E_\mathrm{A}(\mathrm{P}) = E_1+\int_0^\infty g_\mathrm{B}(\rho)\,E_0\,\mathrm{d}\rho+\frac{\pi k}{c}\int_{\rho_0}^\infty g_\mathrm{B}(\rho)\,\mathrm{H}_0^{(2)}(kR_3)\,\mathrm{d}\rho\,. \tag{31}$$

The iteration scheme can now be applied by successively substituting for the unknown g_B the values of the iterations $g_{\mathrm{B}0}$ etc. from (25). For all but the smallest slit widths (e.g. slit widths of half a wavelength or smaller), only one or two iterations will give results of sufficient accuracy, a fact borne out by comparison with experimental measurements of electric field intensity and phase (Tan 1967*b*).

With one iteration, the second term in (31) gives

$$\begin{aligned} \int_0^\infty g_{\mathrm{B}0}(\rho)\,E_0\,\mathrm{d}\rho \simeq \frac{\sin\frac{1}{2}\alpha_0}{\pi}&\left(\frac{\exp(-\mathrm{i}kR_{30})}{\{k(R_\mathrm{A}+2a+R_{30})\}^{\frac{1}{2}}}F(\{k(R_\mathrm{A}+2a-R_{30})\}^{\frac{1}{2}})\right. \\ &\left.+\frac{\exp(-\mathrm{i}kR_\mathrm{B})}{\{k(R_\mathrm{A}+2a+R_\mathrm{B})\}^{\frac{1}{2}}}F(\{k(R_\mathrm{A}+2a-R_\mathrm{B})\}^{\frac{1}{2}})\right), \end{aligned} \tag{32}$$

where we have made the approximations

$$\left.\begin{aligned} g_{\mathrm{B}0}(\rho) = g_{\mathrm{A}0}(\rho) &\simeq c\pi^{-3/2}\sin\tfrac{1}{2}\alpha_0\,(2\mathrm{i}k\rho)^{-\frac{1}{2}}\exp(-\mathrm{i}k\rho)\,, \\ R_3 \simeq R_{30}+\rho\,, &\qquad R_2 \simeq R_\mathrm{B}+\rho\,, \end{aligned}\right\} \tag{33}$$

and have assumed the case of normal plane wave incidence. The approximations (33) are justifiable in that only the current to the edge (ρ small) contributes appreciably to this term.

To evaluate the last term in (31), we replace the Hankel function by the first term of its asymptotic series. However, here we must include the line sources on the entire screen and not just those near the screen edge. Writing

$$g_{\mathrm{B0}}(\rho) = \frac{c}{\pi^{3/2}}\left(\frac{\exp(-\mathrm{i}k\rho)}{(2\mathrm{i}k\rho)^{\frac{1}{2}}}\sin\tfrac{1}{2}\alpha_0 + \exp(\mathrm{i}\tfrac{1}{4}\pi)\sin\alpha_0\exp(\mathrm{i}k\rho\cos\alpha_0)\right.$$
$$\left.\times\int_0^{(2k\rho)^{\frac{1}{2}}\cos\frac{1}{2}\alpha_0}\exp(-\mathrm{i}\mu^2)\,\mathrm{d}\mu\right), \tag{34a}$$

$$\mathrm{H}_0^{(2)}(kR_3) = \{k(R_{30}+\rho)\}^{-\frac{1}{2}}(2/\pi)^{\frac{1}{2}}\exp(\mathrm{i}\tfrac{1}{4}\pi)\exp\{-\mathrm{i}k(R_{30}+\rho)\}, \tag{34b}$$

the two integrals for this reflection effect are

$$\frac{\pi k}{c}\int_{\rho_0}^{\infty}\frac{c}{\pi^{3/2}}\frac{\exp(-\mathrm{i}k\rho)}{(2\mathrm{i}k\rho)^{\frac{1}{2}}}\sin\tfrac{1}{2}\alpha_0\left(\frac{2}{\pi}\right)^{\frac{1}{2}}\exp(\mathrm{i}\tfrac{1}{4}\pi)\frac{\exp(-\mathrm{i}kR_3)}{(kR_3)^{\frac{1}{2}}}\,\mathrm{d}\rho$$
$$\simeq\frac{2^{\frac{1}{2}}}{\pi}\sin\tfrac{1}{2}\alpha_0\frac{\exp(-\mathrm{i}kR_{30})}{\{k(R_{30}+\rho_0)\}^{\frac{1}{2}}}F\big((2k\rho_0)^{\frac{1}{2}}\big) \tag{35}$$

and

$$\frac{\pi k}{c}\int_{\rho_0}^{\infty}\left(\frac{c}{\pi^{3/2}}\exp(\mathrm{i}\tfrac{1}{4}\pi)\sin\alpha_0\exp(\mathrm{i}k\rho\cos\alpha_0)\int_0^{(2k\rho)^{\frac{1}{2}}\cos\frac{1}{2}\alpha_0}\exp(-\mathrm{i}\mu^2)\,\mathrm{d}\mu\right)$$
$$\times\left(\frac{2}{\pi}\right)^{\frac{1}{2}}\exp(\mathrm{i}\tfrac{1}{4}\pi)\frac{\exp(-\mathrm{i}kR_3)}{(kR_3)^{\frac{1}{2}}}\,\mathrm{d}\rho$$
$$\simeq\frac{2^{\frac{1}{2}}}{\pi}\left(\frac{\sin\alpha_0}{1-\cos\alpha_0}\frac{\exp(-\mathrm{i}kR_{30})}{\{k(R_{30}+\rho_0)\}^{\frac{1}{2}}}\right)\left(\exp\{-\mathrm{i}k\rho_0(1-\cos\alpha_0)\}\int_0^{(2k\rho_0)^{\frac{1}{2}}\cos\frac{1}{2}\alpha_0}\exp(-\mathrm{i}\mu^2)\,\mathrm{d}\mu\right.$$
$$\left.+\cos\tfrac{1}{2}\alpha_0 F\big((2k\rho_0)^{\frac{1}{2}}\big)\right). \tag{36}$$

Combining (35) and (36), we have that

$$\frac{\pi k}{c}\int_{\rho_0}^{\infty} j_{\mathrm{B0}}(\rho)\,\mathrm{H}_0^{(2)}(kR_3)\,\mathrm{d}\rho$$
$$\simeq\frac{2^{\frac{1}{2}}}{\pi}\frac{\exp(-\mathrm{i}kR_{30})}{\{k(R_{30}+\rho_0)\}^{\frac{1}{2}}}\left(\frac{F\big((2k\rho_0)^{\frac{1}{2}}\big)}{\sin\frac{1}{2}\alpha_0}+\frac{\sin\alpha_0}{1-\cos\alpha_0}\exp\{-\mathrm{i}k\rho_0(1-\cos\alpha_0)\}\right.$$
$$\left.\times\int_0^{(2\rho_0)^{\frac{1}{2}}\cos\frac{1}{2}\alpha_0}\exp(-\mathrm{i}\mu\,)^2\mathrm{d}\mu\right). \tag{37}$$

Finally, the total electric field at P is given by

$$E = E^{i} + E_{\mathrm{A}}(R_{\mathrm{A}},\theta_{\mathrm{A}}) + E_{\mathrm{B}}(R_{\mathrm{B}},\theta_{\mathrm{B}}), \tag{38}$$

where E_{A} is given by (31), (32), and (37), and E_{B} is similar to E_{A} except for obvious modifications.

From Figure 4, it is seen that the reflection term given by (37) will be present in the illuminated region behind the aperture (in the geometrical optics sense) only for screen inclination angle $\alpha < \frac{1}{3}\pi$. This is because for $\alpha > \frac{1}{3}\pi$ the reflected waves of all the current sources on both screens are confined to the two shadow regions near the screens. In particular the axial field is free from this reflection effect for such cases. When $\alpha < \frac{1}{3}\pi$ this reflection term occurs along the axis for $\rho > \mathrm{OD}$, where D is the intersection on the axis of the reflected ray from sources very far from the

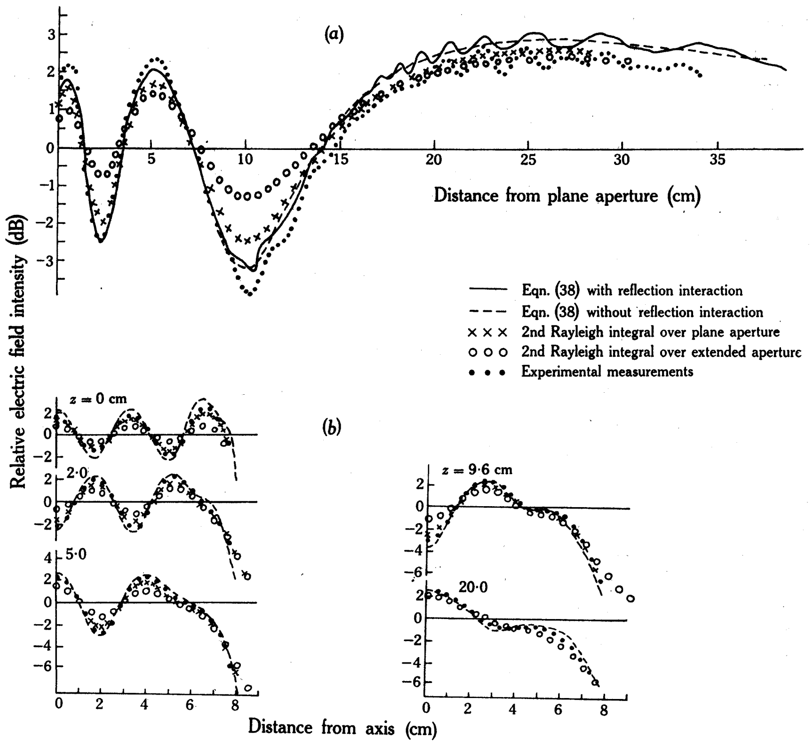

Fig. 5.—Comparison of theoretical and measured values of the electric field intensity (*a*) along and (*b*) transverse to the axis of a slit aperture, with $ka = 5\pi$, $\alpha = \frac{1}{4}\pi$, and $\lambda = 3 \cdot 2$ cm.

edge of B. Near the point D, this reflection would be small since ρ_0 in (31) is large. On the other hand, this term is also small when z is large because, although ρ_0 is now small, the distance from the virtual image sources from which the reflections appear to radiate, R_3, is very large. Thus we expect that the magnitude of this reflection term on the axis increases with z for $z > \mathrm{OD}$, and then begins to decrease as z becomes large. In any case it is a small effect, but its significance lies in the fact that it is essentially due to the two half-planes not being coplanar, and gives one of the main differences between planar and nonplanar diffraction problems. Whereas the planar problem can usually be discussed in terms of edge interactions, the nonplanar case has screen interactions as well.

VI. Results and Discussion

Figure 5(*a*) shows a comparison of theory with measured values of the electric field intensity along the axis of a 5λ slit aperture with $\alpha = \frac{1}{4}\pi$ and $\lambda = 3\cdot 2$ cm. The reflection effect appears to be observed as ripples about three wavelengths behind the aperture. Figure 5(*b*) presents five transverse scans of the same slit aperture.

The computed curves show significant differences between the results obtained with and without reflection interaction. In particular, a characteristic ripple appears in consequence of reflection interaction, and the existence of such ripples has been detected experimentally. For comparison, the second Rayleigh integral over the plane aperture has also been computed. This approximation assumes that the tangential magnetic field in the aperture has the values of the incident field and that it is negligible over the shadow side of the screens. The results in Figure 5 show this approximation to be reasonable, except that it does not predict the reflection effect. A similar approximation, but with the Rayleigh integral taken over the extended aperture (Tan 1967*b*), proves to be less accurate everywhere. Fuller discussions of these results are contained in the author's previous reports.

VII. References

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